Solutions

Chapter 1

Concept Application Exercises

Exercise 1.1

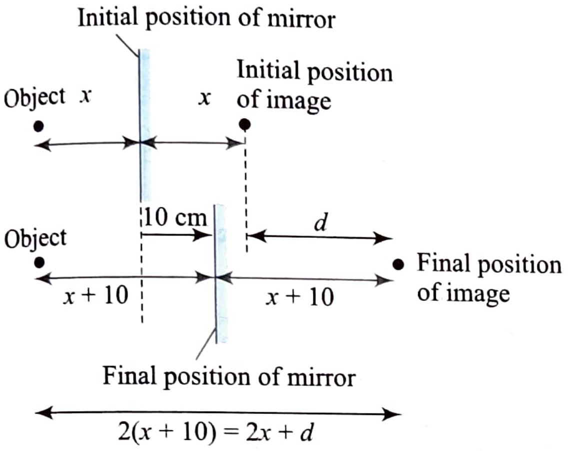
1. We know that

$$x_{im} = -x_{om} \text{ or } x_i - x_m = x_m - x_o$$

or $\quad \Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o$

Here $\quad \Delta x_o = 0; \Delta x_m = 10$ cm

Therefore, $\Delta x_i = 2\Delta x_m - \Delta x_o = 20$ cm


Initial position of mirror
Object x
x
Initial position of image
Object
10 cm
d
Final position of image
x + 10
x + 10
Final position of mirror
2(x + 10) = 2x + d

$\therefore \quad d = 20$ cm

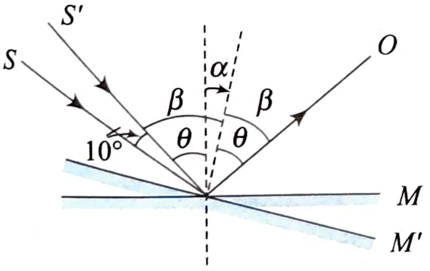
2. The first image at M_1 is formed at a distance of 4 cm behind it.

The first image at M_2 is 6 cm behind it.

The second image at M_2 is at a distance of 4 + 10 = 14 cm behind M_2.

Therefore, the distance between the first image at M_1 and second image at M_2 is 4 + 14 + 10 = 28 cm.

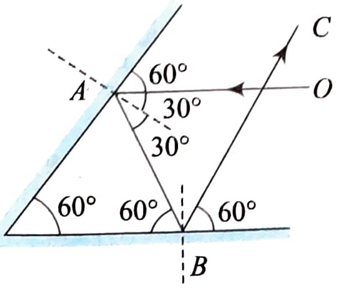
3.


S'
S
O
α
β
β
θ
θ
10°
M
M'

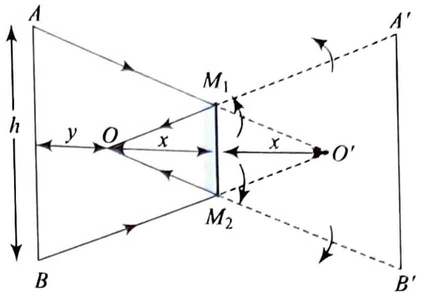
$$\theta + \alpha - 10° = \beta \quad \text{and} \quad \theta = \beta + \alpha$$

$\Rightarrow \quad \alpha = 5°$ clockwise

4. The angle that the ray makes after each reflection is clearly labeled in the following figure. It is clear that final ray makes an angle of 60° with the horizontal, or in other words it emerges parallel to mirror 1.


C
O
A
60°
30°
30°
60°
60°
60°
B

5.


A
A'
M1
h
y
O
x
x
O'
M2
B
B'

From triangles $Q'M_1M_2$ and $O'AB$

$$\frac{M_1M_2}{h} = \frac{x}{2x + y}$$

Size of mirror, $M_1M_2 = \dfrac{hx}{y + 2x}$

6. The image of point A in the mirror is at A' (6, 0).

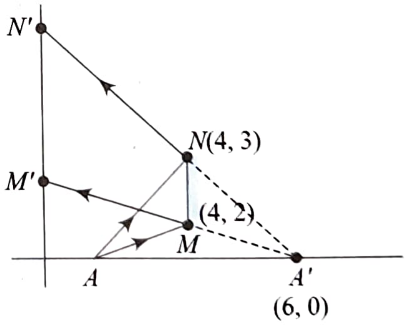
Join $A'M$ and extend to cut Y-axis at M' (ray originating from A which strikes the mirror at M gets reflected as the ray MM' which appears to come from A'). Join $A'N$ and extend to cut Y-axis at N' (ray originating from A which strikes the mirror at N gets reflected as the ray NN' which appears to come from A').

From Geometry:

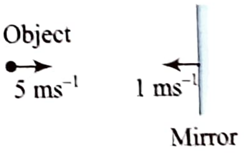
$$M' \equiv (0, 6)$$

and $\quad N' \equiv (0, 9)$

$M'N'$ is the region on Y-axis in which reflected rays are present.


N'
N(4, 3)
M'
(4, 2)
M
A
A'
(6, 0)

7. Take direction toward right as positive.


Object
5 ms⁻¹
1 ms⁻¹
Mirror

$$v_i - v_m = v_m - v_o$$

$$v_i - (-1) = (-1) - 5$$

$\therefore \quad v_i = -7$ m s^{-1}

$\therefore$ The velocity of image is 7 m s^{-1} and direction is toward left.

8. Velocity of point P, $v_p = 10(\hat{i} + \hat{j})$

Velocity of mirror $v_m = -5\hat{i}$

Velocity of particle relative to mirror is

$$\vec{v}_{pm} = -15\hat{i} + 10\hat{j}$$

Velocity of image relative to observer

$$\vec{v}_{PO} = \vec{v}_{pm} - \vec{v}_{Om} = (-15\hat{i} + 10\hat{j}) - (\vec{v}_O - \vec{v}_m)$$

$$= (-30\hat{i} + 10\hat{j})\,\text{ms}^{-1}$$

Angle with X-axis, $\alpha = \tan^{-1}(-1/3)$

Exercise 1.2

1. (a) True. According to the magnification formula,

$$m = \frac{f_0}{f_0 + x}$$

where x is the distance of a real object. For all positive values of x, m is always positive.

(b) False. For a real object, the image formed by a convex mirror is always erect and diminished.

$$m = \frac{f_0}{f_0 + x}$$

For a virtual object lying between pole and focus, the image formed is real, enlarged, and erect.

$$m = \frac{f_0}{f_0 + x}$$

(c) True. For a concave mirror, we know that

$$m = \frac{f_0}{f_0 + x}$$

where x is the object distance.

If $x < f_0$, then $m > 1$, which implies a virtual enlarged image.

(d) False. In case of mirrors (convex, concave, and plane), the object and its image always move in opposite directions.

That is, $\frac{V}{U} = -\left(\frac{f}{u - f}\right)^2$

where V = velocity of image and U = velocity of object.

(e) False. When the object is beyond the centre of curvature, its image moves slower.

(f) True. For a convex mirror, $m = f_0/f_0 + x$

i.e., if $x = f_0$, then $m = 1/2$

(g) True. A concave mirror forms an enlarged image in two situations:

(i) Object is placed between C and F.

(ii) Object is placed between F and P.

(h) False. Convex mirror is used as a rear view mirror.

(i) True. Complete image will be formed, but of less brightness because now the incident rays will be reflected from reduced area to form the image.

(j) True. $m = -v/u$. For a plane mirror, $v = -u$

$\Rightarrow$ $m = +1$. Hence, image is erect and of same size.

(k) True. We see our left hand as right hand and right as left hand in a plane mirror.

(l) True. Virtual object is that point where incident rays seem to be converging.

2. $u = -15$ cm, $f = \frac{+20}{2} = 10$ cm

$$\frac{1}{-15} + \frac{1}{v} = \frac{1}{10}$$

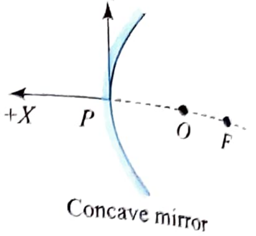
$$\frac{1}{v} = \frac{1}{10} + \frac{1}{15} \Rightarrow v = 6 \text{ cm}$$

$$m = -\frac{v}{u} = \frac{h_2}{h_1}$$

$$\frac{-6}{-15} = \frac{h_2}{5} \Rightarrow h_2 = 2 \text{ cm}$$

So, radius of circular path of the image is 2 cm.

3. Consider the situation shown in the figure below which also includes the coordinate system.



Concave mirror

Here, $u = -10$ cm

$f = -20$ cm

Substituting in the equation,

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

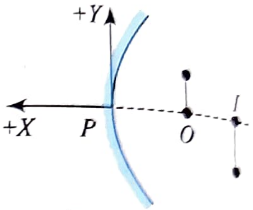
$v = +20$ cm

Thus, the image is formed 20 cm behind the mirror, magnification +2, erect and virtual.

4. Consider the situation shown in the following figure which also indicates the coordinate system.

Here, $u = -15$ cm and $m = -2$.

Note the negative sign for m. The fact that the image is captured on the screen implies the image is real. Therefore, the magnification is to be taken negative, though the problem simply states that the magnification is 2.



Concave mirror

Substituting in equation

$m = -\frac{v}{u}$, we get $v = -30$ cm

Substituting in equation

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get $f = -10$ cm

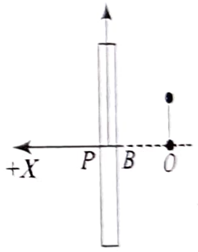
Conclusion: The focal length of the mirror is 10 cm. The negative sign implies it is concave.

5. Consider the situation shown in the following figure where the optical element is represented by a box as we do not know its nature.

Here $u = -15$ cm,

$h_i = +5$ cm, and

$h_o = 15$ cm



Substituting in equation

$$m = \frac{h_i}{h_0} = +\frac{1}{3}$$

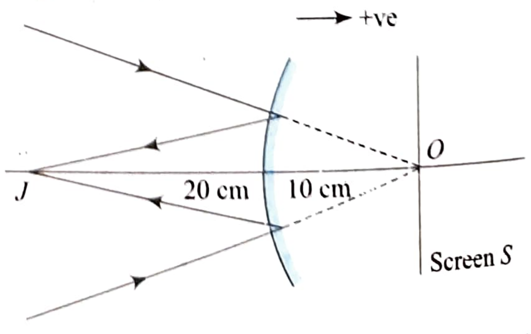
and $m = \frac{-v}{u}$, we get $v = +5$ cm

Substituting in equation

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get $f = +75$

Conclusion: The focal length of the mirror is 7.5 cm. The positive sign implies that the mirror is convex.

6. The mirror can be concave or convex. Let f be the focal length. Let the incident rays be directed toward right as shown. All quantities toward right will be positive.



The initial point of convergence (O) now acts as a virtual object for the mirror.

Hence, $u = +10$ cm and $v = -20$ cm

$$f = \frac{uv}{u+v} = \frac{-200}{10-20} = +20 \text{ cm}$$

As the focal length is positive, the mirror is convex having focal length 20 cm.

7. In this case: $u = +30$ and $v = +120$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{120} + \frac{1}{30}$$

$$\Rightarrow \quad f = 24 \text{ cm}$$

8. We know that, $m = \frac{v}{u} = \frac{f}{u-f}$

In first case, $\frac{f}{u-f} = 4$

$$f = 4u - 4f \text{ and } 5f = 4u$$

In second case, $\frac{f}{(u+3)-f} = +3$

$$f = 3u + 9 - 3f$$

$$\therefore \quad 4u - 4f = 3u + 9 - 3f$$

$$u = f + 9 = \frac{4u}{5} + 9$$

$$u = 45 \text{ cm and } f = 36 \text{ cm}$$

9. Mirror formula: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Here, $v = +4$ cm; $f = +\frac{R}{2} = +\frac{24}{2} = +12$ cm

$$\therefore \quad u = \frac{vf}{v-f} = \frac{(4)(12)}{4-12} = -6 \text{ cm}$$

The negative sign shows that the object is real and it is placed in front of the mirror.

The magnification, $m = -\frac{u}{v} = \frac{-(+4)}{-6} = \frac{2}{3}$

Thus, the image is two-thirds as high as the object.

10. We know that $m = -\frac{v}{u} = \frac{f}{f-u}$

Here, $m_1 = \frac{f}{f-(-25)} = \frac{f}{f+25}$ and

$$m_2 = \frac{f}{f-(-25-15)} = \frac{f}{f+40}$$

Since $\frac{m_1}{m_2} = 4$, therefore $\frac{f+40}{f+25} = 4$. Thus

$$f + 40 = 4f + 100$$

or $f = -20$ cm

The negative sign shows that the mirror is concave.

11. $$\frac{v}{u} = \frac{I}{O} = 3 \qquad \text{...(i)}$$

and $$\frac{v'}{u'} = \frac{I'}{O} = 2 \Rightarrow \frac{v'}{u+6} = 2 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$v' = 2\left(\frac{v}{3} + 6\right) \qquad \text{...(iii)}$$

$$\Rightarrow \quad v' - \frac{2}{3}v = 12$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{1}{v'} + \frac{1}{u'}$$

$$\Rightarrow \quad \frac{1}{v} + \frac{3}{v} = \frac{1}{v'} + \frac{2}{v'}$$

$$\Rightarrow \quad v = \frac{4}{3}v' \qquad \text{...(iv)}$$

From Eqs. (iii) and (iv), $v' - \frac{2}{3} \times \frac{4}{3}v' = 12$

$$\Rightarrow \quad \frac{1}{9}v' = 12 \Rightarrow v' = 108 \text{ cm}$$

$$\Rightarrow \quad v = \frac{4}{3}v' = 144 \text{ cm}$$

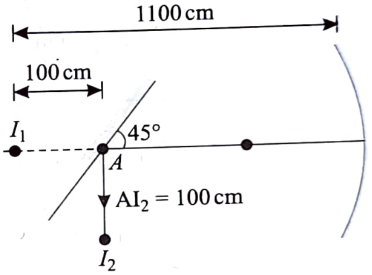
Shifting will be 144 – 108 = 36 cm toward the image.

Exercise 1.3

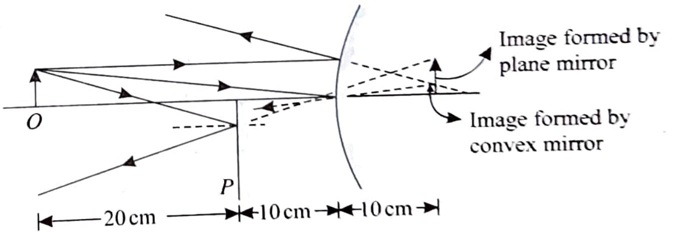
1. Applying mirror formula for concave mirror first $\left(\frac{1}{v} + \frac{1}{u} = \frac{1}{f}\right)$ we have, $\frac{1}{v} - \frac{1}{110} = \frac{1}{-100}$

$$v = -1100 \text{ cm}$$

$AI_1 = 100$ cm. Therefore, final image will be real and at distance 100 cm below point A at I_2.



2. The distance between the object and plane mirror = The distance between image and plane mirror.



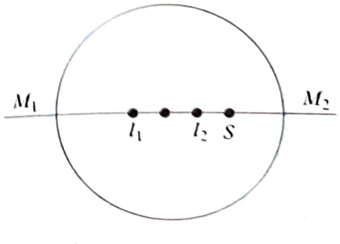
This means, the image is formed behind the convex mirror, at a distance of 10 cm because, $OP = 20$ cm.

$$u = -(20 + 10) = -30 \text{ cm}, v = +10 \text{ cm}, f = ?$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \text{ or } f = \frac{uv}{u+v}$$

$$f = \frac{-30 \times 10}{-30 + 10} = \frac{-300}{-20} = 15 \text{cm}$$

3. For mirror M_1 (the farthest wall)



$$u = -\frac{3R}{2}; f = \frac{-R}{2}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$v = -\frac{3R}{4} \text{ (Image at } I_1)$$

This image acts as an object for mirror M_2

$$u = -\left(2R - \frac{3R}{4}\right) = \frac{-5R}{4}$$

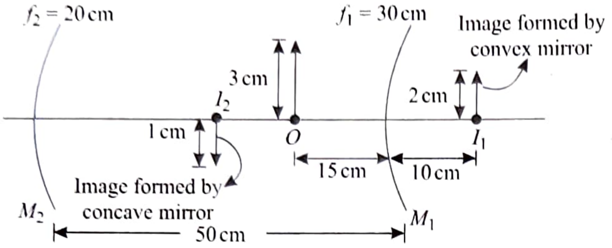
$$\therefore \quad \frac{1}{v} = \frac{-2}{R} + \frac{4}{5R}$$

$$v = -\frac{5R}{6} \text{ (Image at } I_2\text{)}$$

$\therefore$ Distance of image from the nearest wall is $\frac{5R}{6}$.

4. For reflection at the convex mirror, $f = +30$ cm, $u = -15$ cm

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \frac{1}{v} + \frac{1}{(-15)} = \frac{1}{(+30)} \Rightarrow v = +10$ cm



I_1 is the image formed by the convex mirror, which acts as the object for the concave mirror. For reflection at the concave mirror, for sign convention we are taking initial ray travelling direction as positive

$$f = +20 \text{ cm}, \; u = +(50 + 10)\text{cm} = +60\text{cm}$$

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \frac{1}{v} + \frac{1}{(+60)} = \frac{1}{(+20)} \Rightarrow v = +30$ cm

Magnification produced by convex mirror, $m_1 = -\frac{(+10)}{(-15)} = \frac{2}{3}$

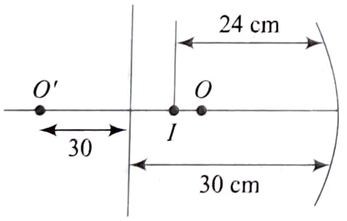
Magnification produced by concave mirror, $m_2 = -\frac{(+30)}{(+60)} = -\frac{1}{2}$

Total magnification $m = m_1 m_2 = \frac{2}{3} \times \left(-\frac{1}{2}\right) = -\frac{1}{3}$

Size of image $= m \times \text{size of the object} = m_1 m_2 = -\frac{1}{3} \times 3 = -1$ cm

The final image is inverted, real and of height 1 cm.

5. (a) If reflection is first taken on the plane mirror:



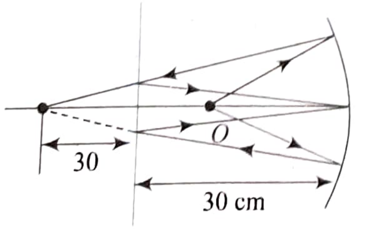
For second reflection at the concave mirror:

$$u = -40 \text{ cm}, f = -15 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{-40} = -\frac{1}{15} \quad \Rightarrow \quad v = -24 \text{ cm}$$

(b) If reflection is first taken on the concave mirror: $u = -20$ cm

$$\frac{1}{v} + \frac{1}{-20} = -\frac{1}{15} \quad \Rightarrow \quad v = -60 \text{ cm}$$



Final image will be formed at the pole of concave mirror.

6. Reflection from concave mirror

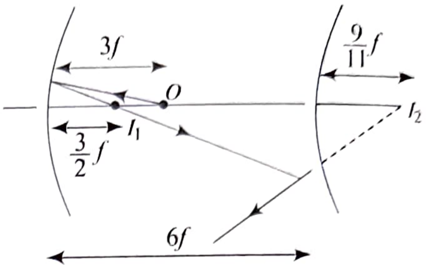
$$u = -3f_{1,} \; f = -f$$

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad v = -\frac{3}{2}f$

Now reflection from convex mirror

$$u = -\left(6f - \frac{3}{2}f\right) = -\frac{9}{2}f, \; f = f$$

Again using mirror formula, we get $v = \frac{9f}{11}$



7. First reflection:

Focus of mirror = −10 cm

$$u = -15 \text{ cm}$$

Applying mirror formula: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$v = -30 \text{ cm}$$

For second reflection on plane mirror:

$$u = -10 \text{ cm} \quad \Rightarrow \quad v = 10 \text{ cm}$$

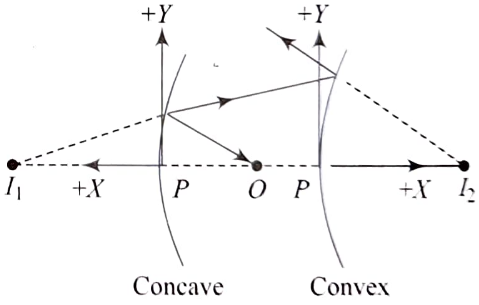
For third reflection on curved mirror again:

$$u = -50 \text{ cm} \quad \text{and} \quad f = -10 \text{ cm}$$

Applying mirror formula: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad v = -12.5 \text{ cm}$$

8. Let us consider the case of rays first striking the concave mirror and then the convex mirror.



Concave mirror: Here, $u = -5$ cm and $f = -10$ cm

Substituting in equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \quad \text{we get} \quad v = 10 \text{ cm}$$

Thus, the image is formed 10 cm behind the mirror.

This image now serves as the object for the convex mirror. However, keep in mind that for convex mirror, all measurements will now be from the pole of this mirror. Furthermore, the sign convention for the convex mirror is shown in the figure.

Convex mirror: Here, $u = -30$ cm and $f = +20$ cm

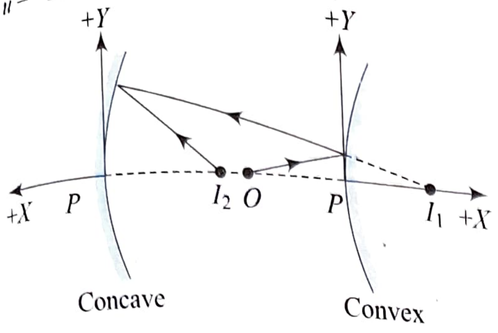
Substituting in equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ we get } v = 12 \text{ cm}$$

Thus, the image is formed 12 cm behind the convex mirror. This image will now be the object for the concave mirror and the process can be repeated.

Refer to the following figure. Here, **for the convex mirror:**

$u = -15$ cm and $f = +20$ cm



Substituting in the equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

we get $v = +(60/7)$ cm

Thus, the image is formed (60/7) cm behind the convex mirror. The image now serves as the object for the concave mirror.

For the concave mirror:

$u = -(20 + 60/7) = -200/7$ and $f = -10$ cm

Substituting in the equation

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get $v = -200/13$

The image is formed (200/13) cm in front of the concave mirror.

9. Now, using $\vec{V}_{i,m} = -m^2 \vec{V}_{o,m}$

or $\left(\vec{V}_i - \vec{V}_m\right) = -m^2\left(\vec{V}_o - \vec{V}_m\right)$

Here, $u = -15$ m and $f = -10$ m

Using mirror formula: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}+\frac{1}{(-15)}=\frac{1}{(-10)} \Rightarrow v = -30 \text{ m}$$

Magnification $m = -\frac{v}{u} = -\frac{(-30)}{(-15)} = -2$

$$\Rightarrow \left[\vec{V}_i - \left(-1\hat{i}\right)\right] = -(-2)^2\left[\left(1\hat{i}\right)-\left(-1\hat{i}\right)\right]$$

$$\Rightarrow \vec{V}_i = -9\hat{i} \text{ m/s}$$

So, the image will move with velocity 9 ms^{-1} towards left.

10. We have mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ ...(i)

Substituting the values, $u = -30$ cm and $f = -20$ cm

In Eq. (i), we have, $\frac{1}{v}+\frac{1}{-30}=\frac{1}{-20}$

Solving this equation, we get $v = -60$ cm

Further, $m = -\frac{v}{u} = -\frac{(-60)}{(-30)} = -2$

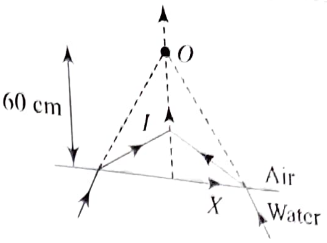
The velocity of image in the direction perpendicular to principal axis,

$$\left(V_{i,m}\right)_y = m\left(V_{o,m}\right)_y = (-2)\times 5\hat{j} = -10\hat{j} \text{ cm/s}$$

So, the image will move with velocity 10 cms^{-1} in downward direction.

Exercise 1.4

1. A ray of light now travels through water, across the interface through air and converges to form an image at I. Here, medium 1 is water and medium 2 is air.



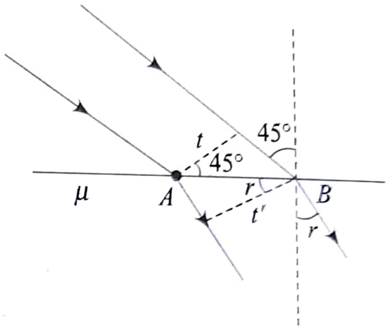
$d_1 = +60$ cm, $\mu_1 = \frac{4}{3}$, $\mu_2 = 1$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1}d_1$, we get

$d_2 = +45$ cm

Thus, the image is formed 45 cm above the interface.

2. $AB = \frac{t}{\cos 45^\circ} = \frac{t'}{\cos r} \Rightarrow t' = t\frac{\cos r}{\cos 45^\circ}$



$$\mu = \frac{\sin 45^\circ}{\sin r} \Rightarrow \sin r = \frac{1}{\sqrt{2}\,\mu}$$

Solving we get, $t' = \frac{t\sqrt{2\mu^2-1}}{\mu}$

3. (a) **When the bird sees the fish:**

The fish and the bird are shown in the following figure along with the corresponding coordinate system. A ray of light now travels from the fish, through water, across the interface, through air and reaches the bird. Here, medium 1 is water and medium 2 is air.

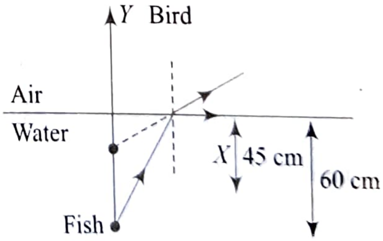


Image of fish at I

$d_1 = -60$ cm, $\mu_1 = \frac{4}{3}$, $\mu_2 = 1$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1}d_1$, we get

$d_2 = -45$cm

Thus, the fish appears to be 45 cm below the water–air interface. Image of fish is formed at I.

(b) **When the fish sees the bird:**

The fish and the bird are shown in the following figure along with the corresponding coordinate system. A ray of light now travels from the bird, through air, across the interface, through water and reaches the fish. Here, medium 1 is air and medium 2 is water.

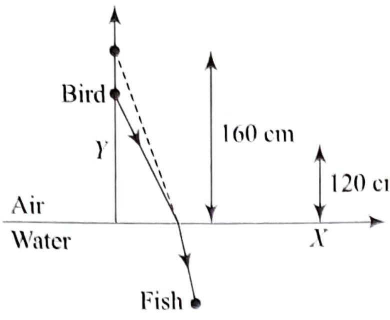


Image of bird at I

$d_1 = +120$ cm, $\mu_1 = 1$, $\mu_2 = \frac{4}{3}$

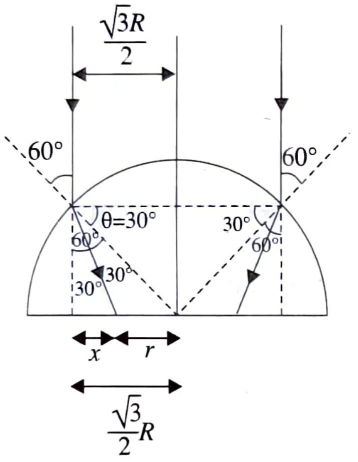
As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$, we get $d_2 = 160$ cm

Thus, the bird appears to be 160 cm above the water–air interface. Image of bird is formed at I.

4. $\mu_1 \sin i = \mu_2 \sin r$

$1 \times \sin 60° = \sqrt{3} \times \sin r$; $r = 30°$

In ΔMNP, $\tan 30° = x/(R/2)$

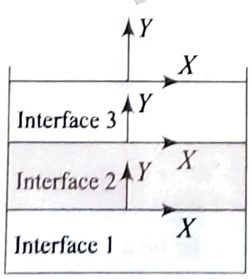


$$x = \frac{R}{2\sqrt{3}}$$

$$r = \frac{\sqrt{3}}{2}R - x = \frac{\sqrt{3}}{2}R - \frac{R}{2\sqrt{3}} = \left(\frac{3-1}{2\sqrt{3}}\right)R = \frac{R}{\sqrt{3}}$$

5. Method 1: Method of interfaces:

A ray of light from the object undergoes refraction at three interfaces: (1) Water–oil, (2) Oil–glycerine, and (3) Glycerine–air. The coordinate system for each of the interfaces is shown in the figure below.



(1) Water–oil interface:

$d_1 = -8$ cm, $\mu_1 = \frac{4}{3}$, $\mu_2 = 1.5$

$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$, we get $d_2 = -9$ cm

(2) Oil–glycerine inteface:

$d_1 = -(9 + 9) = -18$ cm, $\mu_1 = 1.5$, $\mu_2 = 2$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$

$d_2 = -24$ cm

(3) Glycerine–air interface:

$d_1 = -(4 + 24) = -28$ cm, $\mu_1 = 2$, $\mu_2 = 1$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$, we get $d_2 = -14$ cm

Thus, the final image is 14 cm below the glycerine–air interface.

Method 2: Method of shifting

Net shifting

$$s = d_1\left(1 - \frac{1}{\mu_1}\right) + d_2\left(1 - \frac{1}{\mu_2}\right) + d_3\left(1 - \frac{1}{\mu_3}\right)$$

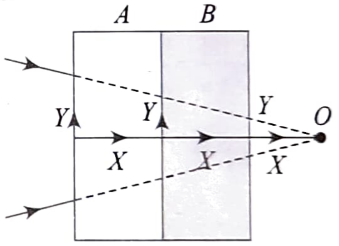
$$= 8\left(1 - \frac{1}{4/3}\right) + 9\left(1 - \frac{1}{3/2}\right) + 4\left(1 - \frac{1}{2}\right) = 7 \text{ cm}$$

The shifting will be in the direction of ray travelling, i.e., upwards.

Hence, apparent depth $h' = (21–7) = 14$ cm.

6. Method 1: Method of interfaces

A ray of light from the object undergoes refraction at three interfaces: (1) Air–medium A, (2) Medium A–medium B, (3) Medium B–air. The coordinate system for each of the interface is shown in the following figure.



(1) Air–medium A interface:

$d_1 = +14$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$, we get

$d_2 = +21$ cm

(2) Medium A–medium B interface:

$d_1 = (21 - 6) = 15$ cm, $\mu_1 = 1.5$, $\mu_2 = 2$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$, we get

$d_2 = +20$ cm

(3) Medium B–air interface:

$d_1 = +(20 - 4) = +16$ cm, $\mu_1 = 2$, $\mu_2 = 1$

As $\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$ or $d_2 = \frac{\mu_2}{\mu_1} d_1$, we get

$d_2 = +8$ cm

Thus, the final image is 8 cm in front of the medium B–air interface.

Method 2: Method of shifting

Net shifting $S = 6\left(1 - \frac{1}{3/2}\right) + 4\left(1 - \frac{1}{2}\right) = 4$ cm

The shifting will be in the direction of ray travelling, i.e., towards right.

Hence, rays will converge finally = 14 + 4 = 18 cm from left surface or 8 cm from right surface.

For the light ray to be reflected from the lower surface *RS*, the angle of incidence must be greater than the critical angle for the glass–air interface (at *N*).

$$i > \sin^{-1}\left(\frac{1}{2}\right) \text{ or } i > 30°$$

At the water–glass interface, $\mu_1 = \frac{4}{3}, \mu_2 = 2$

$$\theta_1 = \theta = ?, \theta_2 = i = 30°$$

Since $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$, we get

$$\theta = \sin^{-1}\left[\frac{\mu_1}{\mu_2}\sin\theta_2\right]$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{3}{4}\right)$$

$$\mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu}$$

Now, $r' >$ critical angle

$$\sin r' > \frac{1}{\mu}$$

$$\sin[90° - r] > \frac{1}{\mu}$$

$$\cos r > \frac{1}{\mu}$$

$$\sqrt{1-\sin^2 r} > \frac{1}{\mu}$$

$$\sqrt{1-\frac{\sin^2 i}{\mu^2}} > \frac{1}{\mu}$$

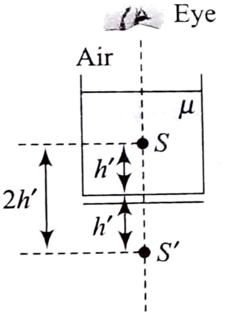
$$\mu^2 - \sin^2 i > 1$$

$$\mu > \sqrt{1+\sin^2 i}$$

$$i_{\max} = 90°, \text{ so } \mu > \sqrt{1+\sin^2 90°}$$

$$\mu > \sqrt{2}$$

9. (a) To observer in air, there are two objects placed in the medium of refractive index μ. One is the actual object *S* and the other object is *S′* which is the image of *S* in the plane mirror.



The observer is able to see two images, one for each object.

(b) For observer in air, apparent depth $h' = \frac{h}{\mu}$

Hence distance between two images will be $2h' = \frac{2h}{\mu}$

The bottom of the beaker appears to be shifted up by a distance

$$\Delta t = \left(1 - \frac{1}{\mu}\right)d$$

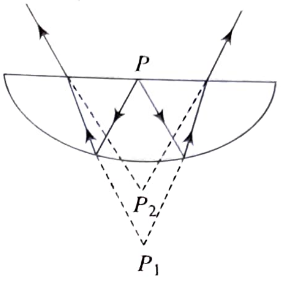
Thus, the apparent distance of the bottom from the mirror is $h - \Delta t = h - [1 - (1/\mu)]d = h - d + (d/\mu)$. The image is formed behind the mirror at a distance $h - d + (d/\mu)$.

11. The ray diagram is shown in the following figure. Let us first locate the image formed by the concave mirror. Let us take vertically upward as the negative axis. Then, $R = -40$ cm. The object distance is $u = -5$ cm. Using the mirror equation:

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

$$\frac{1}{v} = \frac{2}{R} - \frac{1}{u} = \frac{2}{-40 \text{ cm}} - \frac{1}{-5 \text{ cm}} = \frac{6}{40} \text{ cm}$$

or $\quad v = 6.67$ cm



The positive sign shows that the image P_1 is formed below the mirror and hence, it is virtual. These reflected rays are refracted at the water surface and go to the observer. The depth of the point P_1 from the surface is 6.67 cm + 5.00 cm = 11.67 cm. Due to refraction at the water surface, the image P_1 will be shifted above by a distance

$$(11.67 \text{ cm}) \times \left(1 - \frac{1}{1.33}\right) = 2.92 \text{ cm}$$

Thus, the final image is formed at a point $(11.67 - 2.92)$ cm $= 8.75$ cm below the water surface.

12. Let the object be placed at a height '*x*' above the surface of water. The apparent position of the object with respect to the mirror should be at the centre of curvature so that the image is formed at the same position.

Since $\frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$

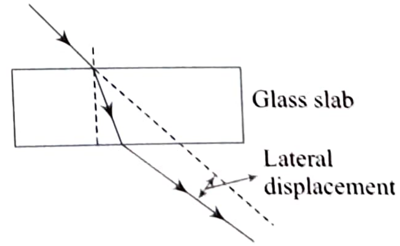
(with respect to the mirror), therefore $\frac{x}{R-h} = \frac{1}{\mu}$

or $\quad x = \frac{R-h}{\mu}$

Exercise 1.5

1. (a) True. The emergent ray is always parallel to the incident ray.

(b) True. See the figure below.



(c) False. The shift produced by a glass slab is always in the direction of incident light, irrespective of the converging or diverging nature of beam.

(d) False. As shifting $s = t\left(1 - \frac{1}{\mu_{\text{rel}}}\right)$

$$\mu_{\text{rel}} = \frac{\mu_{\text{slab}}}{\mu_{\text{medium}}}$$

If $\mu_{\text{medium}} > \mu_{\text{slab}}$, the value of μ_{real} will be less than 1 and shifting will be negative, i.e., opposite to direction of ray travelling.

(e) True. As we know shifting $s = t\left(1 - \frac{1}{\mu_{\text{rel}}}\right)$

$$\mu_{\text{rel}} = \frac{\mu_{\text{slab}}}{\mu_{\text{medium}}} = \frac{\mu_2}{\mu_1}$$

If $\mu_2 > \mu_1$, then $s < t$

If $\mu_1 > \mu_2$, then shift (s) can be greater than t only if $\mu_1/\mu_2 > 2$.

2. (a) Distance of the watcher as seen from fish

$$8 \times \frac{4}{3} + 3 \times \frac{4/3}{8/5} + 10 = \frac{139}{6} \text{ cm}$$

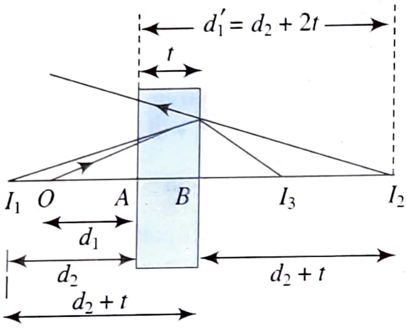
(b) Distance of the fish as seen from watcher is

$$\frac{10}{4/3} + \frac{3}{8/5} + 8 = \frac{139}{8} \text{ cm}$$

3. At first surface,

$$\frac{\mu_1}{d_1} = \frac{\mu_2}{d_2} \Rightarrow \frac{1}{(-8)} = \frac{\mu_2}{d_2} \Rightarrow d_2 = -8\mu$$

Image I_1 will serve as an object for the mirror and form an image I_2 behind it at a distance of $(8\mu + 6)$ cm.



I_2 will serve as an object for the first surface.

The rays will reflect from the mirror.

Again using $\frac{d_1'}{\mu} = \frac{d_2'}{1}$

$$d_1' = d_2 + 2t = -(8\mu + 12)$$

$$d_2' = -(10 + 6) = -16 \text{ cm}$$

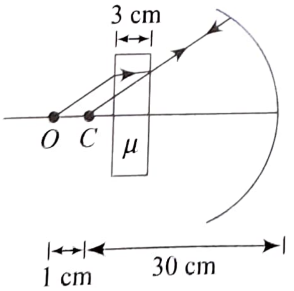
After substituting the values, $\mu = 1.5$.

4. Shift $= 3\left(1 - \frac{1}{3/2}\right)$

For the mirror, object is at a distance $= 21 - 3\left(1 - \frac{1}{3/2}\right) = 20$ cm

Therefore, object is at the centre of curvature of the mirror. Hence, the light rays will retrace and image will be formed on the object itself.

5. (a) Since the apparent shift occurs in the direction of incident light, therefore the mirror should be displaced away from the object.



(b) The magnitude of displacement is equal to the apparent shift,

i.e., $s = t\left(1 - \frac{1}{\mu}\right) = 3\left(1 - \frac{1}{3/2}\right) = 1$ cm

6. $\mu = \mu_0\sqrt{x+1}$, at $x = 0 \rightarrow \mu = \mu_0$

$$\mu \sin\theta = \text{constant}$$

$$\mu_0 \sin 90° = \mu_0\sqrt{x+1}\ \sin\theta$$

$$\sin\theta = \frac{1}{\sqrt{x+1}}$$

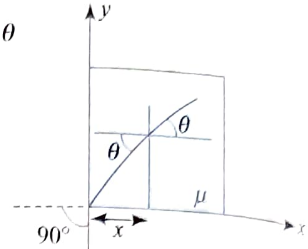
$$\Rightarrow \quad \cos\theta = \sqrt{\frac{x}{x+1}}$$

$$\frac{dy}{dx} = \tan\theta$$

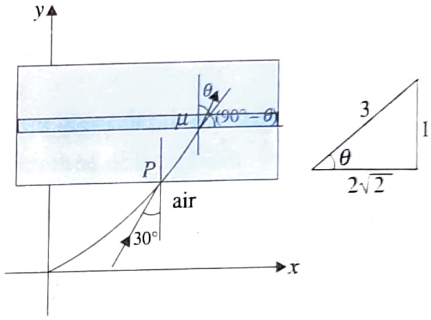
$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{x}}$$

$$\therefore \quad \int dy = \int \frac{1}{\sqrt{x}}\,dx$$

or $\quad y = 2\sqrt{x}$



7. Let RI at $y = y$ is μ and corresponding angle of refraction is θ.



$$\mu \sin\theta = 1 \sin 30° \qquad \text{...(i)}$$

and $\quad \tan\left(\frac{\pi}{2} - \theta\right) = \frac{dy}{dx} \Rightarrow \cot\theta = 8x \Rightarrow \cot\theta = \frac{8y^{1/2}}{2}$

$$\cot\theta = 4y^{1/2}$$

At $\quad y = \frac{1}{2}\text{m} \Rightarrow \cot\theta = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ or $\sin\theta = \frac{1}{2}$

From (i), $\mu \times \frac{1}{3} = \frac{1}{2} \Rightarrow \mu = \frac{3}{2}$

8. We have $\frac{dy}{dx} = \cot(\theta_y) = \sqrt{\frac{\mu_y^2 - \sin^2\theta}{\sin\theta}}$

Here, $\mu = \sqrt{(1+y)}$ and $\theta = 90°$. Therefore, $\frac{dy}{dt} = y^{1/2}$

Integrating with the boundary condition that $y = 0$ at $x = 0$, we get $y = x^2/4$ to be the equation of the path of the ray through the slab. The ray will obviously exit at the point ($2\sqrt{2}$ m, 2 m).

9. The action of slab is to cause a shift in the direction of incident ray.

Shift $s = t\left(1 - \frac{1}{\mu}\right) = 6\left(1 - \frac{2}{3}\right) = 2$ cm

After refraction from the slab, image is formed at distance of 60 cm from the mirror. This acts as object for mirror.

$$\frac{1}{v} + \frac{1}{-60} = \frac{1}{-20}$$

$$\Rightarrow \quad v = -30 \text{ cm}$$

After reflection from the mirror, image is formed at a distance of 30 cm to the left of the mirror. This act as an object for the slab and its image is shifted by 2 cm. Final image is at a distance of 32 cm from the mirror

Exercise 1.6

1. Angle of dispersion: $\delta_v - \delta_r = (\mu_v - \mu_r)A = (1.632 - 1.613)\,5 = 0.095°$

2. (a) $\mu_y = \dfrac{\mu_v + \mu_r}{2} = \dfrac{1.6 + 1.5}{2} = 1.55$

(b) Dispersive power: $\omega = \dfrac{\mu_v - \mu_r}{\mu_y - 1} = \dfrac{1.6 - 1.5}{1.55 - 1} = \dfrac{2}{11}$

3. At minimum deviation, $2r = A$ or $r = 30°$

Also, $\mu = \dfrac{\sin(i)}{\sin(r)}$

Substituting the values, we get $1.5 = \dfrac{\sin(i)}{\sin(30°)}$ or $i = 48.6°$

The angle of deviation, $\delta = 2i - A = 37.2°$

At maximum deviation: $A = 60°$, $\mu = 1.5$ and $i_1 = 90°$

Since $\mu = \dfrac{\sin(i_1)}{\sin(r_1)}$, we get $1.5 = \dfrac{\sin(90°)}{\sin(r_1)}$

or $r_1 = 41.8°$

But $r_1 + r_2 = A$, therefore, $r_2 = 18.2°$

Once again, $\mu = \dfrac{\sin(i_2)}{\sin(r_2)}$

$\therefore \quad 1.5 = \dfrac{\sin(i_2)}{\sin(18.2)}$ or $i_2 = 27.9°$

Therefore, the angle of deviation, $\delta = i_1 + i_2 - A = 57.9°$

4. Given, $\mu = 1.5$ and $A = \delta_{\min}$

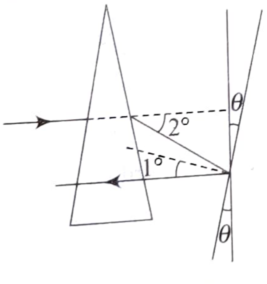
At minimum deviation, $r = A/2$ and $\delta_{\min} = 2i - A$

Substituting for A, we get $i = \delta_{\min}$

$$\text{But } \mu = \frac{\sin\left(\dfrac{\delta_m + A}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \frac{\sin A}{\sin\left[\dfrac{A}{2}\right]} = 2\cos\left[\frac{A}{2}\right]$$

Therefore, $1.5 = 2\cos\left[\dfrac{A}{2}\right]$ or $A = 2\cos^{-1}\left(\dfrac{3}{4}\right)$

5. $\delta = A(\mu - 1) = 4\left(\dfrac{3}{2} - 1\right) = 2°$



Hence, angle of rotation of mirror = 1°.

6. Critical angle between glass and liquid face is

$$\sin\theta_c = \frac{3/2}{\mu} = \frac{3}{2\mu} \quad \text{...(i)}$$

Angle of incidence at face AC is 60°, i.e., $i = 60°$.

For total internal reflection to take place, $i = \theta_c$ or $\sin i > \sin\theta_c$

or $\sin 60° > \dfrac{3}{2\mu}$ or $\dfrac{\sqrt{3}}{2} > \dfrac{3}{2\mu}$

or $\mu > \sqrt{3}$

7. In the case of minimum deviation,

$i_1 = i_2$, $r_1 = r_2$

$\delta = 2i - A$

$r = A/2$

According to problem, $i = 2r = A$

$\delta_{\min} = 2A - A = A$

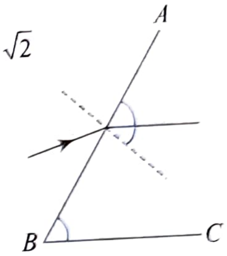
$$n = \frac{\sin\left(\dfrac{A + \delta_{\min}}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \frac{\sin A}{\sin\dfrac{A}{2}} = \frac{2\sin\dfrac{A}{2}\cos\dfrac{A}{2}}{\sin\dfrac{A}{2}}$$

$$\cos\frac{A}{2} = \frac{n}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$\therefore \quad A = 90°$

8. The given parameters are $\delta = 30°$ and $A = 60°$. Let us test whether the prism is in the position of minimum deviation.

$$n = \frac{\sin\left(\dfrac{30° + 60°}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°} = \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$



As $n = \sqrt{2}$, the ray suffers minimum deviation through the prism. Thus,

$$r_1 = r_2 = r = \frac{A}{2} = 30°$$

Inside the prism, the ray makes an angle of 60° with the face AB, so it is parallel to the base.

9. In the figure shown,

$r_2 = C$

$r_1 = 60° - r_2 = 60° - C$

$r_3 = 60° - r_2 = 60° - C$

$r_1 = r_3 = r$ (say)

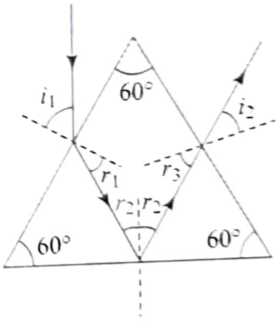
$i_1 = i_2 = i$ (say)

Net deviation, $\delta = (i - r) + (180° - 2r_2) + (i - r) = 108°$

$r_2 = r - i = 36°$

$C + 60 - C - i = 36°$

$i = 24°$, $\sin 24° \approx 0.40$



From Snell's law, we have

$\sin 24° = \mu \sin r$

$0.4 = \mu \sin(60° - C)$

$$0.4 = \mu\left[\frac{\sqrt{3}}{2}\cos C - \frac{1}{2\mu}\right]$$

$$0.8 = \sqrt{3}\mu\sqrt{1 - \frac{1}{\mu^2}} - 1$$

or $\quad 3(\mu^2 - 1) = (1.8)^2$

$\therefore \quad \mu^2 - 1 = 1.08 \quad$ or $\mu = 1.447$

10. As the ray of light grazes the second surface, r_2 is the critical angle.

$$\sin r_2 = \frac{1}{\mu}$$

$r_2 = (45° - r_1)$

$$\sin r_1 = \frac{\sin 45°}{\mu} = \frac{1}{\sqrt{2}\,\mu}$$

$$\sin r_2 = \sin(45° - r_1) = \frac{1}{\sqrt{2}}[\cos r_1 - \sin r_1]$$

$$\frac{1}{\mu} = \frac{1}{\sqrt{2}}\left[\sqrt{1 - \frac{1}{2\mu^2}} - \frac{1}{\sqrt{2}\mu}\right]$$

or $$\frac{1}{\mu} = \frac{1}{\sqrt{2}}\left[\frac{\sqrt{2\mu^2 - 1}}{\sqrt{2}\mu} - \frac{1}{\sqrt{2}\mu}\right]$$

$$2 = \sqrt{2\mu^2 - 1} - 1$$

or $$2\mu^2 - 1 = 9$$

$$2\mu^2 = 10$$

$\Rightarrow$ $$\mu^2 = 5 \quad \text{or} \quad \mu = \sqrt{5}$$

At minimum deviation, $r_1 = r_2 \dfrac{45^\circ}{2} = 22.5^\circ$

$$\mu = \frac{\sin i_1}{\sin r_1} \Rightarrow \sin i_1 = (\sqrt{5})\sin(22.5^\circ)$$

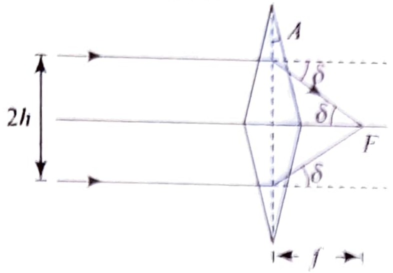
$$i_1 = 58.8^\circ$$

11. For minimum deviation: $\mu = \dfrac{\sin[(A + \delta_m)/2]}{\sin(A/2)}$

$$\frac{3}{2} = \frac{\sin[(60^\circ + \delta_m)/2]}{\sin(60^\circ/2)} = \sin\left(\frac{60 + \delta_m}{2}\right) = \frac{3}{4}$$

$\Rightarrow$ $$i = \frac{A + \delta_m}{2} = \sin^{-1}\left(\frac{3}{4}\right) = 48.5^\circ$$

12. Since $\tan\delta = \dfrac{h}{f} \Rightarrow f = \dfrac{h}{\tan\delta}$

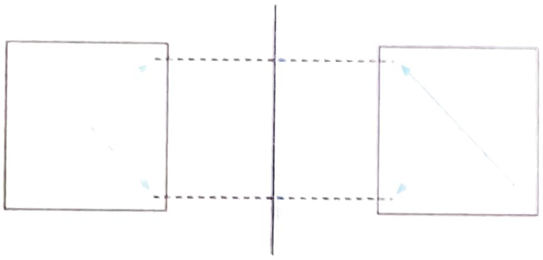


Further, $\delta = (\mu - 1)A \Rightarrow f = \dfrac{h}{(\mu - 1)A}$

Exercises

Single Correct Answer Type

1. **(3)**



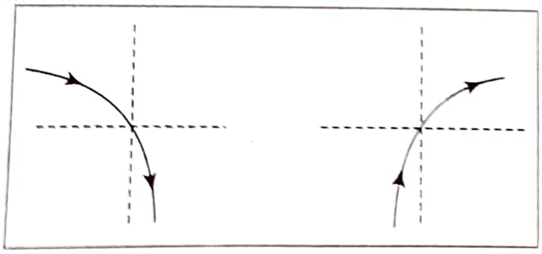
2. **(4)** As the ray moves toward the normal while entering medium 2 from 1, we have $n_2 > n_1$

For total internal reflection at interface of 2 and 3, $n_2 > n_3$.

Besides n_3 should also be less than n_1 or else ray would have emerged in medium 3, parallel to its path in medium 1.

Hence, $n_3 < n_1 < n_2$ is the correct order.

3. **(1)** Since the refractive index is changing, the light cannot travel in a straight line in the liquid as shown in options (3) and (4). Initially, it will bend towards normal and after reflecting from the bottom it will bend away from the normal as shown below.



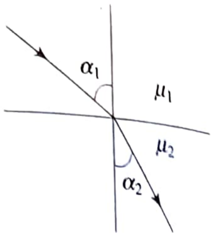
4. **(1)** From Snell's law, $\mu_1 \sin i = \mu_2 \sin r \Rightarrow \dfrac{\mu_2}{\mu_1} = \dfrac{\sin i}{\sin r}$

From the graph, $\tan 60^\circ = \dfrac{\sin i}{\sin r} \Rightarrow \dfrac{\sin i}{\sin r} = \sqrt{3} = \dfrac{\mu_2}{\mu_1}$

$$\mu = \frac{c}{v} \Rightarrow = \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1}$$

$$v_1 = \left(\frac{\mu_2}{\mu_1}\right)v_2 \Rightarrow v_1 = \sqrt{3}\, v_2$$

5. **(2)** $\mu_1 \sin\alpha = \mu_2 \sin\alpha_2$

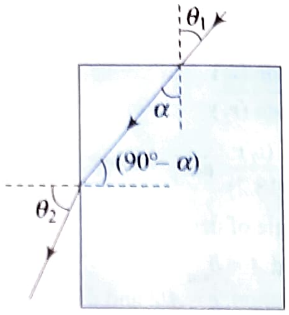


$$\frac{c}{v_1}\sin\alpha_1 = \frac{c}{v_2}\sin\alpha_2$$

$$\frac{\sin\alpha_1}{f\lambda_1} = \frac{\sin\alpha_2}{f\lambda_2}$$

$$\lambda_2 = \frac{\sin\alpha_2}{\sin\alpha_1}\lambda_1$$

6. **(3)**



As θ_1 increases, α also increases. So, $(90^\circ - \alpha)$ decreases and hence θ_2 decreases.

7. **(1)** $\dfrac{\mu_1}{-u} + \dfrac{\mu_2}{v} = \dfrac{\mu_2 - \mu_1}{R}$

For a plane surface, $R = \infty$

$\therefore$ $$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = 0$$

or $$\frac{\mu_2}{v} = \frac{\mu_1}{u} \quad \text{or} \quad \frac{\mu}{v} = \frac{1}{u}$$

or $$v = \mu_2 u$$

Clearly, to the fish, the bird appears farther than its actual distance.

Again, $\dfrac{dv}{dt} = \mu\dfrac{du}{dt}$

or Apparent speed of bird = Refractive index × Actual speed of bird.

8. **(3)** $$\sqrt{3} = \frac{\sin\left(\dfrac{60^\circ + \delta_m}{2}\right)}{\sin\left(\dfrac{60^\circ}{2}\right)}$$

$$\frac{\sqrt{3}}{2} = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

$$\sin 60^\circ = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

or $$\frac{60^\circ + \delta_m}{2} = 60^\circ$$

or $$\delta_m = 60^\circ \Rightarrow i = \frac{A + \delta_m}{2} = \frac{60^\circ + 60^\circ}{2} = 60^\circ$$

9. **(4)** **For A:**

$u = -3$ m, $v_1 = ?$, $f = -1$ m

$$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-3} = \frac{1}{3} - 1 = -\frac{2}{3}$$

or $\quad v_1 = -\frac{3}{2}$

For B:

$$\frac{1}{v_2} = \frac{1}{-1} - \frac{1}{-5}$$

or $\quad \frac{1}{v_2} = \frac{1}{5} - 1 = -\frac{4}{5} \quad$ or $\quad v_2 = -\frac{5}{4}$ m

Now, $\quad v_1 - v_2 = -\frac{3}{2} - \left(-\frac{5}{4}\right) = -\frac{3}{2} + \frac{5}{4} = -\frac{1}{4}$ m $= -0.25$ m

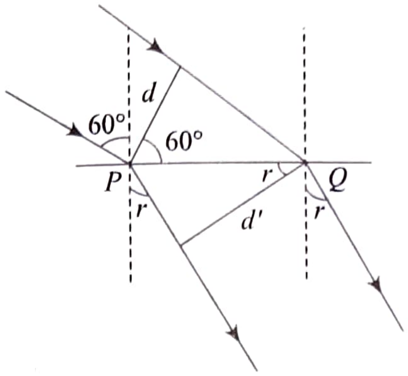
Again, $\quad \frac{I_1}{O} = -\frac{v_1}{u}$

or $\quad I_1 = -\frac{v_1}{u}O = -\left(\frac{-3}{2}\right)\left(\frac{-1}{3}\right) = -1$ m

Again, $\quad \frac{I_2}{O} = -\frac{v_2}{u}$

or $\quad I_2 = -\left(-\frac{5}{4}\right)\left(\frac{1}{-5}\right)2 = -0.5$ m

10. (3) Let d' be the diameter of refracted beam.



Then, $\quad d = PQ \cos 60°$

and $\quad d' = PQ \cos r$

i.e. $\quad \frac{d'}{d} = \frac{\cos r}{\cos 60°} = 2 \cos r$

or $\quad d' = 2d \cos r$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sqrt{3}/2}{3/2} = \frac{1}{\sqrt{3}}$$

$\therefore \quad \cos r = \sqrt{1 - \sin^2 r} = \sqrt{\frac{2}{3}}$

$\therefore \quad d' = (2)(2)\sqrt{\frac{2}{3}} = 4\sqrt{\frac{2}{3}}$ cm ≈ 3.26 cm

11. (2) $v_m = \frac{1}{2}c$

$$\mu = \frac{c}{v_m} = \frac{c}{\frac{1}{2}c} = 2 \text{ or } \frac{1}{\sin i_c} = 2$$

or $\quad \sin i_c = \frac{1}{2}$ or $i_c = 30°$

12. (4) For convex mirror, $u + v = 12 \times 2 = 24$ cm (because for plane mirror, distance of object = distance of image). Also, here for the convex mirror $u = 20$ cm, therefore $v = 4$ cm. Hence, using

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

We find $f = 5$ cm

13. (2) Distance of first image (I_1) formed after refraction from the plane surface of water is $\frac{10}{4/3} = 7.5$ cm from water surface $\left(d_{\text{app}} = \frac{d_{\text{actual}}}{\mu}\right)$.

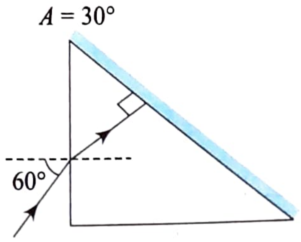
Now, distance of this image is 5 + 7.5 = 12.5 cm from the plane mirror. Therefore, distance of second image (I_2) will also be equal to 12.5 cm from the mirror.

14. (4) The two slabs will shift the image a distance

$$d = 2\left(1 - \frac{1}{\mu}\right)t = 2\left(1 - \frac{1}{1.5}\right)(1.5) = 1.0 \text{ cm}$$

Therefore, final image will be 1 cm above point P.

15. (2) $r_2 = 0°$



$r_1 = A = 30°$ and $i_1 = 60°$

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

16. (1) $A = \delta_m = 60°$

At minimum deviation $i = \left(\frac{A + \delta_m}{2}\right) = 60°$

17. (2) $\quad m = \frac{f}{f - u}$

$$3 = \frac{-24}{-24 - u}$$

or $\quad -24 - u = -8$ or $u + 24 = 8$

or $\quad u = (8 - 24)$ cm $= -16$ cm

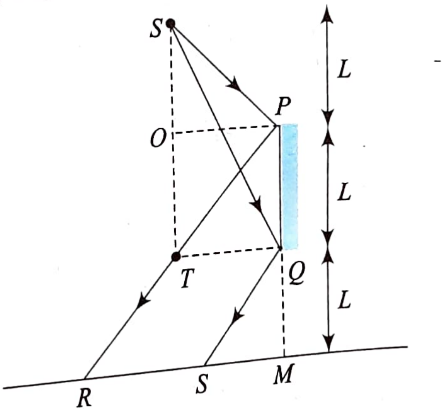
If $m = -3$, then $-3 = \frac{-24}{-24 - u}$

$\quad u + 24 = -8$

or $\quad u = -32$ cm

Note that the magnification is greater than 1, so mirror cannot be convex.

18. (3) Ray diagram is as shown in figure:



Here, $\frac{SO}{OP} = \frac{PM}{RM} \Rightarrow RM = 2L$

Also, $\dfrac{ST}{TQ} = \dfrac{QM}{SM} \Rightarrow SM = \dfrac{L}{2}$

$$RS = RM - SM = \frac{3L}{2}$$

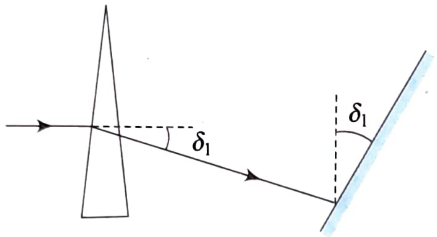
19. (2) The fish can observe the sky only if refraction takes place. If TIR takes place, then image of sky cannot be observed.

i.e., $i < i_c$ and $i_c = 45°$

So, angle subtended $= \dfrac{\pi}{2} = 90°$

20. (4) For light to retrace the path, the light ray falling on the mirror should be along the normal.

Deviation produced by prism, $\delta_1 = (1.5 - 1)4 = 2°$ CW



So, mirror has to be rotated by 2° in CW direction.

21. (1) As object moves from infinity to the pole of convex mirror, image moves from focus to pole. So, $v_i < v_0$, always.

22. (1) In a prism: $r + r' = A \Rightarrow r = A - r'$

$\therefore \quad r = 60° - (10 + t^2) = 50 - t^2$

23. (1) Given $i = 60°$ and $A = \delta = e$

$\delta = i + e - A \Rightarrow \delta = i \; (\because e = A)$

$$\mu = \frac{\sin\left(\dfrac{A + \delta_m}{2}\right)}{\sin\dfrac{A}{2}}$$

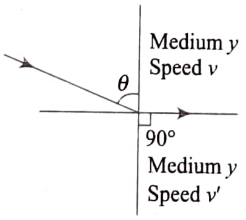
Here, angle of deviation is minimum ($\because i = e$)

$$\mu = \frac{\sin\left(\dfrac{60° + 60°}{2}\right)}{\sin(60°/2)} = 1.73$$

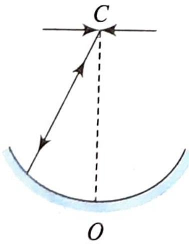
24. (4) Clearly, x is denser medium.

Now, $\dfrac{\sin\theta}{\sin 90^o} = \dfrac{1}{{}_Y\mu_X} = {}_X\mu_Y = \dfrac{v}{v'}$

or $\quad v' = \dfrac{v}{\sin\theta}$



25. (4) When there is no water in the mirror, the rays of light are incident normally on the mirror and retrace their path. So, we get an image coincident with the object as shown in the figure.



When the mirror is filled with water, then the equivalent focal length F is given by

$$\frac{1}{F} = \frac{1}{f_{\text{water lens}}} + \frac{1}{f_{\text{concave mirror}}} + \frac{1}{f_{\text{water lens}}}$$

or $\quad \dfrac{1}{F} = 2 \times \dfrac{1}{f_{\text{water lens}}} + \dfrac{1}{f_{\text{concave mirror}}}$

or $\quad \dfrac{1}{F} = 2(\mu - 1)\left(\dfrac{1}{R}\right) + \dfrac{1}{R}$

or $\quad \dfrac{1}{F} = \dfrac{2(\mu - 1)}{R} + \dfrac{2}{R}$

or $\quad \dfrac{1}{F} = \dfrac{2\mu}{R}$ or $F = \dfrac{R}{2\mu}$

Clearly, focal length of the new optical system in less than the original. So, the object is effectively at a distance greater than twice the focal length. So, the real image will be formed between F and $2F$.

26. (4) A thick glass mirror produces a number of images.

There is an apparent shift of actual silvered surface toward the unsilvered face.

Effective distance of the reflecting surface from unsilvered face $= \dfrac{d}{\mu} = \dfrac{3}{3/2}$ cm = 2 cm.

Distance of the point object from effective reflecting surface = 9 cm + 2 cm = 11 cm.

Distance of image from the point object = 11 cm + 11 cm = 22 cm

Distance of image from unsilvered face = (22 – 9) cm = 13 cm

27. (1) Clearly, the distance of I from P is 15 cm.

Now, $\quad u = -25$ cm, $v = 15$ cm, $f = ?$

$\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$ or $\dfrac{1}{f} = \dfrac{1}{-25} + \dfrac{1}{15}$

or $\quad \dfrac{1}{f} = \dfrac{-3 + 5}{75}$ or $f = \dfrac{75}{2}$ cm = 37.5 cm

28. (2) $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f} \Rightarrow -\dfrac{du}{u^2} - \dfrac{dv}{v^2} = 0$ or $-\dfrac{dv}{v^2} = \dfrac{du}{u^2}$

or $\quad dv = -\dfrac{v^2}{u^2}du = -\dfrac{10 \times 10}{30 \times 30} \times 9 \text{ m s}^{-1} = -1 \text{ m s}^{-1}$

29. (1) For a concave mirror, $u = -\dfrac{15}{2}$ cm, $v = ?$

$f = -\dfrac{10}{2}$ cm $= -5$ cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-5} - \frac{1}{-15/2} = -\frac{1}{5} + \frac{2}{15} = \frac{-1}{15}$$

or $\quad v = -15$ cm

Clearly, the position of the final image is on the pole of the convex mirror.

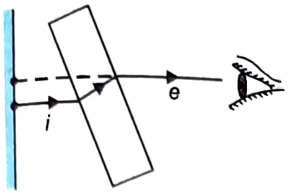
30. (2) For upright portion,

$$m = \frac{f}{f - u} = \frac{\dfrac{-10}{2}}{\dfrac{-10}{2} - (-20)} = \frac{-5}{-5 + 20} = \frac{-5}{15} = -\frac{1}{3}$$

For horizontal portion, magnification is $\left(-\dfrac{1}{3}\right)^2$ i.e., $\dfrac{1}{9}$

Required ratio is $\dfrac{-1/3}{1/9} = -3:1$

31. (1) Emergent ray e will be parallel to incident ray i. So both dots will appear to be shifted upward by same distance. Hence, distance between them remains same.



32. (4) Let $u = -x$

$$\frac{1}{-x} + \frac{1}{-x-10} = -\frac{1}{12}$$

or $$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12}$$

or $$\frac{x+10+x}{x(x+10)} = \frac{1}{12}$$

or $$\frac{2x+10}{x^2+10x} = \frac{1}{12}$$

or $$x^2 + 10x = 24x + 120$$

or $$x^2 - 14x - 120 = 0$$

$$x^2 - 20x + 6x - 120 = 0$$

or $$x(x-20) + 6(x-20) = 0$$

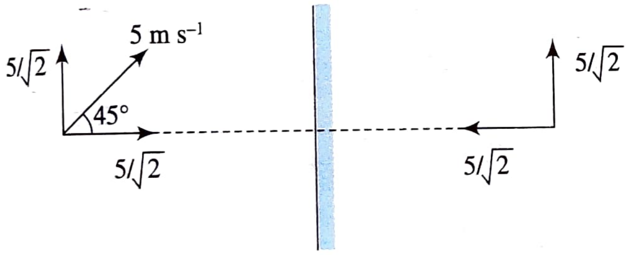
$$(x+6)(x-20) = 0$$

Here $u = -20$ cm

$v = -(20 + 10)$ cm $= -30$ cm

Then magnification $m = -\dfrac{u}{v} = -\left(\dfrac{-30}{-20}\right) = -1.5$

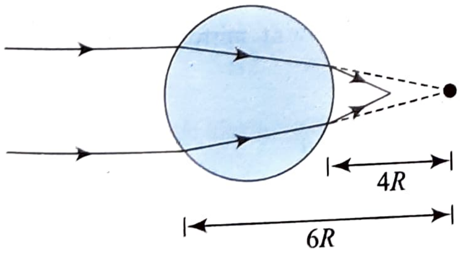
33. (1) Speed of image w.r.t. mirror $= \sqrt{\left(\dfrac{5}{\sqrt{2}}\right)^2 + \left(\dfrac{5}{\sqrt{2}}\right)^2} = 5\ \text{m s}^{-1}$



34. (2) $$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.25}{-(-\infty)} + \frac{1.5}{v} = \frac{1.5 - 1.25}{R} \quad \text{or} \quad \frac{1.5}{v} = \frac{0.25}{R}$$

or $$v = \frac{1.5R}{0.25} = 6R$$



Again, $$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

$$\frac{1.5}{-4R} + \frac{12.5}{v} = \frac{1.25 - 1.5}{-R}$$

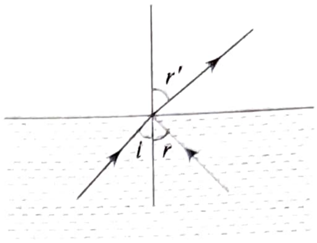
or $$\frac{1.25}{v} = \frac{1}{4R} + \frac{1.5}{4R} = \frac{2.5}{4R}$$

$$v = \frac{1.25 \times 4R}{2.5} = \frac{5R}{2.5}$$

or $v = 2R$

Distance from the centre $= 3R$

35. (3) $$\mu = \frac{\sin r'}{\sin i}$$



But $r' + r = 90°$

or $r' = 90° - r$

$r = 90° - r$

$\therefore$ $$\mu = \frac{\cos i}{\sin i}$$

or $$\frac{1}{\sin i_c} = \frac{1}{\tan i} \quad \text{or} \quad i_c = \sin^{-1}(\tan i)$$

36. (3) $\mu_g \sin \theta_c = \mu_1 \sin 90°$

or $\mu_g \sin \theta_c = 1$

When water is poured,

$\mu_w \sin r = \mu_g \sin \theta_c$

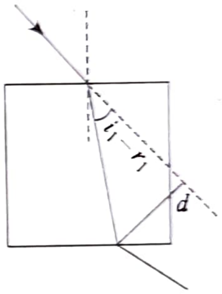
or $\mu_w \sin r = 1$

Again, $\mu_a \sin \theta = \mu_w \sin r$

or $\mu_a \sin \theta = 1$

or $\sin\theta = 1$ or $\theta = 90°$

37. (2) $$\frac{\sin 60°}{\sin r_1} = \sqrt{3}$$



or $$\sin r_1 = \frac{\sin 60°}{\sqrt{3}}$$

or $$\sin r_1 = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$$

or $r_1 = 30°$

Now, $\sin(i_1 - r_1) = \dfrac{d}{5}$

or $d = 5 \sin(i_1 - r_1)$

or $d = 5 \sin(60° - 30°) = 5 \sin 30° = \dfrac{5}{2}$ cm

38. (3) $\sin i_c = \dfrac{1}{\sqrt{2}} \Rightarrow i_c = 45°$

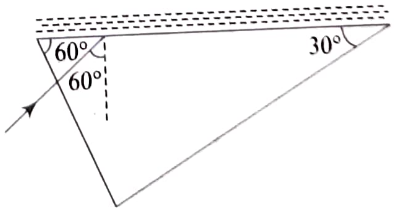
Now, $75° = r + 45°$ or $r = 30°$

Now, $$\frac{\sin i}{\sin 30°} = \sqrt{2}$$

or $$\sin i = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

$\therefore$ $i = 45°$

39. (2)



Clearly, $i_c \le 60°$

So, maximum possible value of i_c is 60°.

Now, $$\mu_g = \frac{1}{\sin i_c}$$

$$\frac{\mu_g}{\mu_1} = \frac{1}{\sin i_c}$$

or $\mu_1 = \mu_g \sin i_c = 1.5 \sin 60° = 1.5 \times \frac{\sqrt{3}}{2}$

$= 1.5 \times 0.866 = 1.299 = 1.3$

40. **(3)** $A = r_1 + r_2$

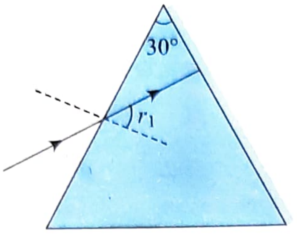
$30° = r_1 + 0$

or $r_1 = 30°$

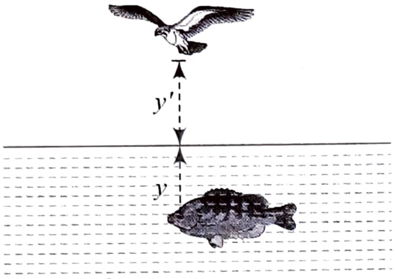
Now, $\frac{\sin i}{\sin 30°} = \sqrt{2}$

or $\sin i = \sqrt{2} \times \frac{1}{2}$

or $i = 45°$



41. **(1)** Effective height of the bird as seen by the fish, $Y = y + \mu y'$



$$\frac{dY}{dt} = \frac{dy}{dt} + \mu \frac{dy'}{dt}$$

$$9 = 3 + \frac{4}{3}\frac{dy'}{dt}$$

or $\frac{dy'}{dt} = \frac{6 \times 3}{4} = \frac{18}{4} = \frac{9}{2} = 4.5 \text{ m s}^{-1}$

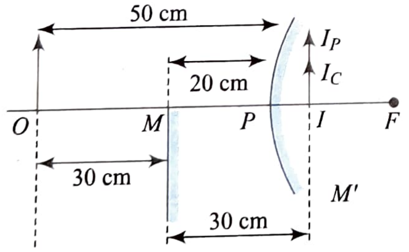
Therefore, actual velocity of bird = 4.5 m s^{-1}

42. **(1)** $\sin i_c = \frac{1}{1.5} = \frac{2}{3}$

$i_c = \sin^{-1}\left(\frac{2}{3}\right) = \sin^{-1}(0.6667) = 41.8°$

The angle of incidence of face AC is 45° which is clearly greater than 41.8°.

43. **(1)** As shown in figure, the plane mirror will form erect and virtual image of same size at a distance of 30 cm behind it. So, the distance of image formed by plane mirror from convex mirror will be $PI = MI - MP$



But as $MI = MO$

∴ $PI = MO - MP = 30 - 20 = 10$ cm

Now, as this image coincides with the image formed by convex mirror, therefore for convex mirror,

$u = -50$ cm; $v = +10$ cm

So, $\frac{1}{+10} + \frac{1}{-50} = \frac{1}{f}$ i.e., $f = \frac{50}{4} = 12.5$ cm

So, $R = 2f = 2 \times 12.5 = 25$ cm

44. **(2)** In first case, if object distance is x, image distance = $x/4$. While it is 2nd case, object distance becomes $(x - 5 \text{ cm})$ and image distance $(x - 5 \text{ cm})/2$. Using mirror formula, we get

In first case, $\frac{1}{f} = -\frac{5}{x}$

In second case, $\frac{1}{f} = -\frac{3}{(x-5)}$

Solving we get, $x = 12.5$ cm and $f = -2.5$ cm and hence $|f| = 2.5$ cm

45. **(3)** For concave mirror, if x and y are object distance and image and distance, respectively, we have

$$-\frac{1}{x} - \frac{1}{y} = -\frac{1}{|f|}$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{|f|}$$

$$\Rightarrow \quad -\frac{1}{x^2}\frac{dx}{dt} - \frac{1}{y^2}\frac{dy}{dt} = 0$$

$$\Rightarrow \quad \left|\frac{V_x}{V_y}\right| = \frac{x^2}{y^2}$$

For $\left|\frac{V_x}{V_y}\right| = \frac{1}{4}, \frac{x}{y} = \pm 2$

For $\frac{x}{y} = 2$, we get $x = \frac{3|f|}{2}$ [for point A]

and for $\frac{x}{y} = -2$, we get $x = \frac{|f|}{2}$ [for point B]

As the middle point happens to be focus of the mirror, we get

$|f| = L$

46. **(3)** $\frac{x}{1} = \frac{x_{\text{rel}}}{\mu} \Rightarrow x_{\text{rel}} = \mu x$

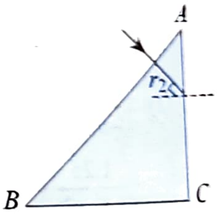
$$\frac{d^2 x_{\text{rel}}}{dt^2} = \mu \frac{d^2 x}{dt^2} \Rightarrow a_{\text{rel}} = \mu g$$

47. **(2)** $A = 90° - \theta$

$\Rightarrow \quad r_2 = A = 90° - \theta > \theta_c$

$\cos\theta > \sin\theta_c = \frac{6/5}{2/3} = \frac{4}{5}$

(θ_c is critical angle)

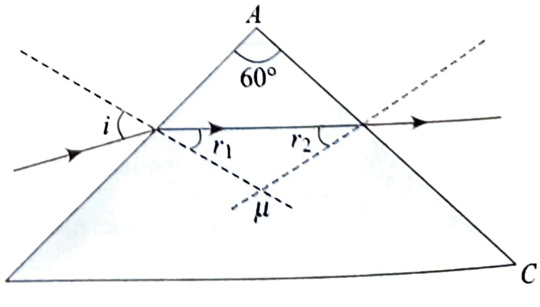


$\theta < \cos^{-1}\frac{4}{5} = 37°$

48. **(2)** Image velocity (w.r.t. mirror) = $-m$ × object velocity (w. mirror)

Here $m = 1$

49. **(1)** $r_2 < \theta_c$; $A - r_1 < \theta_c$



$r_1 > A - \theta_c$

$\sin r_1 > \sin(A - \theta_c)$

$$\frac{\sin i}{\mu} > \sin(A - \theta_c)$$

$$\sin i > \mu(\sin A \cos\theta_c - \cos A \sin\theta_c)$$

$$> \sqrt{\frac{7}{3}}\left(\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{7}} - \sqrt{\frac{3}{7}}\cdot\frac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\sin i > \frac{1}{2} \text{ or } i > 30°$$

(1) Given $i = 60°$, $A = \delta = e$

$\delta = i + e - A \quad \Rightarrow \quad \delta = 1$

$(\because e = A)$ and $\delta = i = e$

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

Here, angle of deviation is minimum $(\because i = e)$

$$\mu = \frac{\sin\left(\frac{60° + 60°}{2}\right)}{\sin(60°/2)} = \sqrt{3}$$

1. (2) Shift $= (l - m)\left(1 - \frac{1}{n_1}\right) + m\left(1 - \frac{1}{n_2}\right) = 0$

2. (1) By mirror formula: $\frac{1}{v} + \frac{1}{-10} = \frac{1}{10}$

$\Rightarrow \quad v = +5$ cm

$\therefore \quad m = +\frac{1}{2}$

The image revolves in a circle of radius 1/2 cm. Image of a radius is erect ⇒ particle will revolve in the same direction as the particle. The image will complete one revolution in the same time 2 s.

Velocity of image v is $\omega r = \frac{2\pi}{2} \times \frac{1}{2} = \frac{\pi}{2}$ cm s^{-1} = 1.57 cm s^{-1}

3. (4) Mirror can be shifted to new position $C'D'$. Distances are shown in figure.

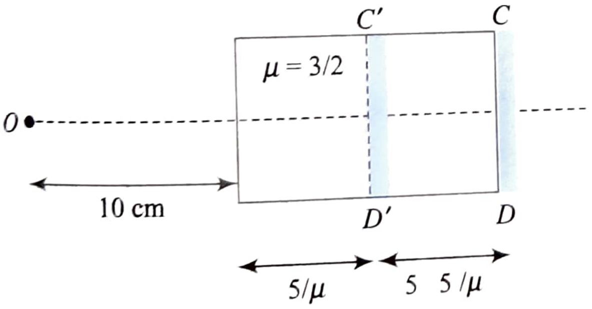


Image will be at equal distance from the mirror $C'D'$ as the object is.

Image distance from $C'D'$ is $10 + \frac{5}{3/2} = 10 + \frac{10}{3} = \frac{40}{3}$ cm

Separation between object and image is $\frac{80}{3}$ cm.

54. (4) Deviation produced by prism is $\delta_1 = (\mu - 1)A = 2°$ CW

Angle of incidence for mirror is δ_1, so deviation produced by the mirror is $\delta_2 = \pi - 2\delta_1 = 176°$ CW

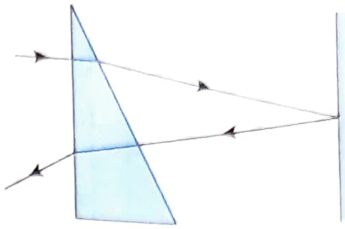
Deviation produced by the prism for 2nd refraction is

$\delta_3 = 2°$ ACW

Net deviation = 174° CW

55. (4) Deviation produced by the mirror is

$\delta_1 = \pi - 2 \times 60 = 60°$ CW



Deviation produced by the prism is

$\delta_2 = (1 - 2) \times 6 = 6°$ ACW

So, net deviation, $\delta = \delta + \delta_2 = (60° - 6°)$ CW = 54° CW

56. (4) From $\sin\left(\frac{A+\delta_m}{2}\right) = \mu \sin\frac{A}{2}$

$$\sin\left(\frac{A+\delta_m}{2}\right) = \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} \times \sin\frac{A}{2}$$

$$\frac{A+\delta_m}{2} = \frac{\pi}{2} - \frac{A}{2}$$

$$\delta_m = \pi - 2A = 180° - 2A$$

57. (4) $u = 21$ cm, $f = \frac{R}{2} = \frac{10}{2} = 5$ cm

On introducing the glass slab, the object as well as the image will be shifted from the mirror through a distance

$d = t\left(1 - \frac{1}{\mu}\right) = 3\left(1 - \frac{1}{1.5}\right) = 1$ cm, so that apparent distance of the object = 20 cm i.e., $u = 20$ cm

By the mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$v = -\frac{20}{3}$ cm $= -6.67$ cm

Distance of the final image from the mirror = 6.67 + 1 = 7.67 cm

58. (4) No change as there would be no deviation in path of rays compared to what it was in air.

59. (2) From similar triangles APB and CBD, point P will be just visible if $CD = 4R$

$\mu \sin i = 1 \times \sin r$

$\mu \sin(\angle CDQ) = 1 \times \sin(\angle CDB)$

$$\mu \times \frac{R}{R\sqrt{17}} = 1 \times \frac{2R}{R\sqrt{20}}$$

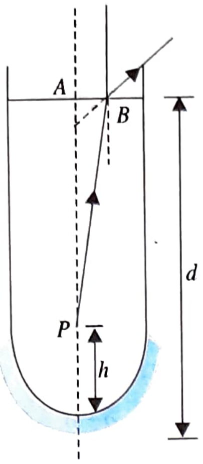
$$\Rightarrow \quad \mu = \frac{2\sqrt{17}}{\sqrt{20}} = \frac{\sqrt{85}}{5}$$

60. (4) Any point to right of P on surface PA will not be visible when the vessel is filled with liquid.

61. (3) There will be two images on by direct observation other by reflection through mirror.

$$\text{Apparent depth} = \frac{\text{Real depth}}{\mu_{\text{rel}}} = \frac{\text{Real depth}}{\left(\frac{\mu_{\text{incident}}}{\mu_{\text{refractive}}}\right)} = \frac{(d-h)}{(\mu\ 1)}$$

$$d_{\text{app}} = \frac{(d-h)}{\mu}$$



Object P is placed at the centre of curvature of the mirror. Its image will be formed at P itself.

So, for image also, $d_{\text{app}} = \dfrac{(d-h)}{\mu} - \dfrac{(d-h)}{\mu} = 0$

62. (3) Given $r = 6$ cm, $r_1 = 4$ cm, $h_1 = 8$ cm

Let h = final height of water column

The volume of the cylindrical water column after the glass piece is put will be

$$\pi r^2 h = 800\pi + \pi r_1^{\,2} h_1$$

$$r^2 h = 800 + r_1^{\,2} h_1$$

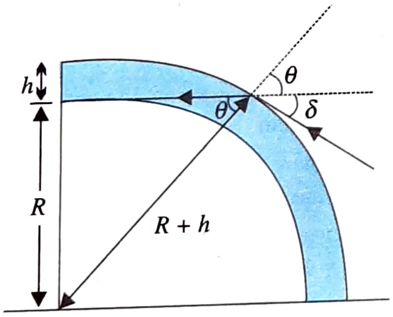
$$(6)^2\, h = 800 + (4)^2\, 8$$

$$h = \frac{800+128}{36} = \frac{928}{36} = 25.7 \text{ cm}$$

There are two shifts due to glass block as well as water

Total shift = (2.66 + 4.44) = 7.1 cm above the bottom.

63. (4) By Snell's law



$$\frac{\sin(\theta+\delta)}{\sin\theta} = n$$

$$\sin\theta = \frac{R}{(R+h)}$$

$$\sin(\theta+\delta) = \frac{nR}{(R+h)}$$

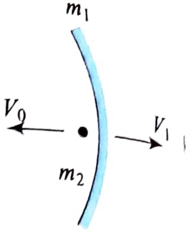
$$\delta = \sin^{-1}\left(\frac{nR}{R+h}\right) - \sin^{-1}\left(\frac{R}{R+h}\right)$$

64. (4) Let v_1 be the speed of gun (or mirror) just after the firing of bullet. From conservation of linear momentum,

$$m_2 v_0 = m_1 v_1$$

$$v_1 = \frac{m_2 v_0}{m_1} \qquad \text{...(i)}$$

Now rate at which distance between mirror and bullet is increasing,



$$\frac{du}{dt} = v_1 + v_0 \qquad \text{...(ii)}$$

$$\frac{dv}{dt} = \left(\frac{v^2}{u^2}\right)\frac{du}{dt}$$

Here, $\dfrac{v^2}{u^2} = m^2 = 1$

(as at the time of firing, the bullet is at pole).

$$\frac{dv}{dt} = \frac{du}{dt} = v_1 + v_0 \qquad \text{...(iii)}$$

Here, dv/dt is the rate at which distance between image (of bullet) and mirror is increasing. So, if v_2 is the absolute velocity of image (towards right), then

$$v_2 - v_1 = \frac{dv}{dt} = v_1 + v_0$$

or $\quad v_2 = 2v_1 + v_0 \qquad \text{...(iv)}$

Therefore, speed of separation of the bullet and the image will be

$$v_r = v_2 + v_0 = 2\,v_1 + v_0 + v_0$$

$$v_r = 2\,(v_1 + v_0)$$

Substituting value of v_1 from Eq. (i), we have

$$v_r = 2\left(1 + \frac{m_2}{m_1}\right)v_0$$

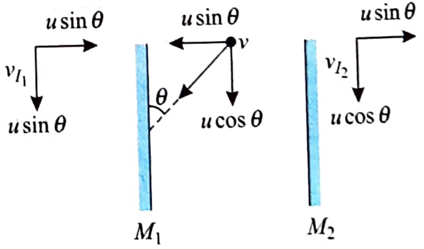
65. (1) For plane surface, $h_{app} = \mu h = \dfrac{3}{2} \times 10 = 15\,\text{cm}$

This first image is at a distance (15 + 3) cm from the plane mirror. So, mirror will make its second image at a distance 18 cm below the mirror or 21 cm below the plane surface.

Now further applying, $d_{app} = \dfrac{d}{\mu} = \dfrac{21}{1.5} = 14\,\text{cm}$

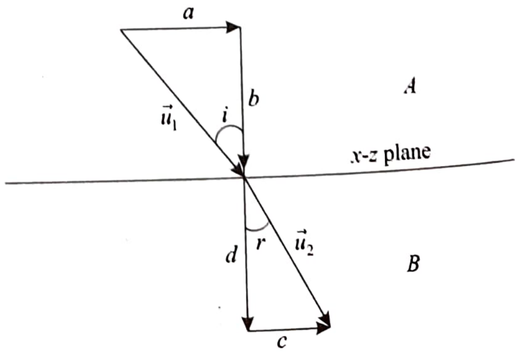
below the plane surface.

66. (4) The image velocity by both the mirror is equal hence the velocity of approach will be zero



$$\Rightarrow \qquad v_{I_1 I_2} = v_{I_2} - v_{I_1} = 0$$

67. (1)



From Snell's law: $1.5 \sin i = 2 \sin r$

$$1.5\times\frac{a}{\sqrt{a^2+b^2}} = 2\times\frac{c}{\sqrt{c^2+d^2}} \Rightarrow \frac{a}{c}=\frac{4}{3}$$

$$\left[\because \sqrt{a^2+b^2}=\sqrt{c^2+d^2}=1(\text{unit vector})\right]$$

68. (1) $\mu_{\text{red}} \sin i = 1 \sin r$

$$1.39\times\frac{1}{1.414} = \sin r$$

$\sin r < 1$ possible (refraction is possible)

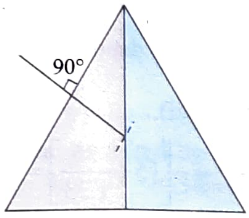
$\mu_{\text{green}} \times \sin i = 1.44\times\frac{1}{1.41} > 1 \Rightarrow$ not possible

Same for blue $1.47\times\frac{1}{1.41} > 1$

$\Rightarrow$ It also not possible.

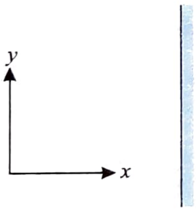
69. (1) $\mu \sin A = 1\times \sin 90° \Rightarrow \mu = \frac{1}{\sin A}$

and $\mu \sin\frac{A}{2} = 1\times\sin e = \frac{1}{\sin A}\times\sin\frac{A}{2} = \sin e$



$$\sin e = \frac{\sin\frac{A}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} \Rightarrow e = \sin^{-1}\left[\frac{1}{2}\sec\frac{A}{2}\right]$$

70. (2)

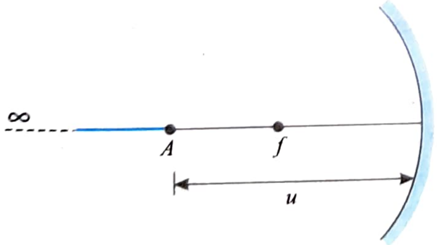


$v_I = 2v_M - v_0 = 2\times 0 - \left(5\hat{i} + (6+2t)\hat{j}\right)$

At $t = 2$ sec, $v_I = -5\hat{i} + 10\hat{j}; v_{I0} = v_I - v_0$

Hence $v_{I0} = -5\hat{i} + 10\hat{j} - \left(5\hat{i}+10\hat{j}\right) = -10\hat{i}$

71. (4)



$u = -u, f = -f$ and $v_A = \frac{uf}{f-u}$

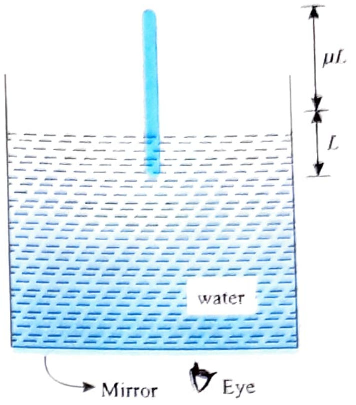
Image of last end of rod, $v_B = -f$

Length $= v_B - v_A = -f - \frac{uf}{f-u} = \frac{-f^2 + uf - uf}{f-u}$

Length $= \frac{f^2}{u-f}$. Hence, (4) is the correct option.

72. (2) When observer in rarer medium and object is in denser medium then the length of immersed portion will be shorter (apparent)

Hence length of image of rod in water

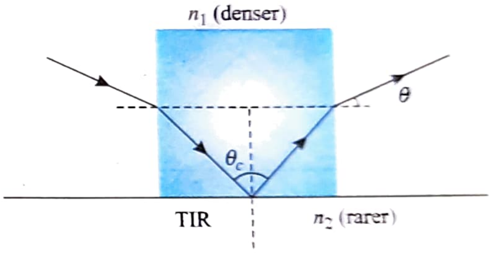


$= 2L + \mu L - L = L + \mu L$

Hence length of image of rod seen by observer in air is

$= \frac{L+\mu L}{\mu} = \frac{L}{\mu} + L$. Hence, (2) is the correct option.

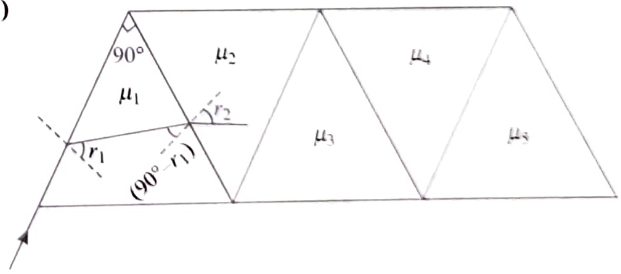
73. (1) $n_1 \sin\theta_C = n_2 \Rightarrow \sin\theta_C = \frac{n_2}{n_1}$



$$1\times\sin\theta = n_1\sin(90° - \theta_C) = n_1\cos\theta_C = n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\theta = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right); \sin\theta < \sqrt{n_1^2 - n_2^2}$$

74. (2)



1st surface: $1 \sin 90° = \mu_1 \sin r_1$

2nd surface: $\mu_1 \cos r_1 = \mu_2 \sin r_2$

3rd surface: $\mu_2 \cos r_2 = \mu_3 \sin r_3$

4th surface: $\mu_4 \sin r_4 = \mu_3 \cos r_3$

5th surface: $\mu_4 \cos r_4 = \mu_5 \sin r_5$

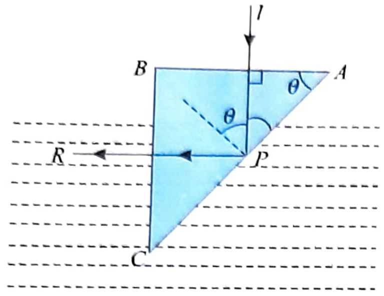
$1.\sin 90° = \mu_5 \cos r_5$

Square and add $1 + \mu_2^2 + \mu_4^2 = \mu_1^2 + \mu_3^2 + \mu_5^2$

75. (2) There will be infinite incident rays from point B, when we made a hole, only reflected of this hole is absent but other reflected rays will be present.

76. **(1)** The phenomenon of total internal reflection takes place during reflection at P.

$$\sin\theta = \frac{1}{{}^{w}_{g}\mu} \qquad \text{...(i)}$$



where θ is the angle of incidence at P.

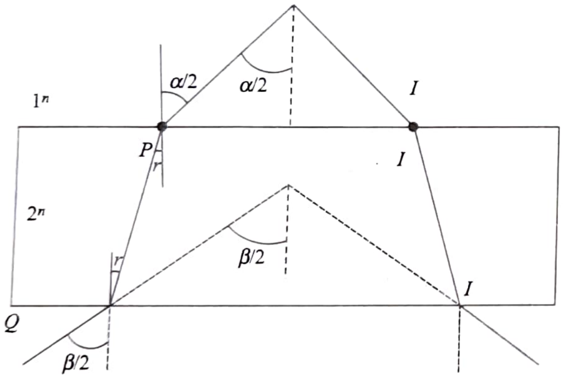
Now, $${}^{w}_{g}\mu = \frac{{}^{a}_{g}h}{{}^{a}_{g}\mu} = \frac{1.5}{4/3} = 1.125$$

Putting in (i), $\sin\theta = \frac{1}{1.125} = \frac{8}{9}$

Therefore, $\sin\theta$ should be greater than $\frac{8}{9}$

77. **(2)** The path of rays becomes parallel to initial direction as they emerge.

Applying Snell's law at P, $${}_{2}^{1}n = \frac{\sin\alpha/2}{\sin r} \qquad \text{...(i)}$$



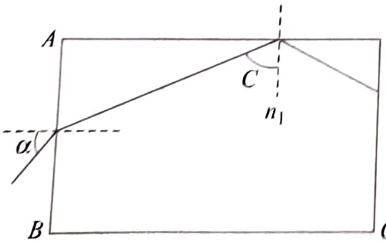
Applying Snell's law at Q, $${}_{2}^{1}n = \frac{\sin\beta/2}{\sin r} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), $a = b$

Therefore, (2) is the correct option.

78. **(1)** See figure. The ray will come out form CD if it suffers total internal reflection at surface AD, i.e., it strikes the surface AD at critical angle C (the limiting case). Applying Snell's law at P,

$n_l \sin C = n_2$ or $\sin C = \frac{n_2}{n_1}$



Applying Snell's law at Q, $n_2 \sin\alpha = n_l \cos C$;

$$\sin\alpha = \frac{n_1}{n_2}\cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}$$

or, $$\alpha = \sin^{-1}\left[\frac{n_1}{n_2}\cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}\right]$$

Hence, (1) is the correct option.

79. **(3)** The image I' for first refraction (i.e., when the ray comes out of liquid) is at a depth of $= \frac{33.25}{1.33} = 25\,\text{cm}$

$$\left[\because \text{Apparent depth} = \frac{\text{Real depth}}{\mu}\right]$$

Now, reflection will occur at concave mirror. For this I' behaves as an object

$\therefore\ u = -(15+25) = -40$ cm, $f = f, v = ?$

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{v} - \frac{1}{40} \Rightarrow v = \frac{40f}{40+f} \qquad \text{...(i)}$$

But $$v = \left[15 + \frac{25}{1.33}\right] \qquad \text{...(ii)}$$

where $\frac{25}{1.33}$ is the real depth of the image.

From Eqs. (i) and (ii),

$$\frac{40f}{40+f} = -\left[15 + \frac{25}{1.33}\right] = -33.79$$

$\Rightarrow \quad f = -18.31$ cm

80. **(2)** Critical angle from region III to region IV

$$\sin\theta_c = \frac{n_0/8}{n_0/6} = \frac{3}{4}$$

Now, applying Snell's law in region I and region III

$$n_0 \sin\theta = \frac{n_0}{6}\sin\theta_c$$

$$\sin\theta = \frac{1}{6}\sin\theta_c = \frac{1}{6}\left(\frac{3}{4}\right) = \frac{1}{8} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{8}\right)$$

Therefore, correct option is (2).

Multiple Correct Answers Type

1. **(3), (4)**

As long as the object moves towards the mirror, image moves away from the mirror (for $u > f$) and $m = -v/u$ $(v > u)$, so image size increases.

2. **(1), (3)**

(1) $n_3 < n_1$ so critical angle decreases. Therefore, TIR will take place at AB.

(2) $n_3 > n_1$, so critical angle increases. Therefore, TIR will not take place.

(3) If $n_3 > n_2$, then TIR will take place at CD and if $n_3 > n_1$, then TIR will take place at AB.

(4) TIR will not take place at face CD for $n_3 < n_1$ because ray is going from rarer to denser medium.

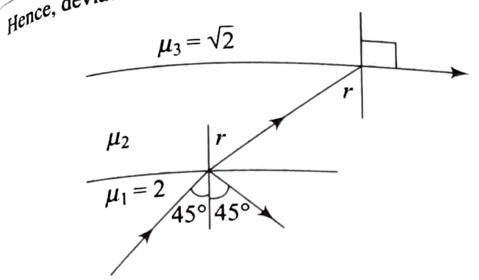
3. **(1), (2)**

For $\mu_2 > \sqrt{2}$ (TIR will not take place)

$2\sin 45° = \mu_2 \sin r$ and $\mu_2 \sin r = \sqrt{2}\,\sin e$

$\Rightarrow e = 90°$

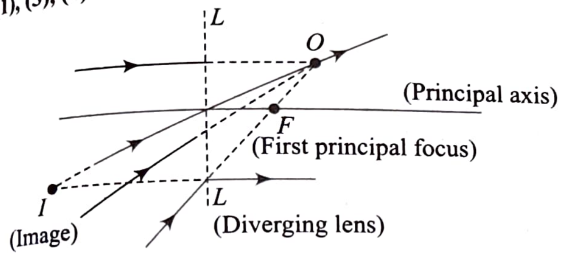
Hence, deviation is 45°



$\mu_2 < \sqrt{2}$. TIR will take place.

Deviation is 90°.

4. **(1), (3), (4)**



5. **(1), (2), (3)**

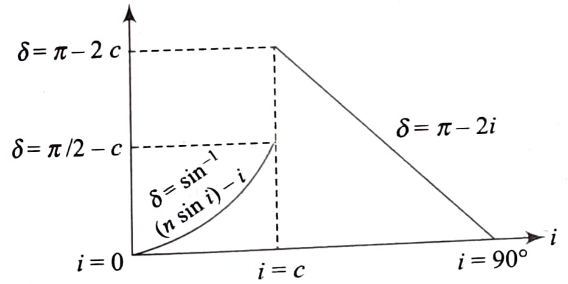
n for liquid = n for glass/yellow light

but n for liquid < n for glass (red light), so it will deviate towards base, for blue light n (liquid) > n (glass) so it will deviate towards vertex.

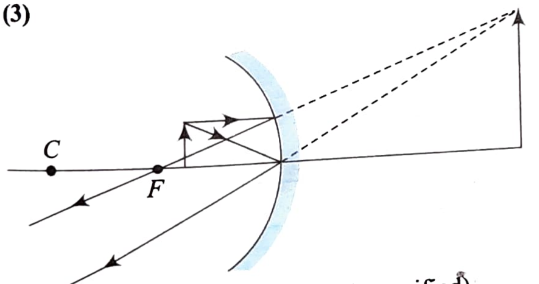
6. **(1), (3)**

The image of a point closer to the focus will be farther. As the transverse magnification of B will be more than A, the image of AB will be inclined to the optical axis.

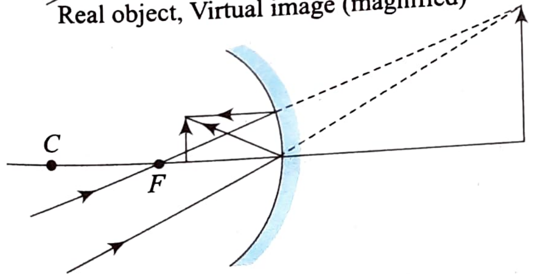
7. **(1), (2), (3), (4)**



8. **(2), (3)**



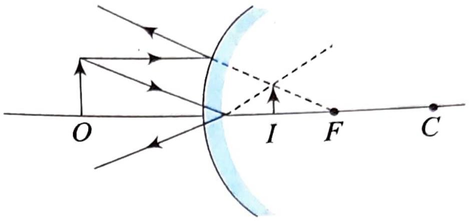
Real object, Virtual image (magnified)



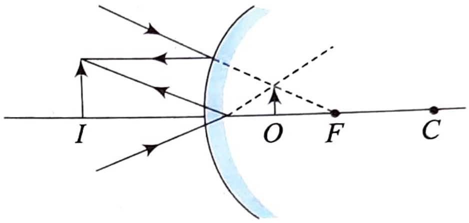
Virtual object, Diminished real image

9. **(1), (2), (3), (4)**

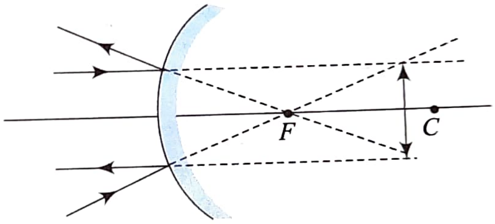
For a real extended object, image formed by convex mirror would be virtual, diminished, erect, and be lying between focus and pole.



For a virtual object, lying between focus and pole, image formed by a convex mirror would be real, magnified, erect, and lying in front of mirror.



For a virtual object beyond focus, image formed by a convex mirror would be virtual, inverted, magnified or diminished (depending on the location of object), and will lie on the same side of mirror.



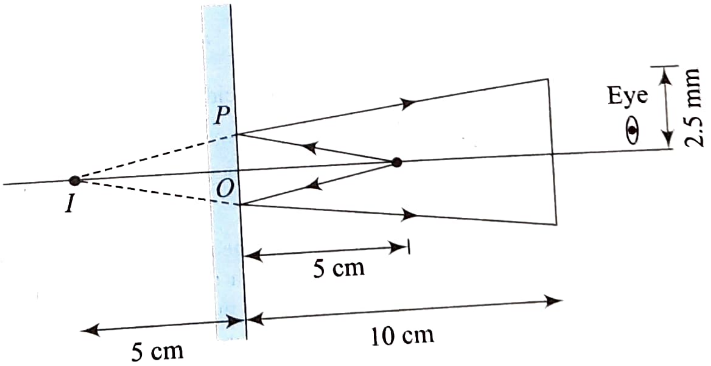
10. **(2), (3), (4)**

If the mirror is concave and a real object is approaching it, the image will move away from the mirror for object distance greater than focal length. If object distance is less than the focal length, virtual image will be formed which moves toward the mirror.

If mirror is convex, as object is approaching the mirror, image will move from focus to pole, i.e., toward the mirror.

11. **(1), (4)**

Here, $\dfrac{OP}{5\text{ cm}} = \dfrac{2.5\text{ mm}}{15\text{ cm}}$



$OP = \dfrac{5}{6}$ mm

Area of mirror used in reflection is $\pi \times OP^2 = \dfrac{25\pi}{36}$ mm²

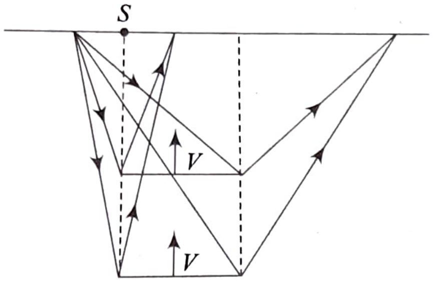
Let the power of source be P.

Intensity in absence of mirror is $I_1 = \dfrac{P}{4\pi \times (5)^2}$

Intensity in presence of mirror is $I_2 = \dfrac{P}{4\pi \times (5)^2} + \dfrac{P/2}{4\pi \times (5)^2}$

$$\frac{I_2}{I_1} = \frac{19}{18}$$

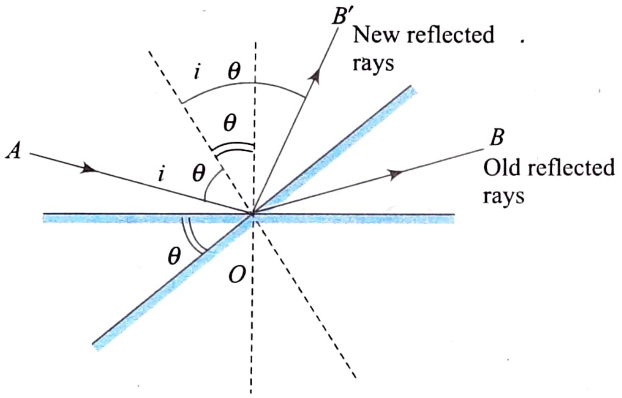
12. (2), (4)



From the above ray diagram, it is clear that the options (2) and (4) are correct.

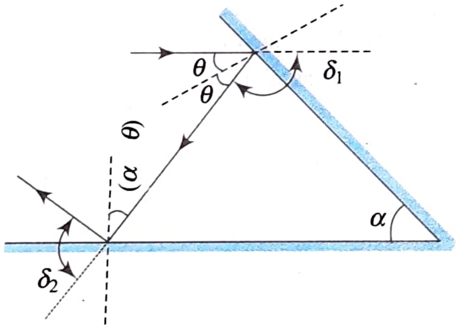
13. (1), (2), (3), (4)

(1) Angle $BOB' = \angle AOB' - \angle AOB' = 2i - (2i - 2\theta) = 2\theta$



(2) Total deviation $\delta = \delta_1 + \delta_2 = (180° - 2\theta) + 180° - 2(\alpha - \theta)$

$= 360° - 2\alpha$



which is independent of angle of incidence.

(3) Power of a plane mirror is zero.

(4) Velocity of the image toward the object $= v + v = 2v$.

14. (2), (3)

If an object is at a distance x from a plane refracting surface and is viewed normally, then it appears at a distance x/μ from the surface, where μ is refractive index of that medium (in which it is situated) with respect to the medium in which observer is situated.

Suppose height of bird from water surface is x and depth of fish from the surface is y, then depth of fish, as observed by the bird, will be equal to

$$r_1 = x + \frac{y}{\mu} \quad \text{...(i)}$$

where μ is refractive index of water with respect to air.

$r_1 = 6.60$ m

Hence, option (2) is correct.

Height of bird, as observed by the fish, will be equal to

$$r_2 = \frac{x}{\mu} + y \quad \text{...(ii)}$$

where μ' is refractive index of air with respect to water.

Now, $\mu' = \dfrac{1}{\mu} = \dfrac{3}{4}$

Hence, $r_2 = 8.80$ m

Hence, option (1) is wrong while option (3) is correct.

15. (2), (4)

In the previous question, velocity of fish, observed by the bird, will be equal to dr_1/dt. Therefore, differentiating Eq. (i), we get

$$\frac{dr_1}{dt} = \frac{dx}{dt} + \frac{1}{\mu}\frac{dy}{dt} = 4.50 \text{ m s}^{-1}$$

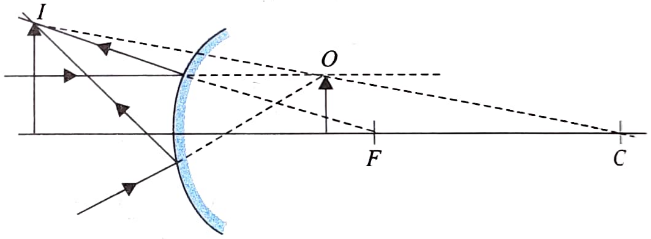
Hence, option (4) is correct. Similarly, velocity of bird, as observed by the fish, will be equal to $\dfrac{dr_2}{dt}$.

$$\frac{dr_2}{dt} = \frac{1}{\mu'}\frac{dx}{dt} + \frac{dy}{dt} = 6.0 \text{ m s}^{-1}$$

Hence, option (2) is correct.

16. (1), (3)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{1}{f} \Rightarrow y = \frac{xf}{x - f}$$

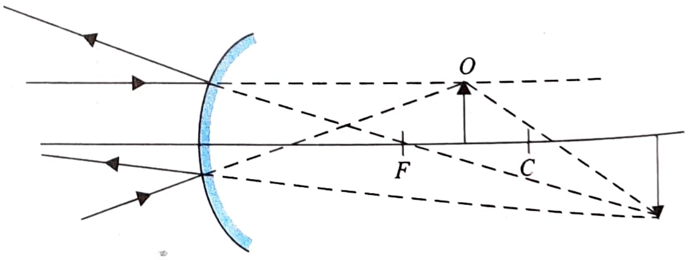


Since $x < f$, so y is negative and hence so image is real.

$m = -\dfrac{v}{u} = -\dfrac{y}{x} \rightarrow$ positive so image is erect.

Image will be enlarged.

17. (2), (4)



$y = \dfrac{xf}{x - f}$ [here $x > f$, so y is positive]

Hence image is virtual

$m = -\dfrac{v}{u} = -\dfrac{y}{x} \rightarrow$ negative, so image is inverted.

Image will be enlarged.

18. (2), (4)

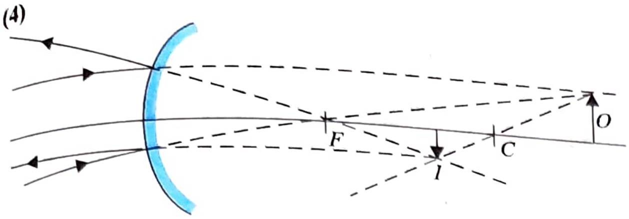


Image is virtual, inverted and diminished.

19. (2), (3)

Image of sun will be formed at the focus of mirror. Hence, focal length of mirror = 12 cm. To use it for shaving purpose, face should be between focus and pole, so that virtual image can be formed.

20. (1), (3)

$$\frac{1}{v}+\frac{1}{-u}=\frac{1}{f}\Rightarrow v=\frac{uf}{u+f} \qquad (v<u \text{ and } v<f)$$

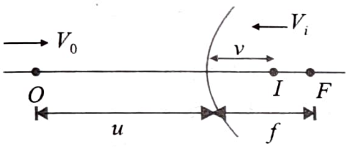
$$V_0=-\frac{du}{dt}, V_i=-\frac{dv}{dt}$$

Differentiating $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad -\frac{1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$$

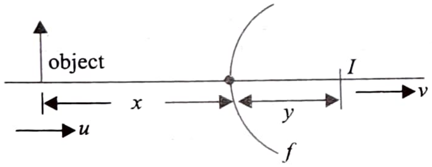
$$\Rightarrow \quad -\frac{1}{v^2}(-V_i)+\frac{1}{u^2}(-V_0)=0 \Rightarrow V_i=\left(\frac{v}{u}\right)^2 V_0$$

Since $v<u$, so $V_i<V_0$ for each case.



21. (1), (3)

$$\frac{1}{y}+\frac{1}{-x}=\frac{1}{f} \quad\Rightarrow\quad \frac{1}{y}=\frac{1}{x}+\frac{1}{f} \quad\Rightarrow\quad y=\frac{xf}{x+f}$$



$\Rightarrow \frac{y}{x}=\frac{f}{x+f}$ as x decreases, so magnification increases.

Now, $\frac{dy}{dt}=\left(\frac{y}{x}\right)^2\frac{dx}{dt} \quad\Rightarrow\quad v=-\left(\frac{y}{x}\right)^2 u$

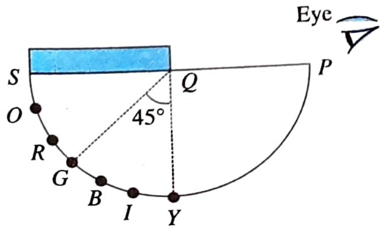
As $\frac{y}{x}$ increases, so v increases.

22. (2), (3)

Let C be the critical angle: $\sin C=\frac{1}{\mu}=\frac{1}{\sqrt{2}}$

$C=45°$

For Y, I and B, angle of incidence at a point on QP can be less than C, so they will be refracted from QP and can be seen by person.



But for G, R and O, angle of incidence at a point on QP will be more than C. Hence, they will be internally reflected.

23. (1), (3)

$$\text{From } \mu=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

We can see that $\delta_m=30°$ for ($\mu=\sqrt{2}$ and $A=60°$)

Further, at minimum deviation, $r_1=r_2=\frac{A}{2}=30°$

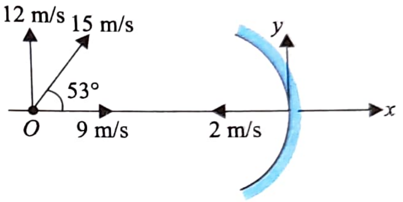
$$\sin i_1=\mu\sin r_1=(\sqrt{2})\sin 30°=\frac{1}{\sqrt{2}}$$

$\Rightarrow \quad i_1=45°$

24. (2), (3)

$\vec{v}_0$ = velocity of object $=(9\hat{i}+12\hat{j})\text{ m s}^{-1}$

$\vec{v}_m$ = velocity of mirror $=-2\hat{i}\text{ m s}^{-1}$



$$m=\frac{f}{f-u}=\frac{-20}{-20-(-30)}=-2$$

For velocity component parallel to optical axis,

$$(\vec{v}_{I/m})_{\parallel}=-m^2(\vec{v}_{O/m})_{\parallel}$$

$$(\vec{v}_{I/m})_{\parallel}=-(-2)^2 11\hat{i}=-44\hat{i}\text{ m s}^{-1}$$

For velocity component perpendicular to optical axis

$$(\vec{v}_{I/m})_{\perp}=m(\vec{v}_{O/m})_{\perp}=-(-2)12\hat{j}=-24\hat{j}\text{ m s}^{-1}$$

$\vec{v}_{I/m}$ = velocity of image with respect to mirror

$$\vec{v}_{I/m}=(\vec{v}_{I/m})_{\parallel}+(\vec{v}_{I/m})_{\perp}=(-44\hat{i}-24\hat{j})\text{ m s}^{-1}$$

Also, $\quad \vec{v}_{I/m}=\vec{v}_I-\vec{v}_m$

or $\quad \vec{v}_I=\vec{v}_{I/m}+\vec{v}_m=(-44\hat{i}-24\hat{j})-2\hat{i}=(-46\hat{i}-24\hat{j})\text{ m s}^{-1}$

25. (1), (2)

If image is virtual. Suppose $u=-x$, then $v=+2x$

From the mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \text{ or } \frac{1}{2x}-\frac{1}{x}=\frac{1}{-f}$$

or $\quad x=\frac{f}{2}$

If image is real, then suppose $u=-y$, then $v=-2y$

Again applying the mirror formula,

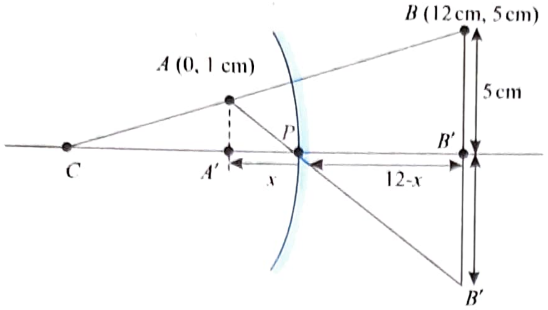
$$\frac{1}{-2y}-\frac{1}{y}=-\frac{1}{f} \text{ or } \frac{3}{2y}=\frac{1}{f} \quad\Rightarrow\quad y=\frac{3}{2}f$$

26. (1), (2), (3)

Join, object and image with a line that intersects the principal axis at point C. (centre of curvature of mirror)

From the similar triangles: $AA'P$ and $BB'P$.

$$\frac{1}{x}=\frac{5}{12-x}\Rightarrow x=2\text{ cm}$$



If A is the object and B is the image then, concave mirror having its pole at (2 cm, 0) must be placed.

Apply mirror formula:

$$\Rightarrow \quad \frac{1}{10}+\frac{1}{-2}=\frac{1}{f}\Rightarrow f=-2.5\,\text{cm}$$

If B is the object and A is the image then, convex mirror must be placed.

Apply mirror formula:

$$\Rightarrow \quad \frac{1}{2}+\frac{1}{-10}=\frac{1}{f}\Rightarrow f=2.5\,\text{cm}$$

27. (1), (2), (3), (4)

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Hence, $f=-\frac{R}{2}$

$$\Rightarrow \quad \frac{1}{v}-\frac{3}{5R}=\frac{-2}{R}$$

$$\frac{1}{v}=\frac{-2}{R}+\frac{3}{5R}=-\frac{7}{5R}\Rightarrow v=-\frac{5R}{7}$$

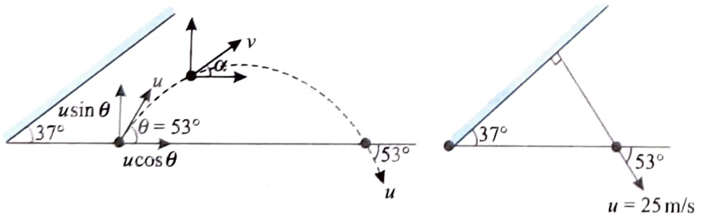
$$m=\frac{I}{O}=-\frac{v}{u}=-\left(\frac{-5R/7}{-5R/3}\right)=-\frac{3}{7}$$

Hence amplitude of oscillation of image $=\frac{3}{7}$ mm

As image is inverted hence phase difference of oscillation is π.

28. (1), (2), (3), (4)

Relative speed will be minimum (zero) when velocity of object will be parallel to plane mirror.



$$\tan\alpha=\frac{u\sin\theta-gt}{u\cos\theta}$$

$$\frac{3}{4}=\frac{25.\frac{4}{5}-10t}{25.\frac{3}{5}}$$

$$\frac{3}{4}=\frac{20-10t}{15}\Rightarrow 45=80-40t$$

$$40t=35\Rightarrow t=\frac{7}{8}\text{s}$$

At maximum relative speed the object moves normal to mirror. And it occurs when the particle hits the ground. Hence, maximum relative speed = 50 m/s.

29. (2), (3)

$$u=-\frac{R}{4};\ f=-\frac{R}{2};\ v=\frac{uf}{u-f}=+\frac{R}{2}$$

$\Rightarrow$ Image is at a distance $R/2$ to the right of the mirror. Its virtual and upright.

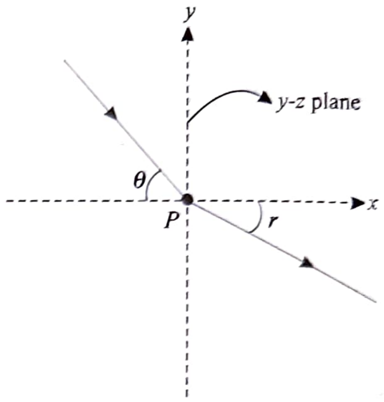
30. (2), (4)

$$|3\sqrt{2}\hat{i}-3\hat{j}-3\hat{k}|=\sqrt{(3\sqrt{2})^2+(3)^2+(3)^2}=6$$

Angle between normal x-axis and incidence ray is

$$\left(3\sqrt{2}\hat{i}-3\hat{j}-3\hat{k}\right)\cdot\left(\hat{i}\right)=6\cdot 1\cdot\cos\theta$$

$$\Rightarrow \quad \frac{3\sqrt{2}}{6}=\cos\theta \text{ or } \cos\theta=\frac{1}{\sqrt{2}}$$



Applying Snell's law at P, $\sin\theta=\mu\sin r$

$$1\cdot\frac{1}{2}=\sqrt{2}\sin r \Rightarrow \sin r=\frac{1}{2}\Rightarrow r=30^\circ$$

Let the angle made by refracted ray with y-axis and z-axis is α

Then, $\cos^2\alpha+\cos^2\alpha+\cos^2 r=1$

$$2\cos^2\alpha=1-\frac{3}{4}\Rightarrow\cos\alpha=\frac{1}{\sqrt{8}}$$

$$\hat{u}_{\text{refracted}}=\cos r\,\hat{i}-\cos\alpha\,\hat{j}-\cos\alpha\,\hat{k}$$

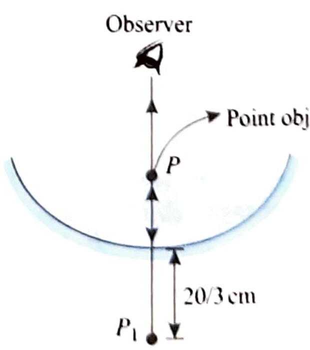
$$\hat{u}_{\text{refracted}}=\cos 30^\circ\hat{i}-\frac{1}{\sqrt{8}}\hat{j}-\frac{1}{\sqrt{8}}\hat{k}=\frac{\sqrt{3}}{2}\hat{i}-\frac{1}{2\sqrt{3}}\hat{j}-\frac{1}{2\sqrt{2}}\hat{k}$$

So unit vector along refracted ray, $\hat{u}_{\text{refracted}}=\frac{\sqrt{6}\hat{i}-\hat{j}-\hat{k}}{\sqrt{8}}$

31. (1), (3)

Image formation due to reflection from concave mirror

Here, $u=-5\,\text{cm}$, $f=-20\,\text{cm}$



Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

or $\quad \frac{1}{5}-\frac{1}{20}=\frac{20}{3}\text{cm}$

$\therefore \quad v=\frac{20}{3}\text{cm}$ i.e., behind the mirror.

The image P_1 formed due to reflection through concave mirror behaves as object for refraction through water surface. For water surface,

$$\mu_1=\frac{4}{3},\mu_2=1,u=-\left(5+\frac{20}{3}\right)=-\frac{35}{3}\text{cm}$$

$$\because\quad \frac{\mu_2}{\mu_1}=\frac{v}{u}$$

$$\therefore\quad v=\frac{\mu_2}{\mu_1}u=\left(\frac{1}{\frac{4}{3}}\right)\left(\frac{-35}{3}\right)$$

$$\because\quad v=\frac{3}{4}\times\left(\frac{-35}{3}\right)=\frac{-35}{4}=-8.75\,\text{cm}$$

32. **(1), (3)**

The object shift due to slab is $x=t\left(1-\frac{1}{\mu}\right)=12\left(1-\frac{3}{4}\right)=3\,\text{cm}$

This shift should be in the direction of ray travelling.

The image formed due to refraction through slab behaves as object for concave mirror.

Now image formation by concave mirror, $u=-60\,\text{cm}$, $f=-20\,\text{cm}$

According to mirror formula,

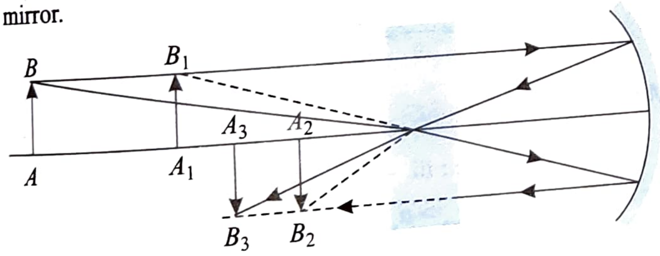
$$\because\quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\therefore\quad v=-30\,\text{cm}$$

$$m=\frac{I}{O}=-\frac{v}{u}=-\frac{(-30)}{-60}=-\frac{1}{2}$$

$\therefore\quad I=-1\,\text{cm}$ (i.e., inverted)

This image (I) behaves as virtual object for second times refraction through slab. As shifting should be in the direction of ray travelling. Hence, the final image I' is 3 cm leftward from I due to second times refraction through slab. Thus, final image is virtual, 1 cm high inverted and 33 cm from the concave mirror.



33. **(1), (3)**

(1) From figure at D, $\theta+30°=90°\Rightarrow\theta=60°$

$\therefore\quad \theta=60°$

Hence angle $\angle CDE=90°-\theta=90°-60°=30°$

$\therefore\quad \angle CED=180°-(30°+30°)$

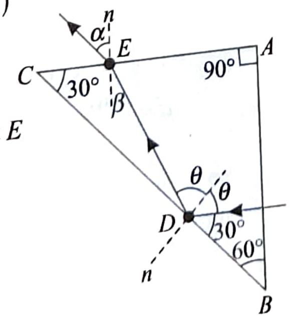
$\therefore\quad \angle CED=120°$

$\therefore\quad \beta=120°-90°=30°$

According to Snell's law at part E

$$\frac{5}{3}\sin\beta=\frac{4}{3}\times\sin\alpha$$

$$\therefore\quad \sin\alpha=\frac{5}{4}\times\frac{1}{2}=\frac{5}{8}$$



(3) Here, $\sin C=\dfrac{5}{2\sqrt{3}\times\frac{5}{3}}=\dfrac{\sqrt{3}}{2}$

$\therefore\quad C=60°$

But $\theta=60°$

Hence, total internal reflection does not take place.

34. **(3), (4)**

For total internal reflection to take place:

Angle of incidence, $i>$ critical angle, $\theta_c\left[\text{where}\sin\theta_c=\frac{1}{n}\right]$

or $\quad \sin 45°>\frac{1}{n}$ or $\frac{1}{\sqrt{2}}>\frac{1}{n}$ or $n>\sqrt{2}$ or $n>1.414$

Therefore, possible values of n can be 1.5 or 1.6 in the given options.

Linked Comprehension Type

For Problems 1–2

1. (2), 2. (1)

The normal shift produced by a glass slab is

$$\Delta x=\left(1-\frac{1}{\mu}\right)t=\left(1-\frac{2}{3}\right)(6)=2\text{ cm}$$

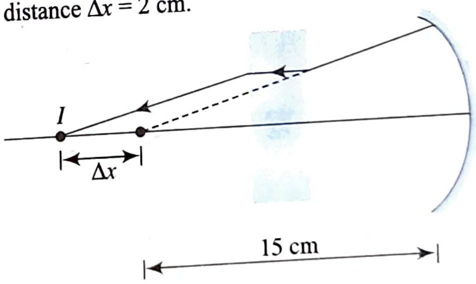
i.e., for the mirror, the object is placed at a distanced $(32-\Delta x)=$ 30 cm from it.

Applying mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\quad\text{or}\quad \frac{1}{v}+\frac{1}{30}=-\frac{1}{10}$$

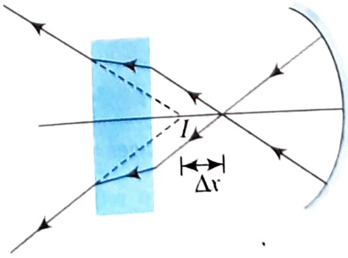
or, $\quad v=-15$ cm

(i) When x = 5 cm: The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance $\Delta x=2$ cm.



Hence, the final real image is formed at a distance $(15+2)=$ 17 cm from the mirror.

(ii) When x = 20 cm: This time also the final image is at a distance of 17 cm from the mirror but it is virtual as shown.



For Problems 3–5

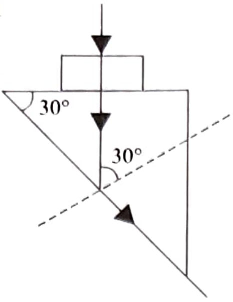
3. (4) $\sin C=\dfrac{\mu}{\mu_0}\Rightarrow\sin[90°-r]=\dfrac{\mu}{\mu_0}\qquad[\because C+r=90°]$

$$\Rightarrow\quad \cos r=\frac{\mu}{\mu_0}\qquad\text{...(i)}$$

$$\mu_0=\frac{\sin i}{\sin r}\Rightarrow\sin r=\frac{\sin i}{\mu_0}\qquad\text{...(ii)}$$

From (i) and (ii) $\left(\frac{\mu}{\mu_0}\right)^2 + \left(\frac{\sin i}{\mu_0}\right)^2 = 1$

$\Rightarrow \quad \mu = \sqrt{\mu_0^2 - \sin^2 i}$



4. **(3)** If ray just fails to emerge, $i = 90°$,

given $\mu_0 = \sqrt{2}$

$$\mu = \sqrt{(\sqrt{2})^2 - \sin^2 90°} = 1$$

5. **(3)** Here $30° > C$

$\Rightarrow \quad \sin 30° > \sin C$

$\Rightarrow \quad \frac{1}{2} > \frac{1}{\mu_0} \Rightarrow \mu_0 > 2$

For Problems 6–8

6. **(1)** $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} + \frac{1}{-30} = \frac{1}{-20} \Rightarrow v = -60$ cm

7. **(4)** Let $u = -x$, $v = -y$

$\frac{dx}{dt} = -2 \text{ cm s}^{-1}; \frac{1}{-y} + \frac{1}{-x} = \frac{1}{f}$

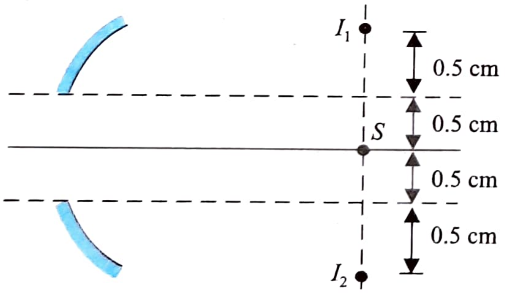
$\Rightarrow \frac{1}{y^2}\frac{dy}{dt} + \frac{1}{x^2}\frac{dx}{dt} = 0$

$\Rightarrow \frac{dy}{dt} = -\left(\frac{y}{x}\right)^2 \frac{dx}{dt} \Rightarrow v_I = -\left(\frac{60}{30}\right)^2 (-2) = 8 \text{ cm s}^{-1}$

8. **(1)** $m = -\frac{v}{u} = -\frac{-60}{-3} = -2$; so image will move twice distance in same time as compared to object. Hence, velocity of image will be double.

For Problems 9–11

9. **(4)** Using $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \Rightarrow \quad v = -50$ cm



and magnification $= \frac{-v}{u} = -1$

So distance between two images = 2 cm.

10. **(2)** For half the mirror, due to reflection being half of the incoming rays, so the intensity of the image is obviously half. More the lateral separation, more will be the gap between the two images formed.

11. **(4)** Both the mirrors will be having the common object at their centre of curvature. Hence the image position also coincides.

For Problems 12–13

12. **(3)** **13.** **(4)**

(i) Given: $u = -30$ cm, $f = 20$ cm

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad v = 12$ cm

Also, $\frac{dv}{dt} = -\left(\frac{v^2}{u^2}\right)\frac{du}{dt} = -0.16 \text{ cm s}^{-1}$

(ii) $H = -\frac{v}{u}h$, H is the height of image and h is height of object.

$$\frac{dH}{dt} = \frac{u\frac{dv}{dt} - v\frac{du}{dt}}{u^2}h = 0.008 \text{ cm s}^{-1}$$

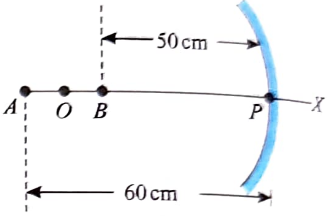
For Problems 14–16

14. **(3)** For extreme position A, $u = -60$ cm, $f = -20$ cm

$\because \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

or $\quad \frac{1}{v} - \frac{1}{60} = -\frac{1}{20}$

or $\quad v = -30$ cm



For extreme position B,

$u' = -50$ cm

$f' = -20$ cm

$\because \quad \frac{1}{v'} - \frac{1}{50} = \frac{1}{-20}$

or $\quad \frac{1}{v'} = \frac{1}{50} - \frac{1}{20} = \frac{2-5}{100}$

$\therefore \quad v' = -\frac{100}{3}$ cm $= -33.33$ cm

$\therefore$ The range of vibration of image is

$A'B' = |v' - v| = (33.33 - 30)$ cm $= 3.33$ cm

15. **(4)** The maximum distance of image from mirror is at that time when object is at extreme position B.

16. **(2)** The time period vibration of image = time period of vibration of object $= \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = 0.2$ s

Matrix Match Type

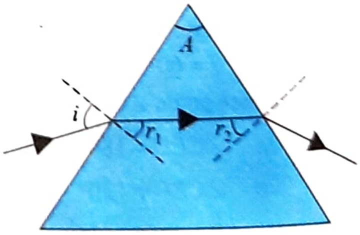
1. **i. → a.; ii. → c.; iii. → b.; iv. → d.**

$\mu_{\text{blue}} > \mu_{\text{green}} > \mu_{\text{yellow}} > \mu_{\text{red}}$

2. **i. → b.; ii. → c., d.; iii. → a.; iv. → c., d.**

Here critical angle C;

$\sin C = \frac{1}{\mu} = \frac{1}{2}$



$\Rightarrow \quad C = 30°$

Let $r_1 = c = 30°$,

$r_1 = A - r_2 = A - 30°$

$\mu = \frac{\sin i}{\sin r_1}$

$\Rightarrow \quad \sin i = \mu \sin r_1 = 2\sin(A - 30°)$

for $i = 90°$, we get $A = 60°$

So for $A = 60°$: for $i = 90°$, we have

$r_2 = C = 30°$

But if $i < 90°$, $r_2 > C$

Hence all the rays are reflected from second surface.

For $A > 60°$: for $i = 90°$, we will get $r_2 = C$

and also for $i < 90°$, we will get $r_2 > C$.

So, all rays will be reflected from second surface.

Similarly for $i = 0$, we get $A = 30°$ and $r_2 = 30° = C$

So, $r_2 < C$. Hence all the rays are refracted from second surface.

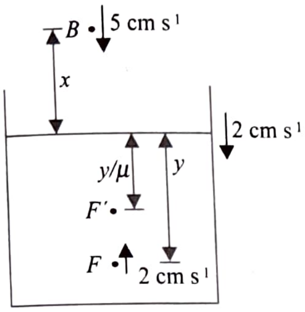
For $30° < A < 60°$, some rays will be reflected and some refracted.

3. **i. → b.; ii. → c.; iii. → a.; iv. → d.**

i. We can write

$$\frac{dx}{dt} = -(5-2) = -3 \text{ cm/s}$$

$$\frac{dy}{dt} = -(2+2) = -4 \text{ cm/s}$$

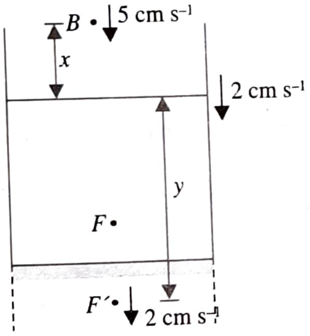


Speed of fish w.r.t. bird

$$= \frac{d}{dt}\left[x + \frac{y}{\mu}\right] = \frac{dx}{dt} + \frac{1}{\mu}\frac{dy}{dt} = -3 + \frac{1}{4/3}(-4) = -6 \text{ cm/s}$$

ii. Here $\frac{dy}{dt} = 0$, so speed of image of fish (F') w.r.t. bird will be

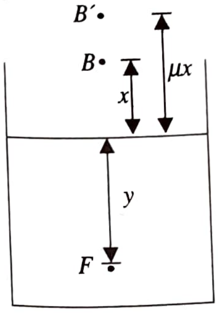
$$\frac{dx}{dt} = -3 \text{ m s}^{-1}$$

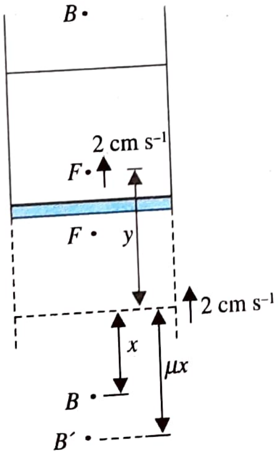


iii. Speed of bird relative to fish $= \frac{d}{dt}(\mu x + y)$

$$= \mu\frac{dx}{dt} + \frac{dy}{dt} = \frac{4}{3}(-3) - 4 = -8 \text{ cm s}^{-1}$$

iv. Here $\frac{dy}{dt} = 0$





$$\frac{d}{dt}(\mu x + y) = \mu\frac{dx}{dt} + \frac{dy}{dt} = \frac{4}{3}(-3) + 0 = -4 \text{ cm s}^{-1}$$

4. **(1)** Virtual object means initial rays converse behind the mirror for which u is positive. We have mirror formula as $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

For concave mirror, f is negative. So, v is always negative. Therefore, image is always real.

$$\frac{1}{v} = \frac{1}{(-f)} - \frac{1}{(+u)}$$

For convex mirror, f is positive. So, v may be positive or negative.

$$\frac{1}{v} = \frac{1}{(+f)} - \frac{1}{(+u)}$$

Hence, image may be virtual or real.

For plane mirror, $f = \infty$. So, v is always negative.

Hence, image is always real.

$$\frac{1}{v} = \frac{1}{(\infty)} - \frac{1}{(+u)}$$

5. **(1)** (i) and (ii)

$$v = \frac{\mu_2}{\mu_1}u$$

$\Rightarrow$ $|v| < |u|$ as $\mu_1 > \mu_2$

i.e. v and u are of same sign.

Or they are on same side of plane surface.

From plane surface, if object is real, image is virtual and vice-versa.

(iii) and (iv)

$$v = \frac{\mu_1}{\mu_2}u \Rightarrow |v| > |u|$$

Other explanations are same.

6. **(2)**

(i) $f = -15$ cm, $u = -30$cm. $V = -30$cm

$\frac{du}{dt} = 0$ $\quad\Rightarrow \frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt} = 0$

Velocity of image with respect to mirror = 0

Velocity of image with respect to mirror = 2 cm/s

(ii) $f = -15$cm, $\Rightarrow u = -30$ cm, $v = -30$ cm

$\frac{du}{dt} = -4$ $\quad\Rightarrow \frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt} = +4$

Velocity of image with respect to mirror = + 4 cm/s

Velocity of image with respect to ground = +6 cm/s

(iii) $f = +20$cm, $u = -20$cm, $v = +10$ cm

$\frac{du}{dt} = +4$ $\quad\Rightarrow \frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt} = -1$ cm/s

Velocity of image with respect to mirror = – 1 cm/s

Velocity of image with respect to ground = – 1 cm/s

(iv) $f = +20$ cm, $u = -20$ cm, $v = +10$ cm

$\frac{du}{dt} = +4$ $\quad\Rightarrow \frac{dv}{dt} = -1$ cm/s

Velocity of image w.r.t. mirror = –1 cm/s

Velocity of image w.r.t. ground = –1 cm/s

7. **(2)** The magnification in car of a mirror, $m = \frac{f}{f-u}$

(i) Convex mirror $u = -\text{ve}$, $f = +\text{ve}$

$$m = \frac{f}{f-(f)} = \frac{1}{2}$$

(ii) Concave $\rightarrow m = \frac{-f}{-f-(-f)} = \infty$

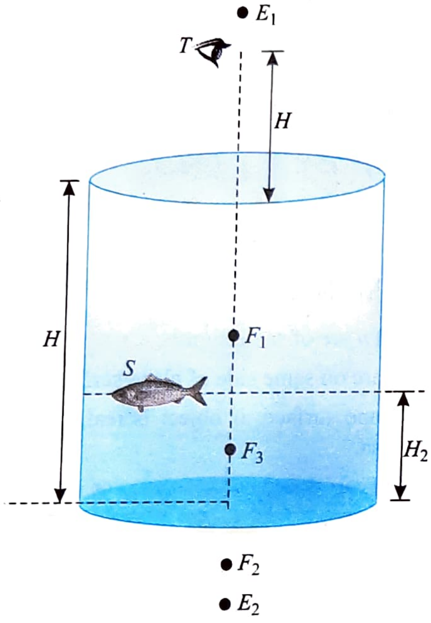
(iii) Concave mirror $u = -R\ (2f)$

$$m = \frac{-f}{-f-(-2f)} = \frac{-f}{f} = -1$$

(iv) Convex mirror $u = -2f$

$$m = \frac{f}{f-(-2f)} = \frac{f}{3f} = \frac{1}{3}$$

8. **(4)**



Fish observing the image of eye:

For d_1: E_1 at $\left(\frac{H}{2} + \mu H\right) = H\left(\mu + \frac{1}{2}\right)$

For d_2: E_2 at $\left(\mu H + H + \frac{H}{2}\right) = H\left(\mu + \frac{3}{2}\right)$

Eye observing the image of fish:

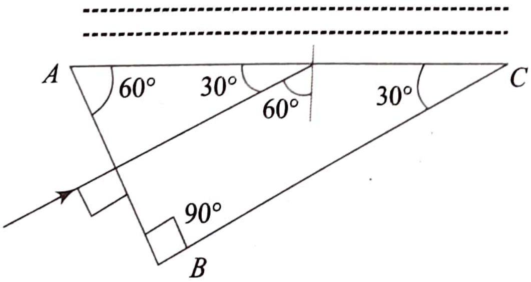
For d_3: F_1 at $\left(\frac{H}{2\mu} + H\right) = H\left(\frac{1}{2\mu} + 1\right)$

For d_4: F_2 at $\left(\frac{3H}{2\mu} + H\right) = H\left(1 + \frac{3}{2\mu}\right)$

Numerical Value Type

1. **(3)** Critical angle between glass and liquid.

$$\sin\theta_C = \frac{\mu}{(3/2)} = \frac{2\mu}{3}$$



Angle of incidence on $AC = 60°$

For TIR, $i > \theta_C \Rightarrow \frac{2}{3}\mu$

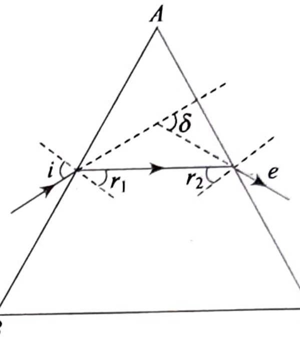
$$\mu < \frac{3\sqrt{3}}{4} = \frac{I\sqrt{3}}{4} \quad \text{(given)} \quad \text{So, } I = 3$$

2. **(3)** Given $i = 60°$, $\delta = 30°$ and $A = 30°$.

We have $\delta = i + e - A$

From Eq. (i), we get $30° = 60° + e - 30°$ or $e = 0$

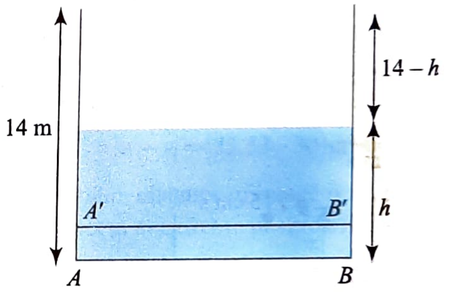
So r_2 is also zero, then $r_1 = A = 30°$



So $\mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$

Hence the value of $a = 3$.

3. **(8)** $A'B'$ is the apparent position of bottom of container, at a distance h/μ from water surface.

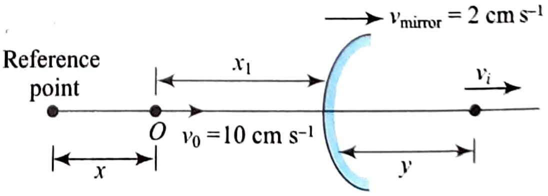


If the container seems to be half filled. Then

$$14 - h = \frac{h}{\mu} \Rightarrow 14 - h = \frac{3h}{4} \Rightarrow h = 8\text{ m}$$

4. **(0)** $\frac{1}{y} + \frac{1}{-x_1} = \frac{1}{10}$ ...(i)

$$\frac{1}{y} - \frac{1}{10} = \frac{1}{10} \Rightarrow y = 5\text{ cm}$$



$v_{\text{mirror}} = 2\text{ cm s}^{-1}$
Reference point
x_1
v_i
O
$v_0 = 10\text{ cm s}^{-1}$
x
y

$$\frac{dx}{dt} = 10\text{ cm s}^{-1}, \quad \frac{d(x + x_1)}{dt} = 2\text{ cm s}^{-1}$$

$$\frac{dx_1}{dt} = -8\text{ cm s}^{-1}$$

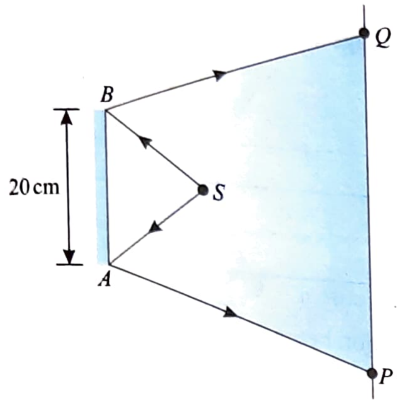
$$\frac{dy}{dt} = v_i = 2$$

From (i): $\frac{dy}{dt} = \left(\frac{y}{x_2}\right)^2 \frac{dx_1}{dt}$

$$\Rightarrow v_i - 2 = \left(\frac{5}{10}\right)^2 (-8) \Rightarrow v_i = 0$$

5. **(6)** Insect can see the image of source S in the mirror, so far as it remains in field of view of image overlapping with the road.

Shaded portion is the field of view, which overlaps with the road up to length PQ.



By geometry we can see that, $PQ = 3AB = 60$ cm

$$\therefore \quad t = \frac{\text{Distance}}{\text{Speed}} = \frac{60}{10} = 6\text{ s}$$

6. **(21)**

Applying $d_{app} = \dfrac{d}{\mu}$

$$d = (\mu) d_{app}$$

$$\therefore \quad d_1 = (1.5)(6) = 9\text{ cm}$$

$$d_2 = (1.5)(8) = 12\text{ cm}$$

Therefore, actual thickness of the glass slab is

$$d_1 + d_2 = 21\text{ cm}$$

7. **(1.95)**

For middle prism, $r + r = A$

$$2r = 60° \Rightarrow r = 30°$$

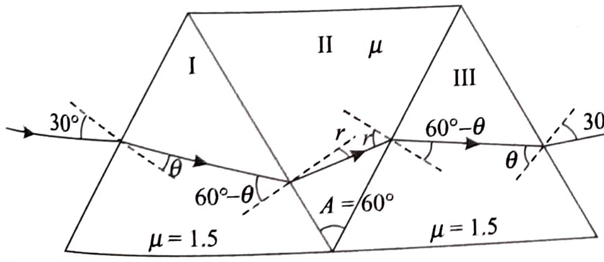
Refraction from Ist surface: $1 \sin 30° = 1.5 \sin\theta$

$$\Rightarrow \sin\theta = \frac{1}{3} \text{ and } \cos\theta = \frac{\sqrt{8}}{3}$$

Refraction from 2nd surface: $1.5\sin(60 - \theta) = \mu \sin 30°$

$$1.5[\sin 60° \cos\theta - \cos 60° \sin\theta)] = \mu \sin 30°$$

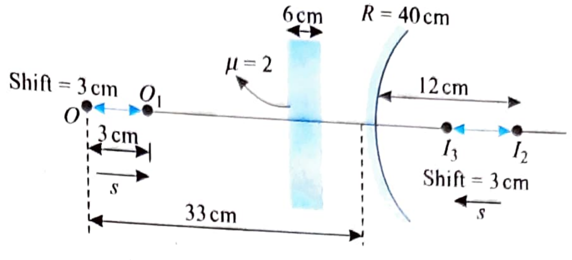
$$\Rightarrow \mu = \sqrt{6} - \frac{1}{2} = \frac{2\sqrt{6} - 1}{2} = 1.95$$



8. **(42)**

I_1 = image formed by the glass slab through the formula of shift
I_2 = image of I_1 formed by the reflection by the convex mirror.
= image of I_2 formed by the refraction through glass slab.

Shift by glass slab in air:



$$\text{Shift, } S = t\left(1 - \frac{1}{\mu}\right) = 6\left(1 - \frac{1}{2}\right) = 3\text{cm}$$

Shift will be in the direction of ray travelling will be zero. Hence for mirror the object distance will be 30 cm.

Apply mirror formula $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ (For calculations of I_2)

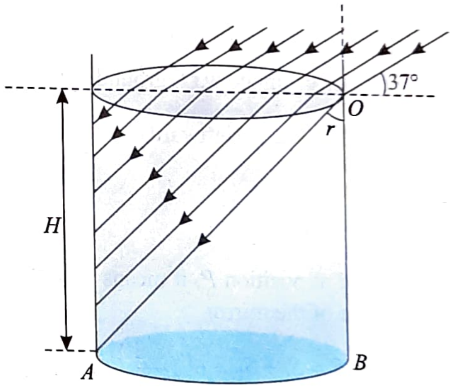
$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{20} \quad (u = -30\text{cm}; f = 20\text{cm, convex mirror})$$

$$\Rightarrow \quad v = +12 \text{ cm}$$

+12 indicates the position of I_2 as shown in figure. Again the rays will pass through slab and produce shift of I_2 towards left.

So, $OI_3 = 33 + 9 = 42$ cm

9. **(4)** By Snell's law, $1 \sin 53° = \dfrac{4}{3} \sin r$,



$$\frac{4}{5} = \frac{4}{5}\sin r, \sin r = \frac{3}{5} \Rightarrow r = 37°$$

In ΔOAB, $\tan r = \dfrac{3}{H} = \tan 37° \Rightarrow H = 4$ metre

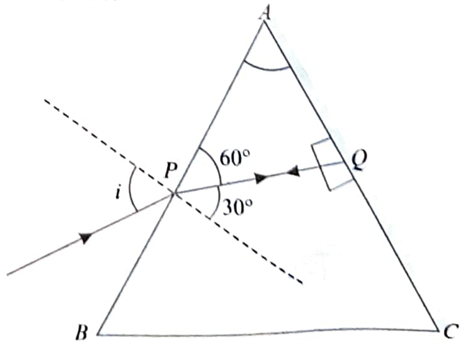
10. **(45)**

Refracting angle = angle of prism = 30° (given)

For any light ray to retrace its path, it must fall normally on the side AC (plane mirror).

By geometry, $\angle APQ = 60°$ $\therefore r = 30°$ (by geometry)

Using Snell's law:



$$1 \times \sin i = \mu \sin 30° \Rightarrow 1 \times \sin i = \sqrt{2} \times \frac{1}{2}$$

$$\sin i = \frac{1}{\sqrt{2}} \Rightarrow i = 45°$$

11. **(3)** Magnification, $m = \frac{f}{f-u}$

Here $f = -f$ and $u = -\frac{3}{2}f$

Hence $m = \frac{-f}{-f+\frac{3f}{2}} \Rightarrow m = -2$

Taking the direction in the right-hand side

$$V_{im} = -m^2 V_{om}$$

Here: V_{im} = velocity of image w.r.t. mirror

V_{om} = velocity of object w.r.t. mirror

$\Rightarrow V_{im} = -4u$

As no external force acting on system 'man + platform' in horizontal direction the linear momentum of the system should always be zero.

$$m(u+V_m)+2mV_m = 0$$

Velocity of platform (mirror)

$$V_m = -\frac{mu}{(m+2m)} = -\frac{u}{3} \text{ (or leftward)}$$

$$\vec{V}_{im} = \vec{V}_i - \vec{V}_m \Rightarrow \vec{V}_i = \vec{V}_{im} + \vec{V}_m$$

Hence, V_i = velocity of image w.r.t. ground

$$\Rightarrow \vec{V}_i = -4u - \frac{u}{3} = -\frac{13}{3}u \text{ (leftward)}$$

$$\Rightarrow V_i = 3 \text{ m/s}$$

12. **(60)**

If image is formed at position P, it means point P should be the centre of curvature of the mirror.

Shift due to slabs $S = S_1 + S_2 = 6\left(1-\frac{2}{3}\right)+5\left(1-\frac{4}{5}\right) = 3\,\text{cm}$

The shift should be in the direction of ray travelling, hence the ray should conserve 17 + 3 = 20 cm from the right face of the slab (2).

Hence $= 17+3+2\times 20 = 60\,\text{cm}$

13. **(12.5)**

Initially observer is mirror for image formation of object O. As O is kept at the bottom, then it sees somewhat above the bottom.

The distance of the object as seen from plane mirror

$$d_1 = 5+\frac{10}{\mu} \quad \Rightarrow d_1 = 5+\frac{10\times 3}{4} = 12.5 \text{ cm (in front)}$$

Hence image from the mirror after reflection is also at 12.5 cm (behind the mirror).

14. **(2)** Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Here $u = -10\,\text{cm}$, $f = -20\,\text{cm}$

Hence $\frac{1}{v}+\frac{1}{(-10)} = \frac{1}{(-20)} \Rightarrow v = 20\,\text{cm}$

The magnification, $m = \frac{I}{O} = -\frac{v}{u}$

From M_1: $O = -1\,\text{mm}, u = -10\,\text{cm}, v = 20\,\text{cm}$

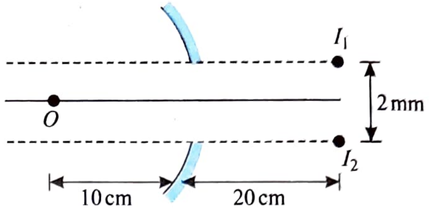
$$\frac{I_1}{(-1\,\text{mm})} = -\frac{(+20\,\text{cm})}{(-10\,\text{cm})} \Rightarrow I_1 = -2\,\text{cm}$$

Hence the image from M_1 will form 2 mm below principal axis behind the mirror.

Similarly, for M_2

$$\frac{I_2}{(+1\,\text{mm})} = -\frac{(+20\,\text{cm})}{(-10\,\text{cm})} \Rightarrow I_2 = +2\,\text{cm}$$

Hence the image from M_2 will form 2 mm above the principal axis.



Hence separation between the images will be 2 mm.

15. **(80)**

Position of the ball 4 sec after dropping,

$$h = 15+20\times 4-\frac{1}{2}\times 10\times 4^2 = 15\,\text{m}$$

It means at this time the ball is at height 15 m from ground and moving down with speed 20 m/s.

Now using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Here $u = -15\,\text{cm}$, $f = -10\,\text{cm}$

$$\frac{1}{v}+\frac{1}{-(15)} = \frac{1}{(-10)} \Rightarrow v = -30\,\text{m}$$

The velocity of image, $V_{\text{image}} = -\left(\frac{v}{u}\right)^2 V_{\text{object}}$

$$\Rightarrow V_{\text{image}} = -\left(\frac{-30}{-15}\right)^2 \times(-20) = 80\,\text{m/s}$$

16. **(22.50)**

The apparent distance (x) of the fly from the interface of the slab and water

Using; $\frac{h_1}{\mu_1} = \frac{h_2}{\mu_2} \Rightarrow \frac{45}{3/2} = \frac{x}{4/3} \Rightarrow x = 40$ cm

Hence the distance of the fly from the mirror,

$$x_{fly} = 20+40 = 60\,\text{cm}$$

Using mirror formula: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}+\frac{1}{(-60)} = \frac{1}{(-20)} \Rightarrow v = -30\,\text{cm}$$

Hence the image will form at a height 30 cm from mirror, or 10 cm from lower surface of the slab.

Now considering separation from lower surface of the slab.

Using, $\frac{h_1}{\mu_1} = \frac{h_2}{\mu_2} \Rightarrow \frac{10}{4/3} = \frac{x_1}{3/2} \Rightarrow x_1 = \frac{45}{4}\text{cm}$

Hence the image will form at a distance $\frac{45}{4}$cm above the lower surface of the slab.

Now finally considering refraction from upper surface of the slab

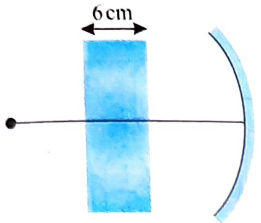
Using, $\frac{h_1}{\mu_1} = \frac{h_2}{\mu_2} \Rightarrow \frac{\left(45 - \frac{45}{4}\right)}{3/2} = \frac{x_1}{1}$

$\Rightarrow \quad x_2 = 45 \times \frac{3}{4} \times \frac{2}{3} = \frac{45}{2} \text{cm} = 22.50 \text{cm}$

17. (42)

Shift due to glass slab,

$$S = 6\left(1 - \frac{1}{3/2}\right) = 6 \times \frac{1}{3} = 2 \text{ cm}$$

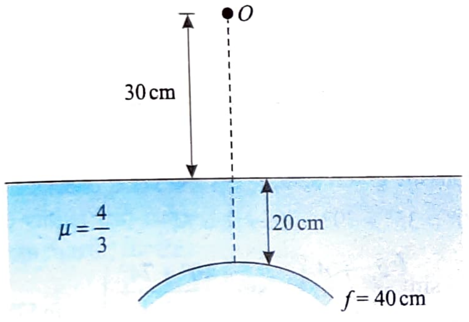


This shift will be in the direction of ray travelling or in rightward direction.

Reflected final image coincides with the object when the effective distance of the object is equal to radius of curvature of mirror $u = 40 + 2 = 42$ cm.

18. (3)

Image formed by partial reflection from water surface at a distance 30 cm below the air water surface.



Secondly image formed by refraction from water surface followed by reflection from mirror and again refraction out of the water surface.

$\therefore$ Distance of the object from mirror $= u = 20 + \frac{4}{3} \times 30 = 60 \text{cm}$

Now considering reflection from the mirror.

Using, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Here $u = -60 \text{cm}, f = +40 \text{cm}$

$$\frac{1}{v} + \frac{1}{(-60)} = \frac{1}{(-40)} \Rightarrow v = +24 \text{cm}$$

Now considering refraction from water surface

Using $\frac{h_1}{\mu_1} = \frac{h_2}{\mu_2} \Rightarrow \frac{(20+40)}{4/3} = \frac{h_2}{1}$

$\Rightarrow \quad h_2 = 44 \times \frac{3}{4} = 33 \text{cm}$

Hence the final image from water surface will form at a depth 33 cm from water surface.

Hence separation between both the images

$\Delta x = 33 - 30 = 3.00 \text{cm}$

19. (5)

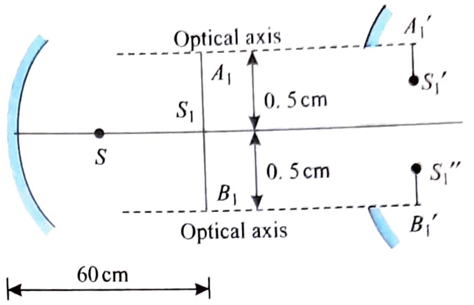
For mirror $M_1, u = -30 \text{cm}, f = -20 \text{cm}$

Using, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = -60 \text{cm}$

The image S_1 behaves as object for two halves.

For convex mirror, $u' = -10 \text{cm} \Rightarrow f' = +10 \text{cm}$

$$\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f'} \Rightarrow v' = +5 \text{cm}$$



$\therefore \quad m' = -\frac{v'}{u'} = \frac{-5}{-10} = \frac{1}{2}$

Thus, $\quad A_1'S_1' = \frac{1}{2} \times 0.5 = 0.25 \text{cm}$

and $\quad B_1'S_1'' = \frac{1}{2} \times 0.5 = 0.25 \text{cm}$

Hence separation between the images

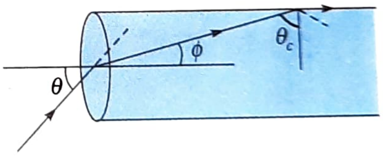
$S_1'S_1'' = 0.5 + 0.5 - 0.25 - 0.25 = 0.5 \text{cm} = 5 \text{mm}$

Archives

JEE Main

Single Correct Answer Type

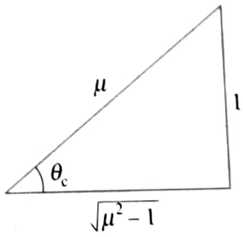
1. (4)



If θ_c has to be the critical angle, $\theta_c = \sin^{-1}\frac{1}{\mu}$

But $\theta_c = 90° - \phi, \theta_i = \theta$

$$\frac{\sin \theta_i}{\sin \phi} = \mu = \frac{2}{\sqrt{3}} \Rightarrow \frac{\sin \theta}{\cos \theta_c} = \mu$$



But $\cos \theta_c = \frac{\sqrt{\mu^2 - 1}}{\mu}$

$\therefore \quad \sin \theta = \mu \frac{\sqrt{\mu^2 - 1}}{\mu} = \sqrt{\mu^2 - 1}$

$\therefore \quad \theta = \sin^{-1}\sqrt{\frac{4}{3} - 1} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

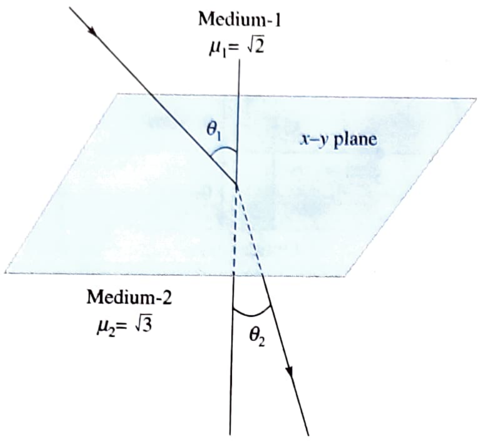
So that θ_c is making total internal reflection.

2. (2) Mirror formula: $\frac{1}{v}+\frac{1}{-280}=\frac{1}{20} \Rightarrow v=\frac{280}{15}$ cm

$$v_{\text{image}}=-\left(\frac{v}{u}\right)^2 v_{\text{object}} \Rightarrow v_{\text{image}}=-\left(\frac{280}{15\times 280}\right)^2 15$$

$$\Rightarrow v_{\text{image}}=-\frac{1}{15}\text{ m/s}$$

3. (2) $\mu_1\sin\theta_1=\mu_2\sin\theta_2$



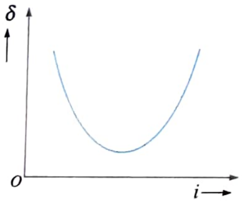
$$\cos\theta_1=\frac{10}{\sqrt{(6\sqrt{3})^2+(8\sqrt{3})^2+100}}=\frac{10}{\sqrt{400}}=\frac{10}{20}$$

$$\cos\theta_1=\frac{1}{2} \Rightarrow \theta_1=60^\circ$$

$$\sqrt{2}\sin 60^\circ=\sqrt{3}\sin\theta_2 \Rightarrow \sqrt{2}\times\frac{\sqrt{3}}{2}=\sqrt{3}\sin\theta_2$$

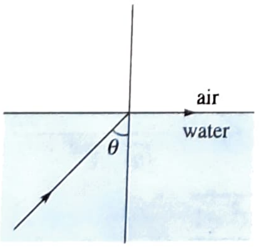
$$\Rightarrow \sin\theta_2=\frac{1}{\sqrt{2}} \Rightarrow \theta_2=45^\circ$$

4. (2) The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is as shown in the following figure.



5. (4) For critical angle we can write, $\sin\theta=\frac{1}{\mu}$

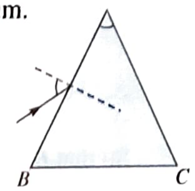
The refractive index (μ) of the medium inversely proportional to the wavelength of the light. μ is less for the greater wavelength (i.e., lesser frequency).



So, θ will be more for lesser frequency of light. Hence the spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

6. (1) $\sin\theta=\mu\sin r_1$

$$\Rightarrow \sin r_1=\frac{\sin\theta}{\mu}$$



$$\Rightarrow r_1=\sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

$$r_2=A-\sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

$$\Rightarrow r_2<\sin^{-1}\left(\frac{1}{\mu}\right)$$

$$A-\sin^{-1}\left(\frac{\sin\theta}{\mu}\right)<\sin^{-1}\left(\frac{1}{\mu}\right)$$

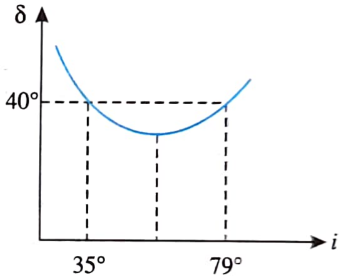
$$\Rightarrow A-\sin^{-1}\left(\frac{1}{\mu}\right)<\sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

$$\Rightarrow \sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)<\frac{\sin\theta}{\mu}$$

$$\Rightarrow \mu\left(\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right)<\sin\theta$$

$$\Rightarrow \sin^{-1}\left(\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right)<\theta$$

7. (1) Given $i=35^\circ$, $\delta=40^\circ$ and $e=79^\circ$



From the given data, $\delta=i+e-A$

$40^\circ=35^\circ+79^\circ-A \Rightarrow A=74^\circ$

$$\mu=\frac{\sin\left(\frac{\delta_m+A}{2}\right)}{\sin\frac{A}{2}}=\frac{\sin\left(\frac{40^\circ+74^\circ}{2}\right)}{\sin 37^\circ}=\frac{\sin 57^\circ}{\sin 37^\circ}=1.44$$

Nearest value is 1.5.

JEE Advanced

Single Correct Answer Type

1. (1) At minimum deviation ($\delta-\delta_m$):

$r_1=r_2=\frac{A}{2}=\frac{60^\circ}{2}=30^\circ$ (for both colors)

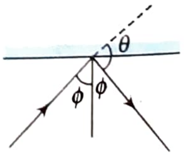
$\therefore$ Correct answer is (1).

2. (3) $V_{\text{ball}}^2=2\times 10\times 7.2 \Rightarrow v=12\text{ m s}^{-1}$

$$X_{\text{image of ball}}=\frac{4}{3}X_{\text{ball}}$$

$$V_{\text{image of ball}}=\frac{4}{3}V_{\text{ball}}=\frac{4}{3}\times 12=16\text{ m s}^{-1}$$

3. (1) Let angle between the directions of incident ray and reflected ray be θ



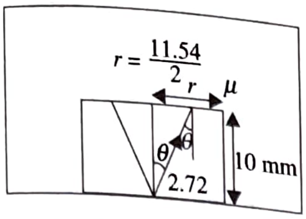
$$\cos\theta=\frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})\cdot\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})=-\frac{1}{2}$$

$\theta=120^\circ$

$2\phi+\theta=180^\circ$

$\phi=60^\circ/2=30^\circ$

4. (3) $\tan\theta = \dfrac{11.54}{2\times 10}$



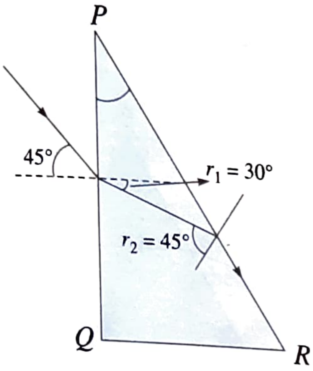
$$\sin\theta = \frac{11.54}{\sqrt{(11.54)^2 + (400)}}$$

$$2.72\sin\theta = \mu \times \sin 90°$$

$$2.72 \times \frac{11.54}{\sqrt{(11.54)^2 + 400}} = \mu$$

$$\mu = 1.36$$

5. (1)



$$1\sin 45° = \sqrt{2}\sin r_1$$

$$r_2 - r_1 = \theta$$

$$\theta = 45° - 30°$$

$$\Rightarrow \quad \theta = 15°$$

6. (4) Distance of point A is $f/2$, the nature of image of the AB will be virtual and behind the mirror. Let A' is the image of A from mirror, for this image

$$\frac{1}{v} + \frac{1}{-f/2} = \frac{1}{-f}$$

$$\frac{1}{v} = \frac{2}{f} - \frac{1}{f} = \frac{1}{f}$$

or $v = f$

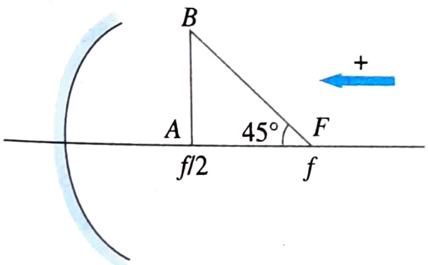
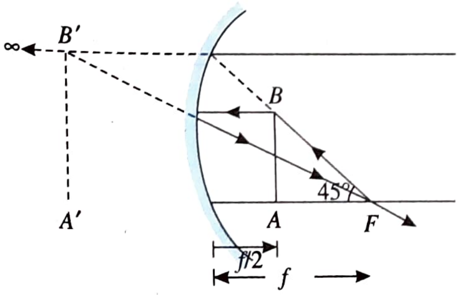


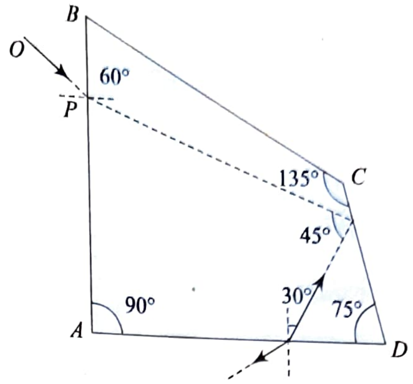
Image of line AB should be perpendicular to the principal axis and image of F will form at infinity, therefore correct image diagram is



The image of part BF will form at infinity. Hence option (4) is correct.

Multiple Correct Answers Type

1. (1), (2), (3)



Using Snell's law, $\sin^{-1}\dfrac{1}{\sqrt{3}} < \sin^{-1}\dfrac{1}{\sqrt{2}}$

Net deviation is 90°

2. (1), (2), (4)

For parallel slab, $n_1 \sin\theta_i = n_2 \sin\theta_f$

and l depends on refractive angle in slab

$\therefore$ l depends on refractive index of slab and independent of n_2

3. (1), (3), (4)

Angular deviation, $\delta = i_1 + i_2 - A$

For minimum deviation, $i_1 = i_2$ so, $i_1 = A$ ...(i)

Also, $r_1 + r_2 = A$

i.e., $r_1 = \dfrac{A}{2}$ ...(ii)

$\therefore$ $r_1 = \dfrac{i_1}{2}$ (option (1) is correct)

By Snell's law, $\sin i_1 = \mu \sin r_1$

i.e., $\sin A = \mu \sin\left(\dfrac{A}{2}\right)$

$$\Rightarrow \quad 2\cos\left(\frac{A}{2}\right) = \mu$$

$$A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

Hence option (2) is incorrect. Option (3) is obviously correct. For tangential emergence, $i_2 \rightarrow 90°$.

So, $\mu \sin r_2 = 1$

$$\cos r_2 = \sqrt{1 - \frac{1}{\mu^2}}$$

Also, $r_1 = A - r_2$

$$\sin r_1 = \sin A \cos r_2 - \cos A \sin r_2 = (\sin A)\frac{\sqrt{\mu^2 - 1}}{\mu} - (\cos A)\frac{1}{\mu}$$

By Snell's law on 1st surface, $\sin i_1 = \mu \sin r_1$

$$\Rightarrow \sin r_1 = \sin A(\mu^2 - 1)^{1/2} - \cos A$$

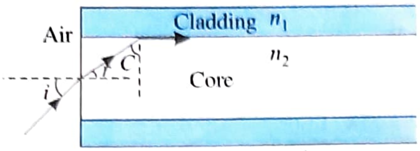
$$\Rightarrow i_1 = \sin^{-1}\left(\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right)$$

i.e., option (4) is correct.

Linked Comprehension Type

1. **(1), (3)**

i_m is the angle of incidence for which $r = 90 - C$



$$\sin C = \frac{n_2}{n_1}$$

$$\Rightarrow \quad n_0 \sin(i_m) = n_1 \sin(90^\circ - C)$$

$$\Rightarrow \quad N_A = \sin(i_m) = \frac{n_1 \cos(C)}{n_0} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$\text{For } S_1,\ NA_1 = \frac{\sqrt{\frac{45}{16} - \frac{9}{4}}}{n_0} = \frac{3}{4n_0}$$

$$\text{For } S_2,\ NA_2 = \frac{\sqrt{\frac{64}{25} - \frac{49}{25}}}{n_0} = \frac{\sqrt{15}}{5n_0}$$

$$\text{For (1) } NA_1 = \frac{3 \times 3}{4 \times 4} = \frac{9}{16};\ NA_2 = \frac{\sqrt{15} \times 3\sqrt{15}}{5 \times 16} = \frac{9}{16}$$

Hence option (1) is correct.

$$\text{For (2) } NA_1 = \frac{3 \times \sqrt{15}}{4 \times 6} = \frac{\sqrt{15}}{8};\ NA_2 = \frac{\sqrt{15} \times 3}{5 \times 4} = \frac{\sqrt{15} \times 3}{20}$$

Hence option (2) is incorrect

$$\text{For (3) } NA_1 = \frac{3}{4 \times 1} = \frac{3}{4};\ NA_2 = \frac{\sqrt{15} \times \sqrt{15}}{5 \times 4} = \frac{3}{4}$$

Hence option (3) is correct.

$$\text{For (4) } NA_1 = \frac{3}{4 \times 1} = \frac{3}{4};\ NA_2 = \frac{3}{4 \times 4} = \frac{9}{16}$$

Hence option (4) is incorrect.

2. **(4)** It is given $NA_2 < NA_1$

$$\Rightarrow \quad i_{m_1} > i_{m_2}$$

Hence if the combination can be placed both ways i.e. 1st structure and then 2nd structure and then reversed also, then the condition of TIR is satisfied for lower i_m then it can be satisfied for all other less angler as well.

Hence NA_2 will be the numerical aperture of the combined structure.

Matrix Match Type

1. **(4) (i) → (b); (ii) → (c); (iii) → (d); (iv) → (a)**

i. $\mu_2 > \mu_1$ ... (towards normal)

$\mu_2 > \mu_3$ ... (away from normal)

ii. $\mu_1 = \mu_2$ ... (no change in path)

$\angle i = 0 \Rightarrow \angle r = 0$ on the block.

iii. $\mu_1 > \mu_2$ ...(away from the normal)

$\mu_2 > \mu_3$ ...(away from the normal)

$$\mu_1 \times \frac{1}{\sqrt{2}} = \mu_2 \sin r \quad \Rightarrow \quad \sin r = -\frac{\mu_1}{\sqrt{2}\mu_2}$$

$$\text{Since } \sin r < 1 \Rightarrow \mu_1 < \sqrt{2}\mu_2$$

iv. For TIR: $45^\circ > C \Rightarrow \sin 45^\circ > \sin C$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \quad \Rightarrow \quad \mu_1 > \sqrt{2}\mu_2$$

Numerical Value Type

1. **(6)** $\sin \theta_C = 3/5$

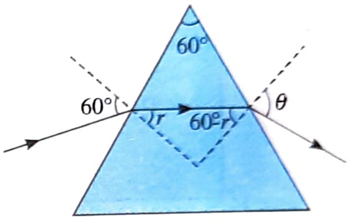
$\therefore \quad R = 6$ cm

2. **(3)** For $v_1 = 50/7$ m, $u_1 = -25$ m, $v_2 = 25/3$ m, $u_2 = -50$ m

$$\text{Speed of object} = \frac{25}{30} \times \frac{18}{3} = 3 \text{ km h}^{-1}$$

3. **(2)** $\sin 60^\circ = n \sin r$ ...(i)

$\sin \theta = n \sin(60^\circ - r)$ ...(ii)



Differentiating Eq. (ii).

$$\cos\theta \frac{d\theta}{dt} = -n\cos(60^\circ - r)\frac{dr}{dt} + \sin(60^\circ - r)$$

Differentiating Eq. (i).

$$n \cos r \frac{dr}{dt} + \sin r = 0$$

$$\cos\theta \frac{d\theta}{dt} = -n\cos(60^\circ - r)\left(\frac{-\tan r}{n}\right) + \sin(60^\circ - r)$$

$$\frac{d\theta}{dt} = \frac{1}{\cos\theta}[+\cos(60^\circ - r)\tan r + \sin(60^\circ - r)]$$

$$\frac{d\theta}{dt} = \frac{1}{\cos 60^\circ}(\cos 30^\circ \times \tan 30^\circ + \sin 30^\circ)$$

$$= 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$$

4. **(8)** Considering Snell's law between first layer and *m*th layer.

$$n \sin\theta = (n - m\Delta n)\sin 90^\circ$$

So, $1.6 \sin(30^\circ) = (n - m\Delta n) \sin 90^\circ$

i.e. $0.8 = n - m\Delta n$ Solving, $m = 8$

Note: Mathematical answer is 8, but this value of refractive index is physically impossible.

Chapter 2

Concept Application Exercises

ercise 2.1

1. (a) True. For plane surfaces, $\frac{\mu_2}{v} = \frac{\mu_1}{u}$

(b) False. In the equation, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

μ_1 is the medium which comes before the boundary and μ_2 is the medium which comes after the boundary.

(c) True. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Here, $\mu_2 = 1.5$, $\mu_1 = 1$, $u = -5R$, $R = +R$

$$\frac{1.5}{v} - \frac{1}{-5R} = \frac{1.5 - 1}{+R}$$

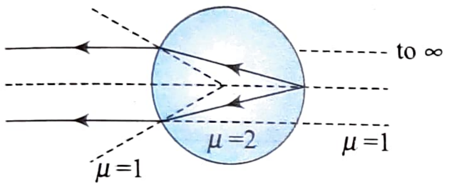
$$\frac{1.5}{v} = \frac{1}{2R} - \frac{1}{5R} = \frac{3}{10R}$$

$$v = 5R$$

(d) False. All the rays coming out from the object are radial. The radial rays are not deviated with the change in medium.

(e) True. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Here, $\mu_2 = 1$, $\mu_1 = 2$, $R = -R$



$$u = -2R$$

$$\frac{1}{v} - \frac{2}{-2R} = \frac{1-2}{-R} = \frac{1}{R}$$

or $\frac{1}{v} = \frac{1}{R} - \frac{1}{R}$

$\Rightarrow$ $v = \infty$

2. (a) $\mu_1 = 1$, $\mu_2 = 1.5$, $R = +20$ mm, $u = +80$ mm

$$\frac{1.5}{v} - \frac{1}{-80} = \frac{1.5 - 1}{-20}$$

$$v = -40 \text{ mm}$$

The image is therefore formed at the left of the vertex.

(b) $\frac{1.5}{v} + \frac{-1.33}{-80} = \frac{1.5 - 1.33}{+20}$, $v = -60$ mm

The fact that v is negative means that the rays after refraction by the surface are not converging but appear to diverge from a point 60 mm to the left of the vertex. The surface forms a virtual image 60 mm to the left of the vertex.

3. We know that $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$u = -12$ cm, $R = 10$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$

$\Rightarrow$ $\frac{1.5}{v} - \frac{1}{-12} = \frac{1.5 - 1}{10}$

$\Rightarrow$ $v = -40$ cm

This image will serve as an object for the second surface.

For the second surface, object distance, $u = 5 + 45 = 50$ cm

For the second surface again,

$u = -50$ cm, $R = -25$ cm, $\mu = 1.5$, $\mu_2 = 1$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{v} - \frac{1.5}{-50} = \frac{1 - 1.5}{-25}$$

or $v = -100$ cm

Final image will be at a distance of –95 cm from the first surface on the same side as the object.

4. Let x be the distance of the object from the pole in medium 1. Here we have, $u = -x$, $v = +x$ and $R = +R$

Using the equation, we get $\frac{\mu_2}{+x} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{+R}$

or $x = \left(\frac{\mu_2 + \mu_1}{\mu_2 - \mu_1}\right)R$

Note that the real image is formed only when $\mu_2 > \mu_1$.

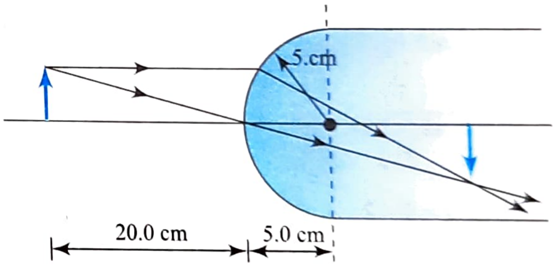
5. Let the object be placed at a distance x from the pole P_1 of the sphere. If a real image is to be formed equidistant from the sphere, then the ray must pass symmetrical through the sphere, as shown in the figure in question.

Applying the equation at the first surface, we get

$$\frac{\mu_2}{+\infty} - \frac{\mu_1}{-x} = \frac{\mu_2 - \mu_1}{+R}$$

or $x = \left(\frac{\mu_1}{\mu_2 - \mu_1}\right)R$

6. Taking the origin at the vertex, $u = -20.0$ cm and $R = 5.0$ cm.



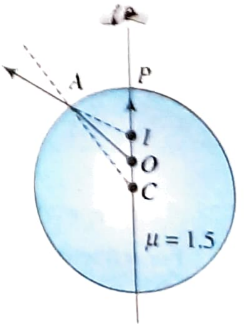
We have, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1.5}{v} = \frac{1}{-20.0\text{cm}} + \frac{0.5}{5.0\text{cm}} = \frac{1}{20\text{cm}}$$

$\Rightarrow$ $v = 30$ cm

The image is formed inside the rod at a distance of 30 cm from the vertex.

7. The observer sees the image formed due to refraction at the spherical surface when the light from the bubble goes from the glass to the air. Here, $u = -4.0$ cm, $R = -10$ cm, $\mu = 1.5$ and $\mu_2 = 1$



$$\frac{1}{v} - \frac{1.5}{-4.0\text{cm}} = \frac{1 - 1.5}{-10\text{cm}}$$

$$\frac{1}{v} = \frac{0.5}{10\text{cm}} - \frac{1.5}{4.0\text{cm}}$$

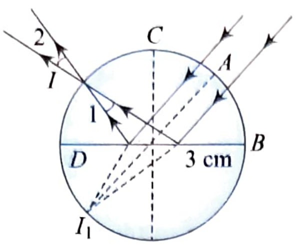
$$v = \frac{-40}{13}\text{ cm}$$

Thus, the bubble will appear 3.0 cm below the surface.

8. (a) Critical angle: θ_c

$$\sin\theta_c = \frac{1}{\mu} = \frac{1}{2} \quad \Rightarrow \quad \theta_c = 30^\circ$$

Since angle of incidence is 45° (> θ_c) at the plane surface, so incident rays will be reflected from the plane surface.



(b) $\frac{2}{v} - \frac{1}{\infty} = \frac{2-1}{3} \quad \Rightarrow v = 6$ cm

If the sphere were completed, then image would form at I_1, but due to reflection at DB, final image will be formed at I.

(c) Final image is formed at I.

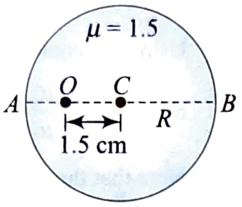
(d) See the figure.

9. (a) $R = 5$ cm for the observer at left:

$$\frac{1}{v} - \frac{\mu}{-AO} = \frac{1-\mu}{-R},$$

$AO = 3.5$ cm

$$\Rightarrow \quad v = -\frac{70}{23} \approx -3 \text{ cm}$$



Hence, image is formed 3 cm to the right of A or 2 cm to the left of C.

(b) For observer to right $\frac{1}{v} - \frac{\mu}{-BO} = \frac{1-\mu}{-R}$

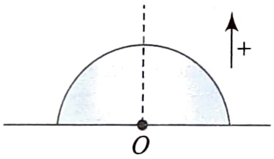
$$v = -\frac{130}{17} = -7.65 \text{ cm}$$

Hence, image is formed 7.65 cm left of B or 2.65 cm left of C.

10. $\frac{1}{v} - \frac{1.5}{-3} = \frac{1-1.5}{-3}$

$v = -3$ cm

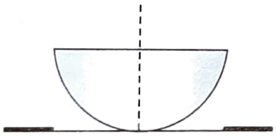
Hence, no shifting.



11. $\frac{1}{v} - \frac{1.5}{-3} = \frac{1-1.5}{\infty}$

$v = -2$ cm

∴ Shifting = 3 − 2 = 1 cm



12. We will have single surface refractions successively at the four surfaces S_1, S_2, S_3, and S_4. Do not forget to shift origin to vertex of the respective surface.

Refraction at first surface S_1: Light travel from air to glass.

$$\frac{1.5}{v_1} - \frac{1}{\infty} = \frac{(1.5-1)}{(+10)}$$

$v_1 = 30$ cm

First image is the object for the refraction at second surface.

For refraction at surface S_2: Light travels from glass to air.

$$\frac{1}{v_2} - \frac{1.5}{(+25)} = \frac{1-1.5}{(+5)}$$

$v_2 = -25$ cm

For refraction at surface S_3: Light travels from air to glass.

$$\frac{1.5}{v_3} - \frac{1}{(-35)} = \frac{(1.5-1)}{(-5)}$$

$v_3 = -35/3$ cm.

For refraction at surface S_4: Light travels from glass to air.

$$\frac{1}{v_4} - \frac{1.5}{-(35/3+5)} = \frac{1-1.5}{-10}$$

$v_4 = -25$ cm

The final image is virtual, formed at 25 cm to the left of the vertex of surface S_4.

Exercise 2.2

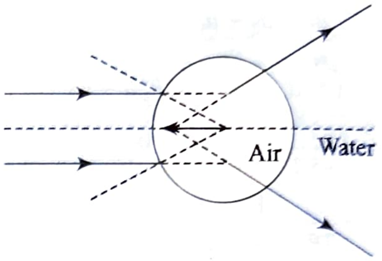
1. (a) False. Real images of real object are always inverted. But for virtual object the image may be erect.

(b) False. Real images are always inverted.

(c) True. From equation, we have $m = \frac{f_0}{x + f_0}$

The value of m is always positive and less than 1.

(d) True. The air bubble in water diverges a parallel beam of light.

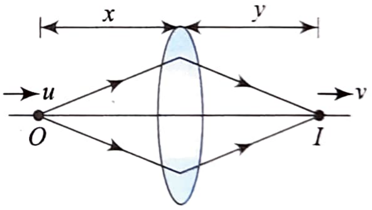


(e) True. The least distance between a real object and its real image is $4f$.

(f) True. $\frac{1}{y} + \frac{1}{x} = \frac{1}{f}$, $\frac{dx}{dt} = -u$, $\frac{dy}{dt} = v$

$$-\frac{1}{y^2}\frac{dy}{dt} - \frac{1}{x^2}\frac{dx}{dt} = 0$$

$$v = \frac{y^2}{x^2}u$$



Initially, x is large and y is small. So, initially v is small.

But as the object moves from ∞ to focus, x decreases and y increases. So, v also increases.

(g) True. Using equation, we have $f = f_0\left[\frac{\mu(\mu_0 - 1)}{\mu_0 - \mu}\right]$

Here, $\mu_0 = \frac{3}{2}$, $\mu = \frac{4}{3}$

$\Rightarrow \quad f = 4f_0$

The effective focal length comes out to be positive.

2. Two images will be formed, one from each type of material.

3. The rays of light diverge from the object; it is a real object. Focal length of a convex lens is positive.

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{2}{R}\right) = \frac{(3/2) - 1}{1}\left(\frac{2}{10}\right) = \frac{1}{10}$$

The given parameters are $u = -30$ cm and $f = +10$ cm

Substituting these values in thin-lens equation, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{v} - \frac{1}{(-30)} = \frac{1}{10}$$

which on solving gives $v = +15$ cm.

The image distance is positive, which implies that the image is real.

The lateral magnification is

$$m = \frac{v}{u} = \frac{15}{(-30)} = -\frac{1}{2}$$

The image is half the size of the object; size of the image is 0.50 cm. The negative sign with magnification shows inverted image.

4. In accordance with cartesian sign convention, the given parameters are

$$f = -10 \text{ cm}, u = -15 \text{ cm}$$

From lens equation, we have

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{(-10)} + \frac{1}{(-15)} = -\frac{1}{6}$$

$$v = -6 \text{ cm}$$

Lateral magnification, $m = \frac{v}{u} = \frac{-6}{-15} = 0.40$

The minus sign with image shows that the image is located on the side of the object. The magnification is positive and $m < 1$, which shows that the image is upright and diminished.

5. We have, $\mu = 1.5$, $R_1 = 20$ cm, and $R_2 = -40$ cm.

Therefore, the focal length of the lens is

$$\frac{1}{f} = (1.5 - 1)\left[\frac{1}{20} - \frac{1}{-40}\right]$$

or $f = \frac{80}{3}$ cm

Now, $f = \frac{80}{3}$, $u = -40$ cm

Therefore, from equation, we get

$$\frac{1}{v} = \frac{3}{80} - \frac{1}{40} \quad \text{or} \quad v = +80 \text{ cm}$$

Therefore, the image is formed 80 cm on the other side of the lens.

6. Here, $\mu = 1.5$, $R_1 = -20$ cm, and $R_2 = +20$ cm.

Substituting in equation, we get $f = -20$ cm

Now, $u = +10$ cm, $f = -20$cm

Substituting in equation, we get

$$\frac{1}{v} = \frac{1}{-20} + \frac{1}{10} \quad \text{or} \quad v = +20 \text{ cm}$$

Thus, the rays now converge to a point 20 cm in front of the lens.

7. Here, $u = -15$ cm, $m = -2$ (Note the negative sign. As the image is on a screen, it is real which implies m must be negative)

$\therefore \quad v = mu = +30$ cm

Applying equation, we get $\frac{1}{f} = \frac{1}{30} - \frac{1}{-15}$

or $\quad f = 10$ cm

Therefore, the lens is convex, with focal length of 10 cm.

8. Here, $f = +10$ cm and $m = +2$. (Magnification is now positive as the image is erect.) Let the object distance from the lens be x.

Then, $u = -x$ and $v = mu = -2x$.

Substituting in equation, we get $\frac{1}{10} = \frac{1}{-2x} - \frac{1}{-x}$

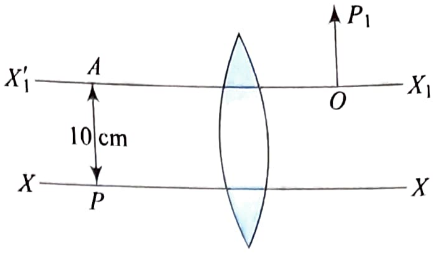
or $\quad x = 5$ cm

Therefore, the lens is to be held 5 cm from the object.

9. $m = 2 = \frac{v}{u} \Rightarrow v = 2u = 2 \times 12 = 24$ cm

Distance between object and image: 12 + 24 = 36 cm.

10. Here, $u = -40$ cm, $f = +20$ cm



$$\therefore \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or $\quad \frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{2-1}{40} = \frac{1}{40}$

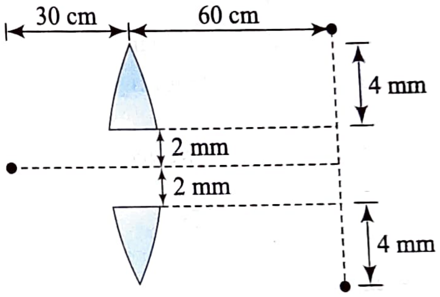
$v = 40$ cm

$$m = \frac{v}{u} = \frac{40}{-40} = -1$$

$\therefore \quad OP_1 = m \times AP = (-1)(-1)$ cm $= 1$ cm

Therefore, the vertical position of P_1 from P is $(1+1) = 2$ cm. As we know, when mirror is rotated by θ, reflected ray is rotated by 2θ. So, in this case, final image is below PO produced at 10 cm.

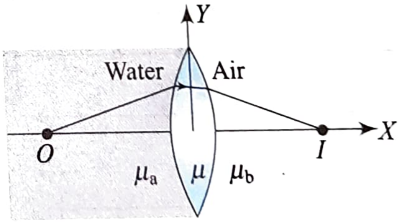
11. $v = \frac{uf}{u+f} = \frac{-30 \times 20}{-30 + 20} = 60$ cm



$$m = \frac{v}{u} = \frac{h_2}{h_1} \Rightarrow h_2 = \frac{vh_1}{u} = \frac{60 \times 2}{30} = 4 \text{ cm}$$

Exercise 2.3

1. Ray diagram: The path of a typical light ray is shown in the following figure.



This problem corresponds to the situation of a different medium on either side of the lens. The governing equation is

$$\frac{\mu_b}{v} - \frac{\mu_a}{u} = \frac{\mu - \mu_a}{R_1} + \frac{\mu_b - \mu_a}{R_2} \qquad \text{...(i)}$$

Here, $\mu = -15$ cm, $\mu_a = 1.33$, $\mu_b = 1$, $\mu = 1.5$

To complete the problem will still need the radius of curvature of the two sides of the lens.

Since it is a biconvex lens, $R_1 = R_2 = R$ and

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$$

or $\quad R = 20$ cm

Therefore, $R_1 = +20$ cm and $R_2 = -20$ cm.

Substituting in Eq. (i), we get $v = 10.3$ cm

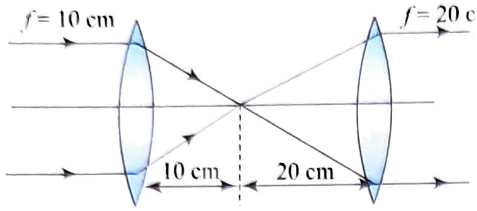
2. For convex lens: $v = \dfrac{uf}{u+f} = \dfrac{-15 \times 10}{-15 + 10} = 30$ cm

For concave lens: $u = (30 - 25) = 5$ cm

$$v = \frac{uf}{u+f} = \frac{5(-10)}{5-10} = 10 \text{ cm}$$

So, final image is formed 10 cm to the right of the concave lens.

3. $d = 10 + 20 = 30$ cm



4. Let the image from the first lens be formed at a distance y to the right of lens.

Refraction at the first lens:

Here, $u = -3$ cm, $f = +20$ cm, $v = +y$

From the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} \quad \text{or} \quad \frac{1}{y} - \frac{1}{-30} = \frac{1}{20} \quad \text{or } y = 60 \text{ cm}$$

Refraction at the second lens:

Here, $u = 60 - 10 = +50$ cm, $f = +25$ cm

Let the image be at a distance z to the right of lens 2.

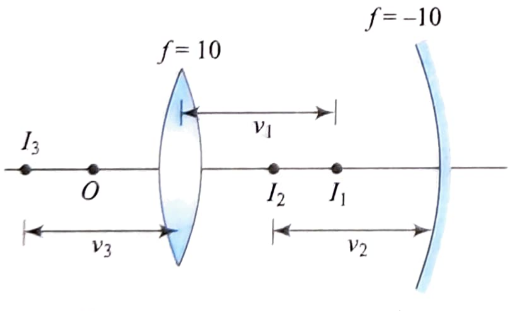
Therefore, $v = +z$.

Substituting in the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2} \quad \text{or} \quad \frac{1}{z} = \frac{1}{25} + \frac{1}{50}$$

or $\quad z = \dfrac{50}{3}$ cm

5. For convex lens: $v_1 = \dfrac{uf}{(u+f)} = \dfrac{-15 \times 10}{-15 + 10} = 30$ cm



For concave mirror: $u = -(43 - 30) = -15$ cm

$$\frac{1}{v_2} + \frac{1}{-15} = \frac{1}{-10}$$

$\Rightarrow \quad v_2 = -30$ cm

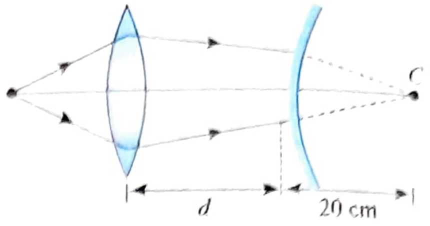
Now, again for convex lens: $u - (45 - 30) = -15$ cm

$\Rightarrow \quad v_3 = \dfrac{-15 \times 10}{-15 + 10} = 30$ cm

So, final image is formed at I_3 at a distance of 30 cm from the lens.

6. For convex lens: $v = \dfrac{-15 \times 10}{-15 + 10} = 30$ cm

The refracted rays from lens should fall normally on the mirror for the image to coincide with the object.



$d + 20 = v = 30$

$\Rightarrow \quad d = 10$ cm

7. The convex lenses and the plane mirror are shown in figure.

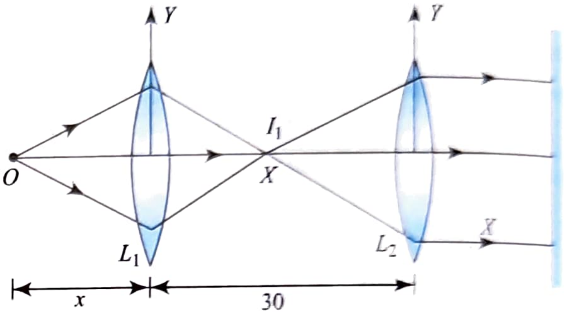
The combination behaves like a concave mirror.

Let the distance of the object from the first lens be x.

For the ray to retrace its path, it should be incident normally on the plane mirror.

From the diagram, we see that for lens L_2

$v = \infty$, $f = +10$ cm, $u = ?$



From the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad u = -10 \text{ cm}$$

From the diagram, we see that for lens L_1

$v = 30 - 10 = 20$ cm, $f = +10$ cm, $u = -x$

From the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{20} - \frac{1}{-x} = \frac{1}{10} \quad \text{or} \quad x = 20 \text{ cm}$$

8. $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{40}$$

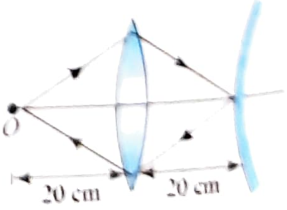
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{(d-30)} - \frac{1}{(-80)} = \frac{1}{40}$$

$\Rightarrow \quad d = 110$ cm

9. Object is placed at a distance of $2f$ from the lens (f = focal length of the lens), i.e., the image formed by the lens will be at a distance of $2f$ or 20 cm from the lens.

So, if the concave mirror is placed in this position, the first image will be formed at its pole and it will reflect all the rays symmetrically to the other side as shown below and the final image will coincide with the object

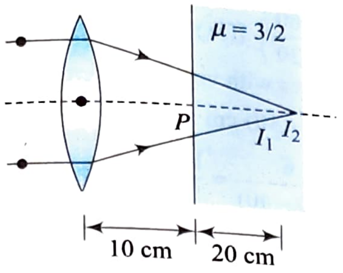


10. The lens will converge the rays at its focus, i.e., 30 cm from the lens or 20 cm from the refracting surface.

$PI_1 = 20$ cm

$\therefore \quad PI_2 = \mu(PI_1)$

$= \frac{3}{2} \times 20 = 30$ cm



So, the rays will converge at a distance of 40 cm from the lens.

11. Focal length of the plano-convex lens is

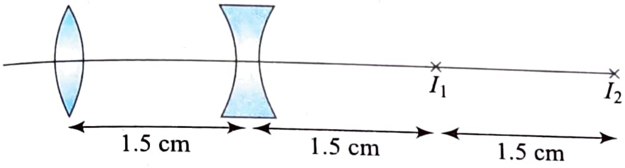
$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} - \frac{1}{\infty}\right)$$

or $\quad f = 20$ cm

If point object O is placed at a distance of 20 cm from the plano-convex lens, rays become parallel and final image is formed at second focus or 20 cm from concave lens which is independent of b.

12. Image formed due to converging lens acts as object for diverging lens $u = +15$ cm,

$$v = (19.2 + 15) \text{ cm} = 34.2 \text{ cm}$$



$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{34.2} - \frac{1}{15} = \frac{15 - 34.2}{34.2 \times 15} = \frac{-19.2}{34.2 \times 15}$$

$f = -26.7$ cm

Exercise 2.4

1. Let F is the effective focal length of this concave mirror. Then,

$$\frac{1}{F} = \frac{1}{f_m} - \frac{2}{f_l}$$

$$f_m = \infty, \quad \frac{1}{f_l} = (\mu - 1)\left(\frac{1}{R}\right)$$

$$\frac{1}{F} = (\mu - 1)\frac{2}{R} \quad \Rightarrow \quad F = \frac{-R}{2(\mu - 1)}$$

$$F = \frac{-30}{2(1.5 - 1)} = -30 \text{ cm}$$

So, object should be placed at $2F = 60$ cm from the lens to coincide the image with the object.

2. Here, $R_1 = +12$ cm, $R_2 = \infty$, $\mu_p = 1.5$, $\mu_a = 1$, therefore

Focal length of the lens, $\frac{1}{f_l} = (1.5 - 1)\left[\frac{1}{12} - \frac{1}{\infty}\right]$

or $\quad f_l = 24$ cm

The plane side of the lens is silvered; focal length of the mirror

$$f_m = \frac{R_2}{2} = \frac{\infty}{2} = \infty$$

Therefore, effective focal length of the silvered lens is

$$\frac{1}{f_e} = \frac{1}{f_m} - \frac{2}{f_l} \quad \text{or} \quad f_e = -12 \text{ cm}$$

The object is placed 24 cm in front of the lens.

$$u = -24 \text{ cm}, f = -12 \text{ cm}$$

And from the mirror Eq. (i), we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \text{...(i)}$$

or $\quad v = -24$ cm

The final image is formed on the object itself. The behavior is like that of a concave mirror.

3. As radius of curvature of silvered surface is 22 cm, so

$$f_M = \frac{R}{2} = \frac{-22}{2} = -11 \text{ cm} = -0.11 \text{ m}$$

And hence, $P_M = -\frac{1}{f_M} = -\frac{1}{-0.11} = \frac{1}{0.11}$ D

Further as the focal length of lens is 20 cm, i.e., 0.20 m, its power will be given by

$$P_L = \frac{1}{f_L} = \frac{1}{0.20} \text{ D}$$

Now as in image formation, light after passing through the lens will be reflected back by the curved mirror through the lens again

$$P = P_L + P_M + P_L = 2P_L + P_M$$

i.e., $\quad P = \frac{2}{0.20} + \frac{1}{0.11} = \frac{210}{11}$ D

So the focal length of equivalent mirror

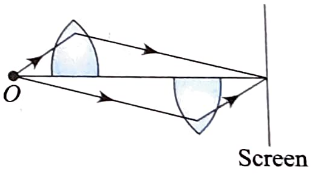
$$F = -\frac{1}{P} = -\frac{11}{210} \text{ m} = -\frac{110}{21} \text{ cm}$$

i.e., the silvered lens behaves as a concave mirror of focal length (110/21) cm. So for object at a distance 10 cm in front of it,

$$\frac{1}{v} + \frac{1}{-10} = -\frac{21}{110} \quad \text{i.e., } v = -11 \text{ cm}$$

i.e., image will be 11 cm in front of the silvered lens and will be real as shown in the figure.

4. Ray diagram: The path of a typical light ray is shown in the following figure.



The element encountered is only a lens in either path of the light rays.

The lens is cut in half along the axis. So, both halves have the same focal length. Since two sharp images are formed on the screen, the result is equivalent to the lens displacement method with $D = 80$ cm and $d = 12$ cm.

For the lens displacement method, the focal length of the lens is given by

$$f = \frac{D^2 - d^2}{4D} \quad \text{or } f = 19.55 \text{ cm}$$

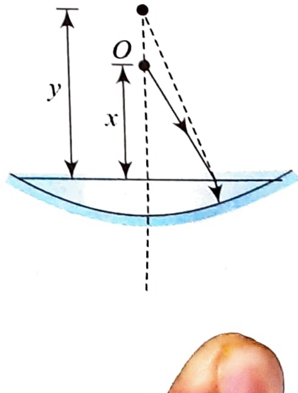
5. $y = \mu x = \frac{4}{3}x$

For the mirror, $u = -y = \frac{4}{3}x$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{3}{4x} = \frac{1}{-30}$$

$$v = -\left(\frac{60x}{2x - 45}\right)$$

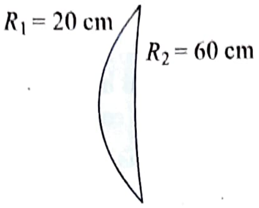


Let the final image is formed at a distance z from the mirror, then

$$\left(\frac{60x}{2x-45}\right)\frac{1}{z}=\mu=\frac{4}{3} \text{ and } \frac{z}{x}=2 \text{ (magnification)}$$

Solving, we get $x = 33.75$ cm

6. $\frac{1}{f_l}=(1.5-1)\left[\frac{1}{20}-\frac{1}{60}\right]=\frac{1}{60}$



$f_l = 60$ cm

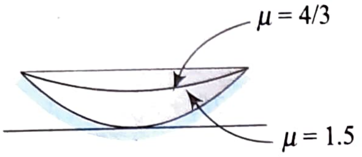
When convex side is silvered and water is not filled:

$$\frac{1}{F}=\frac{2}{f_l}-\frac{1}{f_m}, f_m=\frac{-R_1}{2}=-10 \text{ cm}$$

$$\frac{1}{F}=\frac{1}{-10}-2\left[\frac{1}{60}\right]$$

$$F=\frac{-30}{4}=-7.5 \text{ cm}$$

After water is filled:



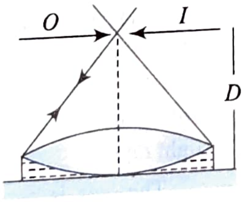
Let f_{l_1} is the focal length of water lens:

$$\frac{1}{f_{l_1}}=\left(\frac{4}{3}-1\right)\left(\frac{1}{60}\right)=\frac{1}{180}$$

$$\frac{1}{F}=\frac{2}{f_{l_1}}-\frac{2}{f_{l_2}}-\frac{1}{f_m}$$

Solving, we get $F=\frac{-90}{13}$ cm

7. If an object has to coincide with its image, then the ray have to retrace its path; hence, it is concluded that object is at the focus of the combination.



Focus of equivalent combination.

$$\frac{1}{F}=\frac{1}{f_g}+\frac{1}{f_l}$$

$$\frac{1}{f_g}=(1.5-1)\frac{2}{R}=\frac{1}{R}$$

$$\frac{1}{f_l}=(\mu-1)\left(-\frac{1}{R}-\frac{1}{\infty}\right)=-\frac{(\mu-1)}{R}$$

$$\frac{1}{F}=\frac{1}{R}-\left(\frac{\mu-1}{R}\right)=\left(\frac{2-\mu}{R}\right)$$

Since, $F = D$

$$\therefore \quad \mu=2-\frac{R}{D}=2-\frac{20}{30}=\frac{4}{3}$$

8. Velocity of image w.r.t. lens along the principal axis is given by

$$\vec{V}_{\text{image,lens}}=m^2\vec{V}_{\text{object,lens}} \quad \ldots\text{(i)}$$

$$\Rightarrow \quad \left(\vec{V}_{\text{image}}-\vec{V}_{\text{lens}}\right)=\left(\frac{v}{u}\right)^2\left(\vec{V}_{\text{object}}-\vec{V}_{\text{lens}}\right)$$

Using: $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{(-20)}=\frac{1}{(+30)} \Rightarrow v=-60\text{cm}$

Substituting, $u=-20\text{cm}, v=-60\text{cm}$

$\vec{V}_{\text{object}}=5\hat{i}, \vec{V}_{\text{lens}}=0$ in Eq. (i),

$$\vec{V}_{\text{image}}=\left(\frac{-60}{-20}\right)^2[(5\hat{i})-0]+0=45\hat{i} \text{ cm/s}$$

Hence, image moves with velocity 45 cm/s. away from the lens.

$$u_2=+(160-40) \text{ cm}$$

$$\frac{1.7}{v_2}-\frac{1.6}{+(160-40)}=\frac{1.6-1.7}{(-20)} \Rightarrow v_2=+204\text{cm}$$

Exercise 2.5

1. As chromatic aberration $=\omega f$,

$$\Rightarrow f=\frac{0.351 \text{ cm}}{0.026}=13.5 \text{ cm}$$

2. As $\frac{1}{f_b}=(\mu_b-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$,

$$\frac{1}{f_b}=(1.523-1)\left(\frac{1}{-25}-\frac{1}{30}\right)=-\frac{0.523\times 11}{150}$$

or $\quad f_b=-26.073$ cm

Similarly, $\frac{1}{f_r}=-\frac{0.5204\times 11}{150}$

or $\quad f_r=-26.203$

Longitudinal chromatic aberration $=f_r-f_b$

$=-26.203 \text{ cm}-(-26.073 \text{ cm})=-0.130$ cm

3. Here; $f = 6.25$ cm, $u=-D=-25$cm

(i) From $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$;

$$\frac{1}{u}=\frac{1}{v}-\frac{1}{f}=\frac{1}{-25}-\frac{1}{6.25}=-\frac{1}{25}-\frac{4}{25}=-\frac{1}{5}$$

or $\quad u=-5$ cm

(ii) Angular magnification, $M=1+\frac{D}{f}=1+\frac{25}{6.25}=1+4=5$

When the image is formed at infinity, $M=\frac{D}{f}=\frac{25}{6.25}=4$

4. Here, $f = 5$ cm

(a) Let $u_{\min}$ be the closest distance, at which he can read the book through the magnifying glass. In that case, the image of the book will be formed at a distance of 25 cm in front of him.

$\therefore \quad u=-u_{\min}$ and $u=-25$ cm

From the lens formula, we have $u=\frac{vf}{f-v}$

or $\quad -u_{\min}=\frac{-25\times 5}{5-(-25)}=-\frac{125}{30}=-4.17\text{cm}$

or $\quad u_{\min}=4.17$ cm

(b) Let $u_{\max}$ be the farthest distance, at which he can read the book through the magnifying glass. In that case, the image of the book will be formed at infinity.

$\therefore \quad u=-u_{\max}=$ and $v=\infty$

From the lens formula, we have $u=\frac{vf}{f-v}=\frac{f}{f/v-1}$

or $\quad -u_{\max}=\frac{5}{5/\infty-1}=\frac{5}{0-1}=-5\text{cm}$

or $\quad u_{\max}=5$ cm

Here, $f_o + f_e = 101$ cm ...(i)

$M = -100$ (magnifying power is negative)

Now, $M = -\frac{f_0}{f_e} \Rightarrow -100 = -\frac{f_0}{f_e}$

or $f_o = 100 f_e$ ...(ii)

From the Eqs. (i) and (ii), we have

$$100 + f_e = 101 \text{ or } f_e = 1 \text{ cm}$$

Again, $f_o = 100 f_e = 100 \times 1 = 100$ cm

The distance between the objective and eye-piece,

$$L = v_0 + |u_e| \qquad \text{...(i)}$$

To find v_0: $u_o = -6$ cm, $f_o = 4$ cm

Now, $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$

$\Rightarrow$ $-\frac{1}{-6} + \frac{1}{v_0} = \frac{1}{4} \Rightarrow v_0 = 12\text{cm}$

To find u_e: $v_e = -25$ cm, $f_e = 6$ cm

Using, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$ or $u_e = -4.84$ cm

From the Eq. (i), we have

$$L = 12 + 4.84 = 16.84 \text{ cm}$$

7. Magnification produced by a compound microscope,

$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

Here, $M = -20$ (final image is inverted)

and $m_e = 5$ (magnification produced by eye-piece)

Therefore, magnification produced by the objective,

$$m_0 = \frac{M}{m_e} = -\frac{20}{5} = -4$$

Now, $m_e = 1 + \frac{D}{f_e} \Rightarrow 5 = 1 + \frac{20}{f_e} \Rightarrow f_e = 5\text{cm}$

Also, $m_e = \frac{f_e}{u_e + f_e}$

or $u_e = -4$ cm

The distance between the objective and eye-piece,

$$v_o + |u_e| = 14 \text{ or } v_o + 4 = 14$$

or $v_o = 10$ cm

Now, magnification produced by the objective, $m_0 = \frac{v_0}{u_0}$

or $-4 = \frac{10}{u_0}$ or $u_0 = -2.5$ cm

Also, $-\frac{1}{u_0} + \frac{1}{v_0} = \frac{1}{f_0} \Rightarrow -\frac{1}{-2.5} + \frac{1}{10} = \frac{1}{f_0} \Rightarrow f_0 = 2\text{cm}$

8. Magnification produced by a telescope in normal adjustment,

$$M = -\frac{f_0}{f_e} \Rightarrow -5 = -\frac{f_0}{f_e} \Rightarrow f_0 = 5f_e$$

Also, distance between two lenses (when telescope is in normal adjustment),

$$L = f_0 + f_e$$

or $24 = f_0 + f_e$

or $24 = 5f_e + f_e$

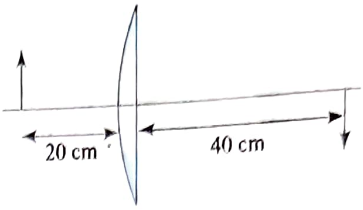
or $f_e = 4$ cm

Also, $f_0 = 5 \times 4 = 20$ cm

Exercises

Single Correct Answer Type

1. (1) $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

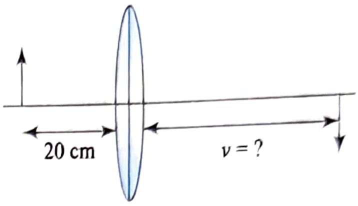


$\Rightarrow$ $\frac{1}{f} = \frac{1}{40} - \frac{1}{(-20)}$

$\Rightarrow$ $f = \frac{40}{3}$

Focal length for the combination,

$$f_{eq} = \frac{20}{3}$$



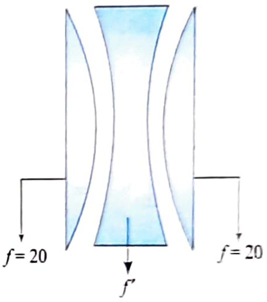
$$\frac{1}{f_{eq}} = \frac{1}{v} - \frac{1}{u}$$

$\Rightarrow$ $\frac{30}{20} = \frac{1}{v} + \frac{1}{20}$

$\Rightarrow$ $\frac{1}{v} = \frac{3}{20} - \frac{1}{20}$

$v = 10$ cm (to the right)

2. (3) $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$



$R = 10$ cm

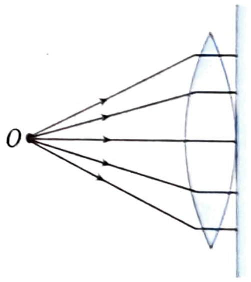
$f' = (3 - 1)\left(\frac{1}{-10} - \frac{1}{10}\right)$

$f' = \frac{-10}{4}$

$$\frac{1}{f_{eq}} = \frac{1}{20} - \frac{4}{10} + \frac{1}{20} = \frac{2}{20} - \frac{4}{10}$$

$f_{eq} = -10/3$ cm

3. (3) "O" act as the focal point.



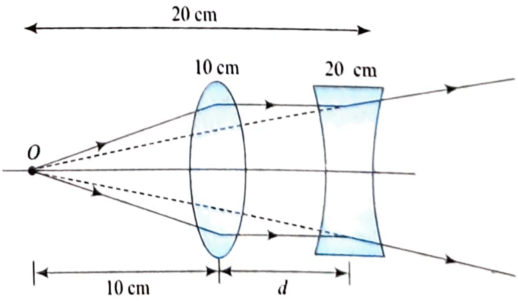
Also, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow$ $\frac{1}{40} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{10}\right)$

$\frac{1}{40} = (\mu - 1)\left(\frac{2}{10}\right)$

$\Rightarrow$ $\mu = 9/8$

4. (1)



$d = (20 - 10)$ cm $= 10$ cm

5. (3) $\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$

or $f = \frac{R}{2(\mu - 1)}$

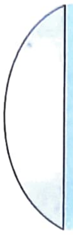
Now, $f > R$

$$\therefore \quad \frac{R}{2(\mu-1)} > R$$

$$\text{or} \quad \frac{1}{2(\mu-1)} > 1 \text{ or } 2(\mu-1) < 1$$

$$\text{or} \quad \mu - 1 < \frac{1}{2} \text{ or } \mu < \left(1 + \frac{1}{2}\right) \text{ or } \mu < 1.5$$

6. (2)



$$\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_l} + \frac{1}{f_m}$$

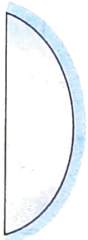
$$\frac{1}{F} = \frac{2}{f_l} + \frac{2}{f_m}$$

$$\text{or} \quad \frac{1}{F} = 2(\mu-1)\left(\frac{1}{R}\right) + \frac{1}{\infty}$$

$$\text{or} \quad F = \frac{R}{2\mu(-1)}$$

$$\text{Now, } -60 = \frac{R}{2(\mu-1)}$$

$$\text{or} \quad 120(\mu-1) = R \qquad \text{...(i)}$$



$$\text{Again, } \frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_l} + \frac{1}{f_m}$$

$$= \frac{2}{f_l} + \frac{1}{R/2}$$

$$= 2(\mu-1)\left(\frac{1}{R}\right) + \frac{2}{R}$$

$$= \frac{2}{R}(\mu - 1 + 1)$$

$$\text{or} \quad F = \frac{R}{2\mu}$$

$$\text{Now, } -20 = \frac{-R}{2\mu}$$

$$\text{or} \quad 40\mu = R \qquad \text{...(ii)}$$

Dividing (i) by (ii), we get

$$\frac{120(\mu-1)}{40\mu} = \frac{R}{R} = 1$$

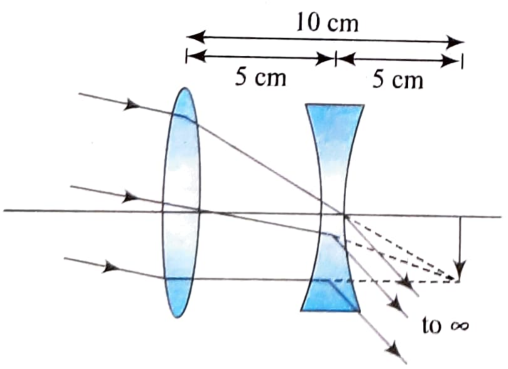
$$\text{or} \quad 120(\mu-1) = 40\mu$$

$$\text{or} \quad 120\mu - 40\mu = 120$$

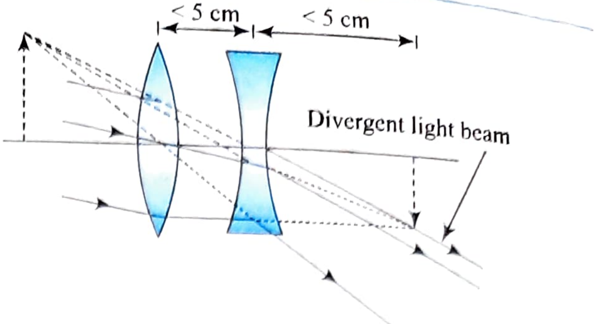
$$\text{or} \quad 80\mu = 120$$

$$\text{or} \quad \mu = \frac{120}{80} = \frac{3}{2} = 1.5$$

7. (4)



At 5 cm from the lens, the second lens has a virtual object (image of the first lens) at its focal length. The emergent rays are therefore parallel.



When the second lens is closer than 5 cm to the first lens, its object is outside the focal length of the diverging second lens. This produces a virtual image outside $2f$ of the second lens. The emergent rays are therefore *divergent.*

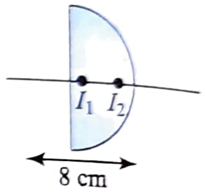
8. (1) A diverging lens is ruled out because both x and y are positive values. Both x and y equal 20 cm at their smallest sum, which occurs when

$$x + y = 40 \text{ cm} = 4f$$

$$\therefore \quad f = 10 \text{ cm}$$

This indicates a converging lens of focal length = 10 cm

9. (4) Distance of image from the plane surface is as follows:



$$x_1 = \frac{4}{1.6}$$

$$= 2.5 \text{ cm}\left(d_{\text{app}} = \frac{d_{\text{actual}}}{\mu}\right)$$

For the curved side

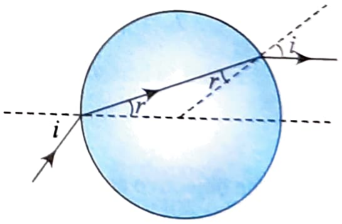
$$\frac{1.6}{4} + \frac{1}{x_2} = \frac{1-1.6}{8}$$

$$\therefore \quad x_2 \approx -3.0 \text{ cm}$$

The minus sign means the image is on the object side.

$$\therefore \quad I_1 I_2 = (8 - 2.5 - 3.0) \text{ cm} = 2.5 \text{ cm}$$

10. (2) Deviation by a sphere is $2(i - r)$



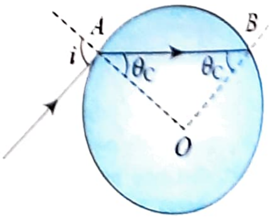
Here, deviation $\delta = 60° = 2(i - r)$

$$\text{or} \quad i - r = 30°$$

$$\therefore \quad r = i - 30° = 60° - 30° = 30°$$

$$\therefore \quad \mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$$

11. (3) $\angle ABO = \angle OAB = \theta_C$



$$\sin\theta_C = \frac{1}{\mu} = \frac{2}{3}$$

Applying Snell's law at A

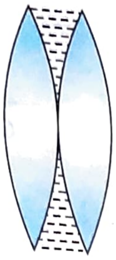
$$\frac{\sin i}{\sin\theta_C} = \frac{3}{2}$$

$$\sin i = \left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$$

$$\text{or} \quad i = 90°$$

(4) Let R be the radius of curvature of each surface. Then

$$\frac{1}{f}=(1.5-1)\left(\frac{1}{R}+\frac{1}{R}\right).$$



For the water lens,

$$\frac{1}{f'}=\left(\frac{4}{3}-1\right)\left(-\frac{1}{R}+\frac{1}{R}\right)=\frac{1}{3}\left(-\frac{2}{f}\right);$$

$$\frac{1}{f'}=-\frac{2}{3f}$$

Now, using $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$,we have

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}+\frac{1}{f'}=\frac{2}{f}-\frac{2}{3f}=\frac{4}{3f}$$

$$\Rightarrow \quad F=\frac{3f}{4}$$

(4) Only convex lens can form a real as well as virtual image. So, the given lens is a convex lens.

Let f is the focal length of the lens and n the magnitude of magnification.

In the first case, when the image is real: $u=-16$

$$v=+16n$$

So, applying $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{16n}+\frac{1}{16}=\frac{1}{f}$$

or $$1+\frac{1}{n}=\frac{16}{f} \qquad \text{...(i)}$$

In the second case, when image is virtual:

$$u=-6$$

$$v=-6n$$

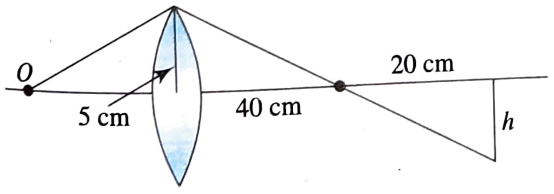
$$f=+f$$

$$\therefore \quad \frac{1}{-6n}+\frac{1}{6}=\frac{1}{f}$$

$$\therefore \quad 1-\frac{1}{n}=\frac{6}{f} \qquad \text{...(ii)}$$

Adding (i) and (ii), we get $2=\frac{22}{f}$ or $f=11$ cm

14. (3)



$$\frac{h}{20}=\frac{5}{40}=\frac{1}{8}$$

or $$h=\frac{20}{8}\text{ cm}=\frac{5}{2}\text{ cm}=2.5\text{ cm}$$

15. (4) If the refractive index of the material of the lens is greater than the refractive index of the surrounding medium, then a concave lens would behave as a concave lens.

16. (4) Clearly, power of system is zero.

$$\therefore \quad 0=\frac{1}{20}+\frac{1}{f}-\frac{5}{20f}$$

or $$-\frac{1}{20}=\frac{15}{20f} \text{ or } f=-15\text{ cm}$$

17. (3) $P=(1.5-1)\left(\frac{200}{5}\right)=20$ D

$$P'=(1.5-1)\left(\frac{200}{5}\right)=16.67\text{ D}$$

Decrease in power = 20 D – 16.67 D = 3.33 D

18. (4) Applying lens formula,

$$\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$$

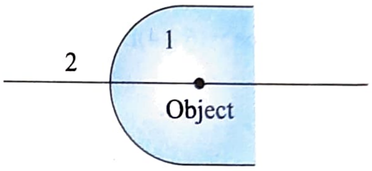
$$\frac{1}{v}-\frac{1}{-15}=\frac{1}{f}$$

or $$\frac{1}{v}-\frac{1}{15}+\frac{1}{10}$$

or $$\frac{1}{v}=\frac{-2+3}{30}$$

or $v=30$ cm

19. (1) Applying $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

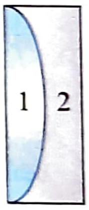


$$\frac{1}{v}-\frac{1.5}{(-u)}=\frac{1-(1.5)}{-R} \quad \text{or} \quad \frac{1}{v}+\frac{3}{2u}=\frac{1}{2R}$$

For v to be positive, $\frac{1}{2R}>\frac{3}{2u}$

or $u>3R$

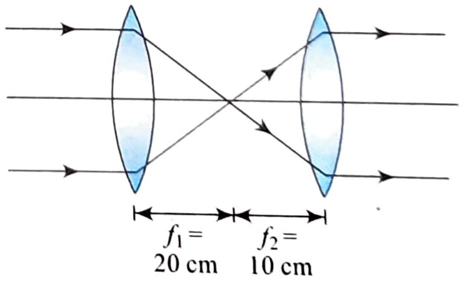
20. (1) $\frac{1}{F}=(\mu-1)\left(\frac{1}{\infty}+\frac{1}{R}\right)+(\mu_2-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)$



$$=\frac{\mu_1-\mu_2}{R}$$

or $$F=\frac{R}{\mu_1-\mu_2}$$

21. (2)



As shown in figure, the distance between the lenses should be 30 cm.

22. (4) Image will be formed at infinity if object is placed at focus of the lens, i.e., at 20 cm from the lens. Hence,

$$\text{shift}=25-20=\left(1-\frac{1}{\mu}\right)\mu$$

or $$5=\left(1-\frac{1}{1.5}\right)t \quad \text{or} \quad t=\frac{5\times1.5}{0.5}=15\text{ cm}$$

23. (1) Only one image will be formed by this lens system because optic axis of both the parts coincide. Two images would have formed if their optic axis were different.

24. (3) $m = \frac{f}{f+u}, \frac{1}{2} = \frac{f}{f-2}$; or $2f = f - 2$ or $f = -2$ m

$|f| = 2$ metre. Since the image is virtual as well as diminished, therefore the lens is concave.

25. (2) Power of the system decreases due to separation between the lenses. So, the focal length increases.

26. (1) In the first case, neither the radii of curvature nor the material of the lens is affected. In the second case,

$$\frac{1}{f''} = (\mu - 1)\left(\frac{1}{R}\right) \text{ or}$$

$$\frac{1}{f''} = \frac{1}{2}(\mu - 1)\left(\frac{2}{R}\right) = \frac{1}{2f}$$

or $f'' = 2f$

27. (4) $\frac{I}{O} = \frac{v}{u}$

$$\frac{I}{15} = \frac{-25}{-10}$$

$I = 15 \times 2.5 \text{ cm} = 37.5 \text{ cm}$

28. (2) Power of combination = (6 – 4)D = 2D

$$\text{Power} = \frac{100}{f\,(\text{in cm})}$$

$$\Rightarrow \quad 2 = \frac{100}{f} \text{ (in cm)}$$

or $f\,(\text{in cm}) = 50$

Since the net power is positive, therefore the combination shall behave like a convex lens.

29. (1) Power of system,

$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{1} + \frac{1}{-0.25} - \frac{0.75}{(1)(-0.25)}$$

$$= 1 - 4 + 3 = -3 + 3 = 0$$

Since power of the system is zero, therefore the incident parallel beam of light will remain parallel after emerging from the system.

30. (2)

$$P = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$0 = \frac{1}{20} - \frac{1}{5} - \frac{d}{20(-5)}$$

$$\frac{d}{100} = \frac{1}{5} - \frac{1}{20} = \frac{4-1}{20} = \frac{3}{20}$$

or $d = 15$ cm

31. (4) Clearly, $2f = 20$ cm

or $f = 10$ cm

Now, $u = -15$ cm, $v = ?, f = 10$ cm

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

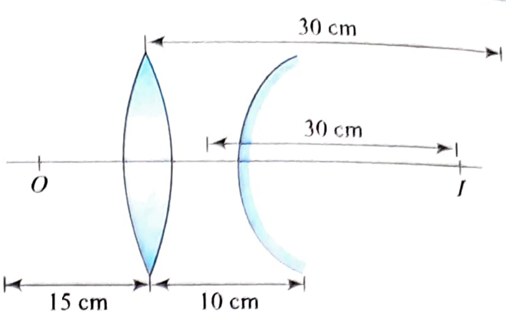
or
$$\frac{1}{v} + \frac{1}{15} = \frac{1}{10}$$

or
$$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30}$$

or $v = 30$ cm

The change in image distance is (30 – 20) cm, i.e., 10 cm.

32. (2)



$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

or
$$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} \text{ or } \frac{1}{v} = \frac{3-2}{30}$$

or $v = 30$ cm

Clearly, the rays coming from the convex lens should fall normally on the convex mirror. In other words, the rays should be directed toward the center of curvature of the convex mirror.

$\therefore \quad 2f = 20$ cm or $f = 10$ cm

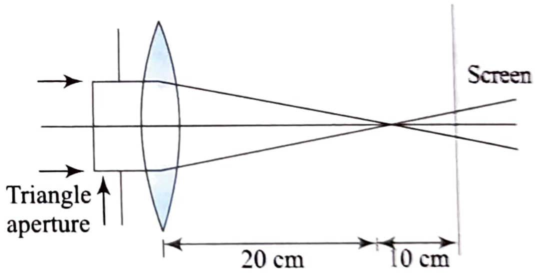
33. (4) For lens $\frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1}$

$$\frac{u_1}{f} = \frac{u_1}{v_1} - 1 \quad \Rightarrow \quad m_1 = \frac{v_1}{u_1} = \frac{f}{u_1 + f}$$

And $m_2 = \frac{f}{u_2 + f}$

$$\frac{1}{m_2} - \frac{1}{m_1} = \frac{(u_2 - u_1)}{f} \quad \Rightarrow \quad f = \frac{(u_2 - u_1)}{(m_2)^{-1} - (m_1)^{-1}}$$

34. (4)



The image on the screen is real and inverted.

The size of the image on the screen has aperture size given by

$$\text{Size} = 1.0\left(\frac{10}{20}\right) = 0.5 \text{ cm}$$

Hence, the path of light on the screen is best represented diagram (4).

35. (2) Power of liquid lens $= (1.6 - 1)\left(\frac{2}{0.20}\right) = \frac{6}{10} \times 10 = 6$ D

Power of concave lens $= -(1.5 - 1) \times 10 = -0.5 \times 10$ D

Total power of two concave lenses = –10 D

Power of system = –10 D + 6 D = –4 D

Focal length $= \frac{1}{-4} = -0.25$ m

36. (1) Since the refractive index of the liquid is greater than refractive index of glass, therefore the focal length of system has to be negative. So, options (2) and (4) are exclud

Now, $\frac{1}{24} = (1.5 - 1)\frac{2}{R}$

or $R = 24$ cm

Again, for liquid concave lens,

$$\frac{1}{f} = -(1.6-1)\left(\frac{2}{24}\right)$$

$$\frac{1}{f} = -\frac{3}{5}\times\frac{1}{12}$$

or $\frac{1}{f} = -\frac{1}{20}$ or $f = -20$ cm

Now, $\frac{1}{F} = \frac{1}{24} - \frac{1}{20} = \frac{5-6}{120}$

or $F = -120$ cm

3) $m = 30$

$v + u = xu$

or $\frac{v}{u} + 1 = x$ or $m + 1 = x$

or $x = 31$

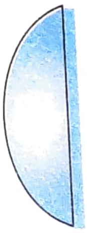
(3) $u = -100$ cm

$v = -120$ cm

$F = ?$

$$\frac{1}{F} = \frac{1}{u} + \frac{1}{v} = \frac{1}{-100} + \frac{1}{-120}$$

$$\frac{1}{F} = -\frac{6+5}{600} = -\frac{11}{600}$$



Now, $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m} = \frac{2}{f_l}$

or $f_l = 2F$

or $f_l = 2 \times \frac{600}{11}$

[Negative sign is not to be used here.]

or $f_l = \frac{1200}{11} = 109.1$ cm

(3) $\frac{\mu_2}{-\mu} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R} = 0$

In the first case, $\frac{\mu}{-d} + \frac{1}{-v} = \frac{\mu_1 - \mu_2}{\infty} = 0$

or $\frac{1}{-v} = \frac{\mu}{d}$

or $v = -\frac{d}{\mu}$

In the second case, $\frac{\mu}{-d} + \frac{1}{-v'} = \frac{1-\mu}{-R}$

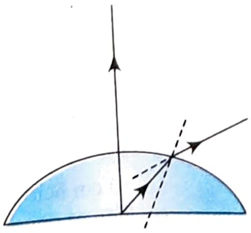
or $\frac{1}{-v'} = \frac{\mu}{-d} - \frac{1-\mu}{-R}$

or $\frac{1}{-v'} = \frac{1}{-v} + \frac{1-\mu}{-R}$

Clearly, $\frac{1}{-v'} > \frac{1}{-v}$

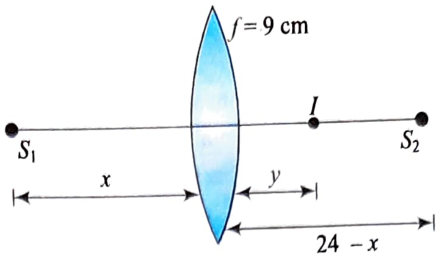
or $\frac{1}{-v'} < \frac{1}{-v}$

or $v' > v$



40. (1) In this case, one of the image will be real and the other virtual. Let us assume that image of S_1 is real and that of S_2 is virtual.

Applying $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



For S_1: $\frac{1}{y} + \frac{1}{x} = \frac{1}{9}$ ...(i)

For S_2: $\frac{1}{y} + \frac{1}{24-x} = \frac{1}{9}$ ...(ii)

Solving Eqs. (i) and (ii), we get $x = 6$ cm

41. (3) Let f_1 and f_2 be the focal length in water.

Then $\frac{1}{f_1} = \left(\frac{\mu_1}{\mu_w} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \left(\frac{\mu_1}{\mu_w} - 1\right)\left(\frac{2}{R}\right)$ ...(i)

$\frac{1}{f_2} = \left(\frac{\mu_2}{\mu_w} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right) = \left(\frac{\mu_2}{\mu_w} - 1\right)\left(\frac{-2}{R}\right)$ ...(ii)

Adding (i) and (ii), we get $\frac{1}{f_1} + \frac{1}{f_2} = \frac{2(\mu_1 - \mu_2)}{\mu_w R}$

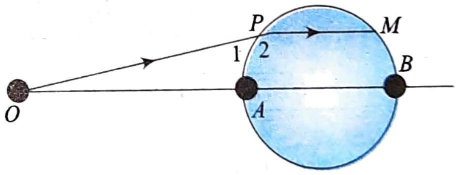
or $\frac{1}{30} = \frac{2(\mu_1 - \mu_2)}{\mu_w R}$

$\therefore$ $(\mu_1 - \mu_2) = \frac{\mu_w R}{60}$

Substituting the values, $(\mu_1 - \mu_2) = \frac{4\times 15}{3\times 60} = \frac{1}{3}$

42. (3) A ray of light starting from O gets refracted twice. The ray of light is traveling in a direction from left to right. Hence, the distances measured in this direction are taken positive.

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, twice with proper signs.



We have, $\frac{3/2}{v} - \frac{4/3}{-20} = \frac{3/2 - 4/3}{10}$

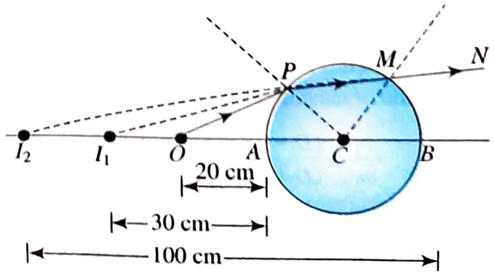
or, $v = -30$ cm

Now, the first image I_1 acts as an object for the second surface, where

$BI_1 = u = -(30 + 20) = -50$ cm

$$\frac{4/3}{v'} - \frac{3/2}{-50} = \frac{4/3 - 3/2}{-10}$$

Therefore, $v' = -100$ cm, i.e., the final image I_3 is virtual and is formed at a distance of 100 cm (toward left) from B. The ray diagram is as shown in figure.

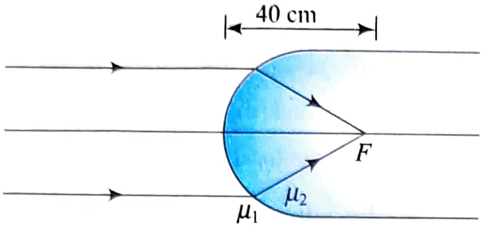


Following points should be noted while drawing the ray diagram.

i. At P, the ray travels from a rarer to a denser medium. Hence, it will bend toward normal MC.

ii. PM ray when extended backward meets at I_1 and MN ray when extended meets at I_2.

43. (2) Let us see where does the parallel rays converge or diverge on the principal axis. Let us call it the focus and the corresponding length the focal length f. Using



$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

With proper values and signs, we have

$$\frac{4/3}{f} - \frac{1.0}{\infty} = \frac{4/2 - 1.0}{+10}$$

or $\quad f = 40 \text{ cm} = 0.4 \text{ m}$

Since the rays are converging, its power should be positive. Hence,

$$P(\text{in dioptre}) = \frac{+1}{f\,(\text{meter})} = \frac{1}{0.4}$$

or $\quad P = 2.5 \text{ D}$

44. (2) Let the object distance be x. Then, the image distance is $D - x$.

From lens equation, $\dfrac{1}{x} + \dfrac{1}{D-x} = \dfrac{1}{f}$

On algebraic rearrangement, we get $x^2 - Dx + Df = 0$

On solving for x, we get $x_1 = \dfrac{D - \sqrt{D(D-4f)}}{2}$

$$x_2 = \frac{D + \sqrt{D(D-4f)}}{2}$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D-4f)}$$

45. (2) If the object is at $u = x_1$, $m_1 = \dfrac{I_1}{O} = \dfrac{D - x_1}{x_1}$

Now, $x_1 = \dfrac{1}{2}(D - d)$, where $d = \sqrt{D(D-4f)}$

$$m_1 = \frac{D - (D-d)/2}{(D+d)/2} = \left(\frac{D+d}{D-d}\right)$$

Similarly, when the object is at x_2, the magnification is

$$X_2 = \frac{1}{2}(D+d)$$

$$m_2 = \frac{I_2}{O} = \frac{D - x_2}{x_2} = \frac{D - (D+d)/2}{(D+d)/2} = \frac{D = d}{D+d}$$

The ratio of magnification is

$$\frac{m_2}{m_1} = \frac{(D-d)(D+d)}{(D+d)(D-d)} = \left(\frac{D-d}{D+d}\right)^2$$

46. (3) As $m_1 = \dfrac{I_1}{O} = \dfrac{v_1}{u_1}$ and $m_2 = \dfrac{I_1}{O} = \dfrac{v_2}{u_2}$

$$m_1 \times m_2 = \frac{I_1 I_2}{O^2} = \frac{v_1}{u_1} \times \frac{v_2}{u_2} = 1$$

Hence, $O = \sqrt{I_1 I_2} = \sqrt{h_1 h_2}$

47. (2) As $d = \sqrt{D(D-4f)}$

$$d^2 = D^2 - 4Df$$

$$f = \frac{D^2 - d^2}{4D}$$

48. (4) As $m_1 - m_2 = \dfrac{D+d}{D-d} - \dfrac{D-d}{D+d} = \dfrac{4Dd}{D^2 - d^2}$

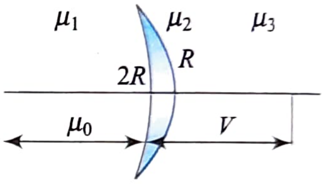
Hence, $m_1 - m_2 = \dfrac{d}{f}$

$$f = \frac{d}{m_1 - m_2}$$

49. (2) $\dfrac{\mu_2}{V} - \dfrac{\mu_1}{-u_0} = \dfrac{\mu_2 - \mu_1}{-2R}$

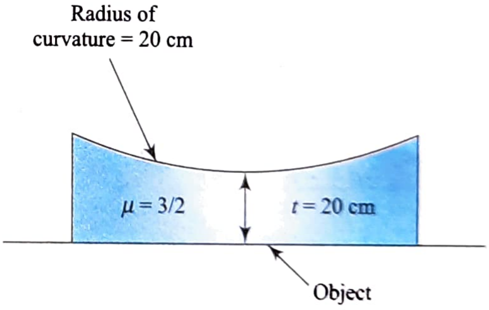
$$\frac{\mu_3}{V_f} - \frac{\mu_2}{V} = \frac{\mu_3 - \mu_2}{-R}$$

$$\frac{\mu_3}{V_f} + \frac{\mu_1}{u_0} = \frac{\mu_1 - \mu_2}{2R} + \frac{\mu_2 - \mu_3}{R}$$



$$\therefore \frac{\mu_1 - \mu_2}{2} < \mu_3 - \mu_2 \Rightarrow \mu_1 - \mu_2 < 2\mu_3 - 2\mu_2 \Rightarrow \mu_1 + \mu_2 < 2\mu_3$$

50. (1)



Considering refraction at the curved surface,

$$u = -20; \quad \mu_2 = 1$$

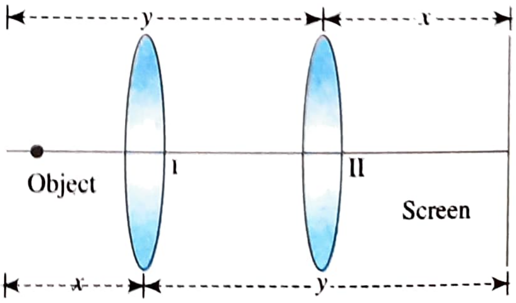
$$\mu_1 = 3/2; \quad R = +20$$

Applying $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$$\frac{1}{v} - \frac{3/2}{-20} = \frac{1 - 3/2}{20} \Rightarrow v = -10$$

i.e. 10 cm below the curved surface or 10 cm above the actu position of flower.

51. (2)



At first position of lens, let the distance of lens from object and screen be x and y, respectively.

$\therefore \quad x + y = 100 \quad$ ...(i)

$\therefore$ At second position of lens, the distance of lens from object and screen shall be y and x, respectively.

$\therefore \quad y - x = 40 \quad$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$y = 70 \text{ cm} = \frac{700}{100} \text{ m and } x = 30 \text{ cm} = \frac{30}{100} \text{ m}$$

Therefore, the power of lens is

$$\frac{1}{f} = \frac{1}{y} + \frac{1}{x} = \frac{100}{70} + \left(\frac{100}{30}\right) = \frac{100}{21} \approx 5 \text{ diopter}$$

52. (1) Use refraction formulae separately; that is for air and $\mu = 1.6$; and for air and $\mu = 2.0$ and positions of the two images.

53. (2) Cutting a lens in transverse direction doubles their focal length, i.e., $2f$.

Using the formula of equivalent focal length,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \frac{1}{f_4}$$

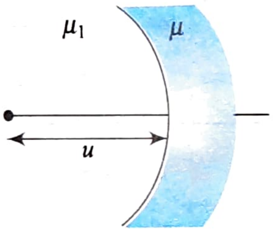
We get equivalent focal length as $\frac{f}{2}$.

54. (4) We have $\frac{\mu}{v} - \frac{\mu_1}{-u} = \frac{\mu - \mu_1}{-R}$

$$\frac{\mu}{v} = -\frac{\mu_1}{-u} + \frac{\mu_1 - \mu}{R}$$

$$\frac{\mu}{v} = \frac{-\mu_1 R + \mu_1 u - \mu u}{uR}$$

$$v = \frac{\mu u R}{(\mu_1 - \mu)u - \mu_1 R}$$



Radius = R

For real image, v should be positive

So, $(\mu_1 - \mu_2)u - \mu_1 R > 0$ for real image

i.e., $\mu_1 - \mu > \frac{\mu_1 R}{u}$

or $\quad \mu_1 \left(1 - \frac{R}{u}\right) > \mu$

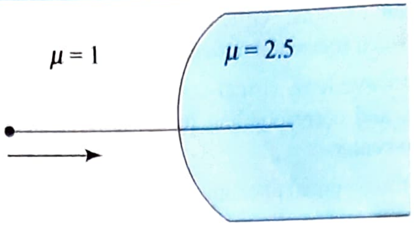
$$\mu_1 > \mu \left[1 - \frac{R}{u}\right]^{-1}$$

which depends upon the location of object.

55. (3) $\frac{2.5}{v} - \frac{1}{-u} = \frac{2.5 - 1}{R}$

$$\frac{2.5}{v} = \frac{1.5}{R} - \frac{1}{u}$$

$$v = \frac{2.5uR}{1.5u - R}$$



At $\quad v = \infty, 1.5\,u - R = 0$

$$u = \frac{2R}{3}$$

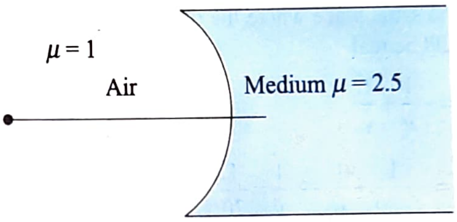
For $\quad u < \frac{2R}{3}, v = 0$, virtual image

$\quad u > \frac{2R}{3}, v = 0$, real image

So, image is changing from real to virtual at $u = \frac{2R}{3}$

56. (2) $\frac{2.5}{v} - \frac{1}{-u} = \frac{1.5}{-R}$

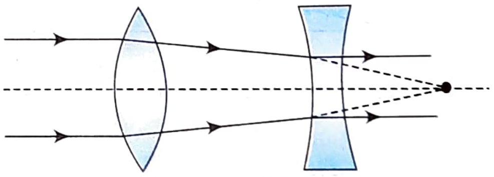
$$v = -\frac{2.5uR}{1.5u + R}$$



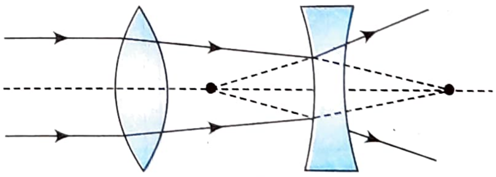
$v < 0$ for all values of u, so the image remains virtual.

57. (2) Initially, the object for lens B (the image formed by lens A) is at its focus, so rays are parallel.

As the separation between A and B decreases, the object for B is lying away from focus and hence, the rays are diverged.



Initial position



Later position

58. (1) Due to the insertion of glass plate, the image has to be shifted by 0.3 cm.

Shift produced by a glass slab of thickness t and refractive index μ is given by $t\left(1 - \frac{1}{\mu}\right)$.

So, $\quad t\left[1 - \frac{1}{1.6}\right] = 0.3$

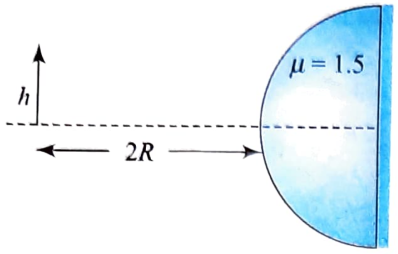
$\Rightarrow \quad t = 0.8$ cm

59. (4) Both the lenses (convex or concave) form virtual image of a real extended object. In both the cases, image would be erect.

For convex lens, virtual image will form when object is in-between focus and optical center.

For concave lens, virtual image will form for any location of object, and corresponding image is lying between focus and optical center.

60. (2) Here, three optical phenomena take place—first refraction, then reflection, and finally refraction.



For 1st refraction, $\dfrac{1.5}{v} - \dfrac{1}{-2R} = \dfrac{1.5-1}{R}$

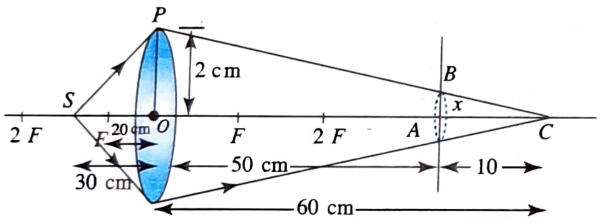
$$\frac{1.5}{v} = 0 \Rightarrow v = \infty$$

i.e., rays after refraction get parallel to the axis and strike the mirror normally, get retraced back, and final image is formed at the same place where the object is and of same size. Image would be real.

61. (2) $\dfrac{1}{f} = -\dfrac{1}{u} + \dfrac{1}{v}$

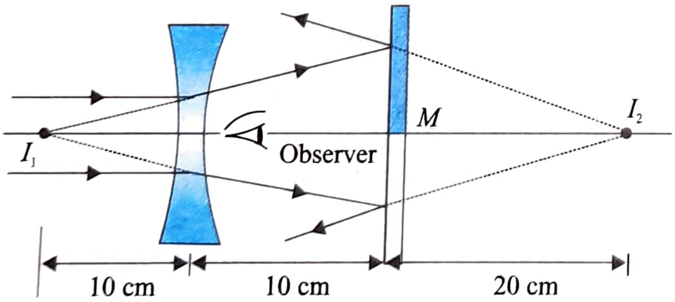
$$\frac{1}{20} = -\frac{1}{-30} + \frac{1}{v} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}$$

$v = 60$ cm



As triangles OPC and ABC are similar, $\dfrac{2}{60} = \dfrac{x}{10} \Rightarrow x = \dfrac{1}{3}$ cm

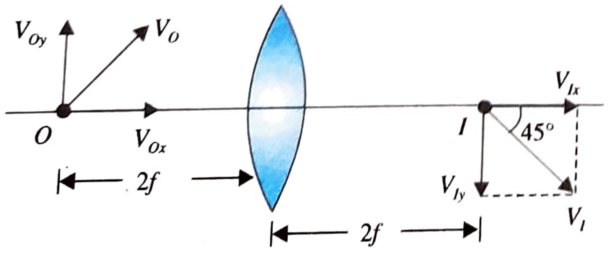
62. (3)



I_1 is the image formed by lens which will act as object for mirror. Mirror will form image I_2 of I_1.

Such that $MI_1 = MI_2$.

3. (4) Object is at $2f$, so image is also formed at $2f$.



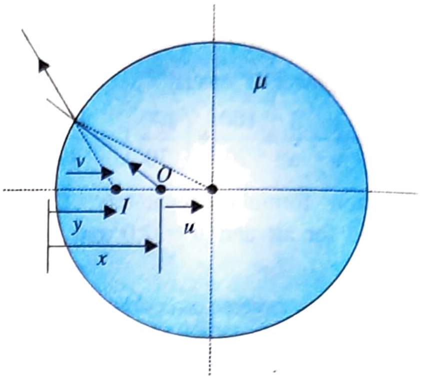
$V_{Iy} = V_{Oy}$ and $V_{Ix} = V_{Ox}$

64. (3) $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$. Here, $v = 15$ cm and $u = 10$ cm.

Hence, $\dfrac{1}{f} = \dfrac{1}{15} - \dfrac{1}{10} = -\dfrac{1}{30}$ cm

$|f| = 30$ cm

65. (2) $\dfrac{1}{-y} - \dfrac{\mu}{-x} = \dfrac{1-\mu}{-R} \Rightarrow \dfrac{1}{y} = \dfrac{1-\mu}{R} + \dfrac{\mu}{x}$ ...(i)



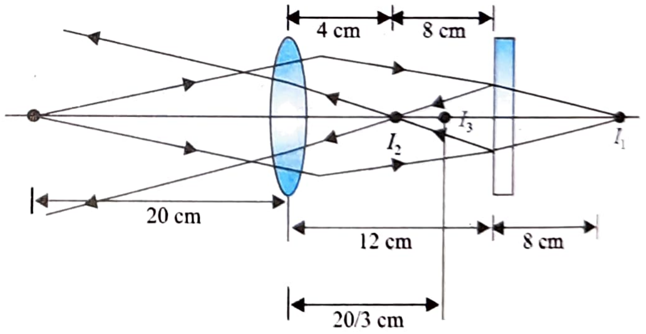
When object is at center: $x = R$.
Putting $x = R$, we get $y = R$.
Differentiating (i), we get

$$-\frac{1}{y^2}\frac{dy}{dt} = -\frac{\mu}{x^2}\frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \mu\left(\frac{y}{x}\right)^2\frac{dx}{dt}$$

$$\Rightarrow \quad v = \mu\left(\frac{R}{R}\right)^2 u \Rightarrow v = \mu u$$

66. (1) For diverging lens, all refracted rays corresponding to an incident beam consisting of parallel rays meet at a point in focal plane when produced backward.

67. (2) I_1 is the image of O by lens. I_2 is the image of I_1 by mirror. Now for lens $\dfrac{1}{v} - \dfrac{1}{-4} = \dfrac{1}{10}$



$$\frac{1}{v} = \frac{1}{10} - \frac{1}{4} \Rightarrow v = \frac{-20}{3}\text{cm}$$

Observer will see between lens and mirror I_3.

68. (2) For initial diverging lens, $\dfrac{1}{-F} = (\mu - 1)\left(\dfrac{2}{-R}\right)$

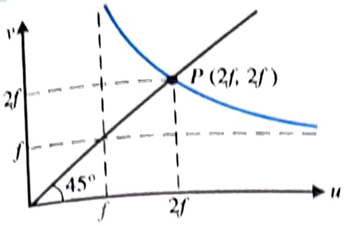
$$\Rightarrow \frac{1}{F} = \frac{(\mu-1)2}{R} \qquad \text{...(i)}$$

For final combination: $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3}$

$$\Rightarrow \frac{2(\mu-1)}{R} = (\mu-1)\left[-\frac{1}{R}\right] + (\mu_{\text{liq}} - 1)\frac{2}{R} + (\mu-1)\left[-\frac{1}{R}\right]$$

$$\Rightarrow \mu = \frac{\mu_{\text{liq}}+1}{2} = \frac{3+1}{2} = 2$$

(1)



(3) For plano convex lens; $\frac{1}{f}=\frac{1}{40}=\frac{(\mu-1)}{R}$ ...(i)

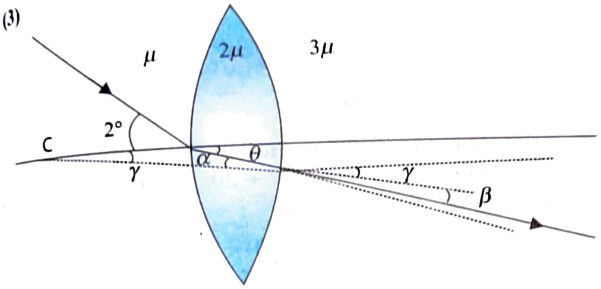
For left lens: $\frac{1}{v_1}-\frac{1}{-10}=\frac{1}{20} \Rightarrow v_1=-20 \text{ cm}$

So image is formed 20 cm to left of lens. Distance of image from plano-convex lens is $d+20$. For left surface of plano-convex lens:

$$\frac{\mu}{\infty}-\frac{1}{-(d+20)}=\frac{\mu-1}{R} \Rightarrow \frac{1}{d+20}=\frac{1}{40}$$

$\Rightarrow$ $d=20$ cm

1. (3)



$$2\mu\theta=\mu(2^\circ) \Rightarrow \theta=1^\circ$$
$$\alpha=\theta-\gamma$$
$$\beta(3\mu)=(2\mu)\alpha \Rightarrow \beta=\frac{2}{3}(\theta-\gamma)$$

Angle made by emergent ray with optic axis is

$$=\beta+\gamma$$
$$=\frac{2}{3}(\theta+\gamma)+\gamma=\frac{2}{3}\theta-\frac{\gamma}{3}\simeq\frac{2}{3}\times 1^\circ=\left(\frac{2}{3}\right)^\circ$$

[because $\gamma\simeq 0^\circ$ for thin lens]

72. (3) The lens is concave $m=\frac{v}{u}=\frac{1}{4}=\frac{x}{4x}$

Distance between object and image $=3x$

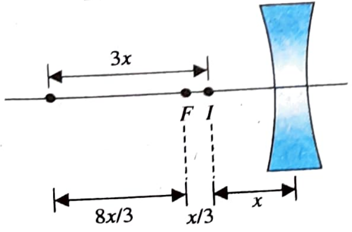
$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=-\frac{1}{x}+\frac{1}{4x} \Rightarrow f=-\frac{4x}{3}$$

Distance between object and focus $=4x-\frac{4x}{3}=\frac{8x}{3}$

Distance between image and focus $=\frac{4x}{3}-x=\frac{x}{3}$

Thus, required ratio is 8:1

See the diagram, $\frac{OF}{FI}=\frac{8}{1}$



73. (3) $m=+2;\ u=-x$

$v=-2x$

$$-\frac{1}{2x}+\frac{1}{x}=\frac{1}{f} \quad ...(i)$$

Similarly, $m=-4,\ u=-y$, and $v=+4y$

$$\frac{1}{4y}+\frac{1}{y}=\frac{1}{f} \quad ...(ii)$$

Equating (i) and (ii), $\frac{1}{2x}=\frac{5}{4y}$

$x=0.4y$ or $y=2.5x$

Required ratio of images $=4y:2x=(4)(2.5x):2x=5:1$

74. (4) When cut horizontally, focal length remains same as is clear from the following formula:

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R}-\frac{1}{(-R)}\right]=\frac{2(\mu-1)}{R} \quad ...(i)$$

When cut vertically, focal length is doubled.

$$\frac{1}{f'}=(\mu-1)\left[\frac{1}{R}-\frac{1}{\infty}\right]=\frac{(\mu-1)}{R}=\frac{1}{2f}$$

Combining these factors, $f'=2f$

75. (2) $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$\mu_2=1,\ \mu_1=\mu,\ u=-v,\ R=-R$

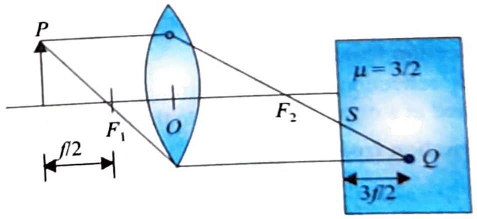
$$\frac{1}{v}-\frac{\mu}{(-u)}=\frac{1-\mu}{(-R)}$$
$$\frac{1}{v}=\frac{u(\mu-1)-\mu R}{uR}$$
$$v=\frac{uR}{u(\mu-1)}$$

For $v=\infty$, $u(\mu-1)-\mu R=0$

$$u=\frac{\mu R}{(\mu-1)}$$

76. (4) $u=-\left(f+\frac{f}{2}\right)=-\frac{3f}{2}$

Distance of final image from slab is $\frac{3}{2}f$. Distance of image formed due to convex lens from faces of the slab facing the lens $=\frac{(3/2)f}{\mu}=f$



Let $F_2S=x$

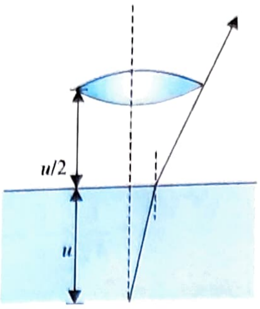
$v=2f+x$

From $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{(x+2f)}-\frac{1}{(-3f/2)}=\frac{1}{f}$$

or $x=f$

77. (3) For plane water surface,



$$\frac{1}{v}-\frac{4/3}{(-u)}=\frac{1-4/3}{\infty}=0$$

or $v=-0.75\,u$

For convex lens.

$$\mu_1=1,\ \mu_2=1,$$
$$u_1=-(0.75+0.5)u=-1.25u$$
$$v_1=-(u+0.5u)=-1.5u$$

Rays are coming from air and are viewed through air.

From $\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f}\Rightarrow-\frac{1}{1.5u}-\frac{1}{(-1.25u)}=\frac{1}{f}$

or $f=7.5u$

78. (2) $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$v=-3$ cm; $u=-8$ cm

$$\frac{1}{-3}-\frac{1}{-8}=\frac{-8+3}{24}=\frac{-5}{24}$$

$$f=\frac{-24}{5}\text{cm}=-4.8\text{ cm (diverging lens)}$$

79. (4) We know that $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

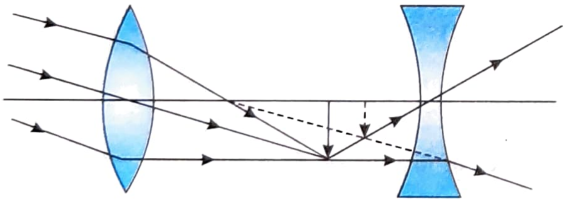
For divergence, $\mu_2>\mu_1$

Here, $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ is negative.

Therefore, (4) is the correct option.

80. (2) For the concave lens, $\frac{1}{v}-\frac{1}{4}=\frac{1}{20}\Rightarrow v=5\text{ cm};\ \frac{h_2}{h_1}=\left|\frac{v}{u}\right|$

$$\Rightarrow\quad h_2=2\times\frac{5}{4}=2.5\text{ cm}$$



81. (2) Refraction from lens: $\frac{1}{v_1}-\frac{1}{-20}=\frac{1}{15}$

$v=60$ cm +ve direction

i.e, first image is formed at 60 cm to the right of lens system.

Reflection from mirror:

After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens:

$$\frac{1}{v_3}-\frac{1}{60}=\frac{1}{15}\leftarrow+\text{ve direction}$$

or $v_3=12$ cm

Therefore, the final image is formed at 12 cm to the left of the lens system.

82. (3) From lens formula:

$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$, we have $\frac{1}{f}=\frac{1}{10}-\frac{1}{-10}\Rightarrow f=+5$

Further, $\Delta u=0.1$

and $\Delta v=0.1$ (from the graph)

Now, differentiating the lens formula, we have

$$\frac{\Delta f}{f^2}=\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}$$

$$\Delta f=\left(\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}\right)f^2$$

Substituting the values, we have $\Delta f=\left(\frac{0.1}{10^2}+\frac{0.1}{10^2}\right)(5)^2=0.05$

$\therefore\quad f\pm\Delta f=5\pm0.5$

83. (4) As given $\mu_2>\mu_1>\mu_3$

$\therefore\ \mu_2>\mu_1\Rightarrow$ lens act as a concave lens for upper half portion of lens and $\mu_3<\mu_1$

$\Rightarrow$ Lower half portion act as a convex lens.

84. (3) From figure (given in question) 1st behave as diverging and 2nd behave as converging.

In 1st, incident ray appears to pass at 5 cm distance point and after refraction it becomes parallel to principal axis i.e. the focal length $f=-5$ cm and for second $f=5$ cm.

85. (3) For plano-convex lens

$$\frac{1}{f}=\frac{1}{40}=\frac{(\mu-1)}{R}\qquad\text{...(i)}$$

For left lens: $\frac{1}{v_1}-\frac{1}{-10}=\frac{1}{20}\quad\Rightarrow v_1=-20$ cm

So image is formed 20 cm to left of lens. Distance of image from plano-convex lens is $d+20$. For left surface of plano-convex lens:

$$\frac{\mu}{\infty}-\frac{1}{-(d+20)}=\frac{\mu-1}{R}$$

$$\Rightarrow\quad\frac{1}{d+20}=\frac{1}{40}$$

$$\Rightarrow\quad d=20\text{ cm}$$

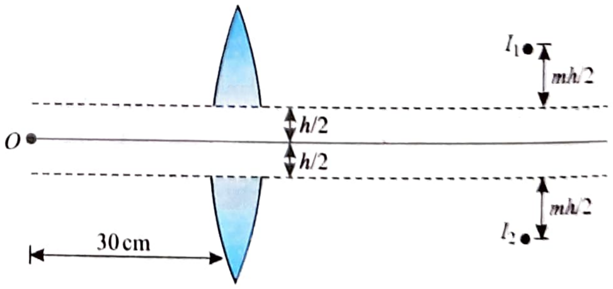
86. (1) Using lens formula: $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$. Here $u=-30$ cm

$$\frac{1}{v}-\frac{1}{-30}=\frac{1}{f}\Rightarrow v=\frac{30f}{30-f}$$

And magnification, $m=\frac{v}{u}=\frac{-f}{30-f}$

Hence separation between the images,

$$d=\frac{h}{2}+\frac{h}{2}+(m)\left(\frac{h}{2}\right)+m\left(\frac{h}{2}\right)=(m+1)h$$



$$\Rightarrow\quad d=\left(\frac{2f-30}{f-30}\right)h\qquad\text{...(i)}$$

From given graph, the slope of the line $=\frac{6}{2}=3$

$$\frac{2f-30}{f-30}=3\Rightarrow f=60\text{ cm}$$

(3) A convex mirror and a concave lens always produce virtual image for the objects. Therefore, option (2) and (4) are not correct. The image by a convex lens is diminished when the object is placed beyond $2f$.

Let $u = 2f + x$.

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$,

$$\Rightarrow \quad \frac{1}{v} - \frac{1}{-(2f+x)} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{2f+x} = \frac{2f+x-f}{f(2f+x)} = \frac{(f+x)}{f(2f+x)}$$

But $u = v = 1$ (given) $(2f + x) + \frac{f(2f+x)}{f+x} \le 1$

$$\Rightarrow \quad 2f = x\left[1 + \frac{f}{f+x}\right] \le 1 \Rightarrow \frac{(2f+x)^2}{f+x} \le 1$$

$$\Rightarrow \quad (2f+x)^2 \le f + x$$

The above is true for $f < 0.25$ m. Hence option (3) is the correct answer.

ultiple Correct Answers Type

1. (1), (2), (3), (4)

$$\frac{h_1}{h_2} = 4$$

$$\frac{h_1}{h_2} \times \frac{h_2}{h_0} = 1 \Rightarrow \frac{h_2}{h_0} = \frac{1}{4}$$

When shorter image is formed, magnification = 2

$$\Rightarrow \quad \left|\frac{v}{u}\right| = 2 \text{ and } |v| + |u| = 96$$

$$\Rightarrow \quad |v| = 64 \text{ and } |u| = 32$$

Hence, object distance = 32 cm and second lens position can be 96 – 32 = 64 cm

Distance between two positions = 32 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{64} - \frac{1}{(-32)} = \frac{1}{f} \quad \Rightarrow \quad f = \frac{64}{3} \text{ cm}$$

2. (2), (3)

$$\frac{1}{100-x} = \frac{1}{-x} = \frac{1}{21}$$

⇒ x, The distance of object from the lens is 30 cm, 70 cm

$$m_1 = \frac{70}{-30},\ m_2 = \frac{30}{-70} \quad \therefore |m_1 - m_2| = \frac{40}{21}$$

3. (1), (2), (3), (4)

$$v + u = D \text{ and } v - u = x$$

$$\Rightarrow \quad v = \frac{D+x}{2},\ u = \frac{D-x}{2}, \text{ and } f = \frac{D^2 - x^2}{4D}$$

$$m_1 = \frac{D+x}{D-x},\ m_2 = \frac{D-x}{D+x}$$

4. (1), (3)

$$\frac{v}{u} = \frac{3}{1} \Rightarrow v = 3u$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f};\ \frac{1}{3u} + \frac{1}{u} = \frac{1}{f}$$

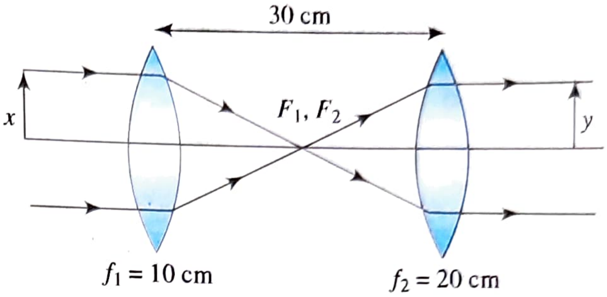
$f = 30$ cm.

5. (1), (2), (3)

For a real object, a real image can be formed by convex lens only (and not by concave lens) and depending on the location of object from infinity to focus, the image can be enlarged or diminished. But when object is in between focus and optical center, image would be virtual. So, $f \le 25$ cm.

6. (1), (3)

Here, $\frac{x}{10} = \frac{y}{20}$



$y = 2x$

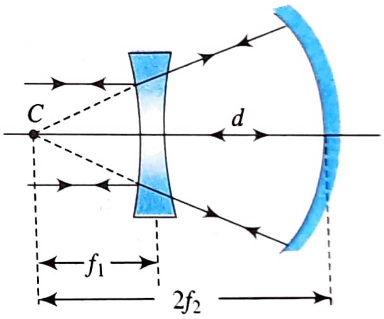
So, if $x = 1$ cm, $y = 2$ cm

7. (1), (3)

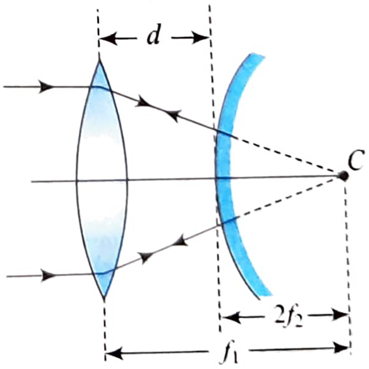
In case of part A, radii of the two surfaces will remain same. Hence, focal length of part A will be same as that of complete lens, i.e., power of part A will remain unchanged, i.e., power of A is P.

In case of part B, radius of one surface will remain same while that of other plane surface will be ∞. Hence, focal length of part B will be double that of whole lens, i.e., power of part B is $P/2$.

8. (1), (2)



9. (1), (2)



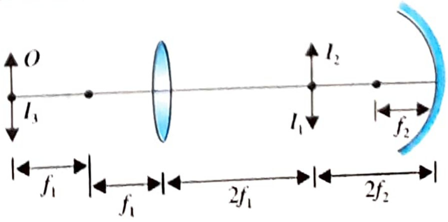
10. (1), (3)

I_1 is image of O through lens.

I_2 is image of I_1 through mirror.

I_3 is image of I_2 through lens.

I_3 is of same size as O and inverted w.r.t. O.



11. (1), (3)

$$\frac{1}{f_{\text{air}}}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_{\text{water}}}=\left(\frac{3/2}{4/3}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

From these two equations we get, $f_{\text{water}}=4f_{\text{air}}=4f$

In air image was inverted, real and magnified. Therefore, object was lying between f and $2f$. Now the focal length has changed to $4f$. Therefore, the object now lies between pole and focus. Hence, the new image will be virtual and magnified.

12. (1), (2), (4)

For plano convex lens $\frac{1}{f}=\frac{(\mu-1)}{R}$

For equiconvex lens $\frac{1}{f}=(\mu-1)\frac{2}{R'}$

$R=\frac{R}{2}$ [choice (1) is correct]

Let F be the focal length of silvered plano-convex lens

$$\frac{1}{F}=\frac{2(\mu-1)}{R}+\frac{1}{\infty}=\frac{2(\mu-1)}{R}$$

Given $\frac{1}{f}=\frac{1}{20}=\frac{(\mu-1)}{R}$

$F=10$ cm (it acts on concave mirror) [choice (4) is correct]

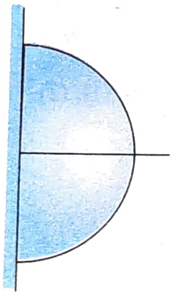
$$\frac{1}{u}+\frac{1}{v}=\frac{1}{F}$$

If $u=-15$ cm, $\frac{1}{v}=-\frac{1}{10}+\frac{1}{15}=\frac{-3+2}{30}=\frac{1}{-30}$

$v=-30$ cm [choice (2) is correct]

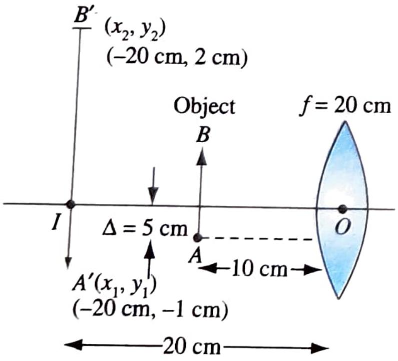
If $u=-20$ cm, $\frac{1}{v}=\frac{1}{-10}+\frac{1}{20}=\frac{1}{-20}$

$\Rightarrow v=-20$ cm [choice (3) is wrong]



13. (1), (2), (4)

$u=-10$ cm, $f=20$ cm



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{20}-\frac{1}{10}=\frac{1-2}{20}=\frac{-1}{20}\Rightarrow v=-20\text{ cm}$$

Magnification, $m=\frac{v}{u}=\frac{-20}{-10}=2$

Height of image $=1.5\times2=3$ cm

The y_1 coordinate of a point A on the image will be

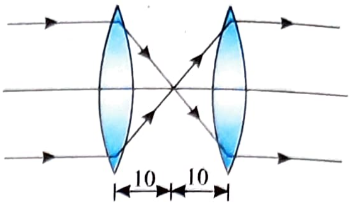
$y_1=(-0.5)\times2=-1$ cm

The y_2 coordinate of a point B on the image will be

$y_2=1\times2=2$ cm

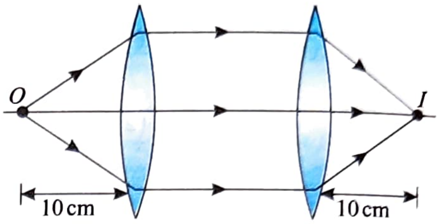
14. (1), (3)

The focal length of each of the lens should be 10 cm.

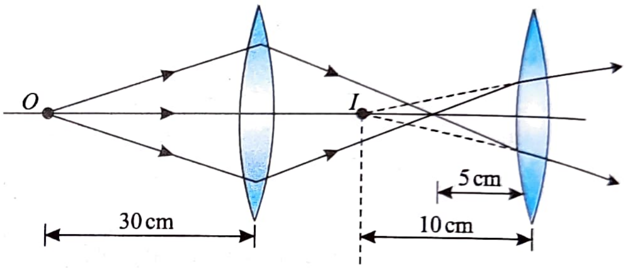


$f_1=f_2=10$ cm

Option (1): From figure it is clear, option (1) is correct and option (2) is wrong.



Option (3): From first lens, $\frac{1}{v}-\frac{1}{-30}=\frac{1}{10}\Rightarrow v=+15$ cm



From second lens $\frac{1}{v}-\frac{1}{-5}=\frac{1}{10}\Rightarrow v=-10$ cm

Hence option (3) is correct and option (4) is wrong.

15. (1), (3), (4)

Parallel ray will pass through focus after passing through lens.

$\therefore$ Slab will shift the image by an amount of

$$=t\left[1-\frac{1}{\mu}\right]=3\left[1-\frac{2}{3}\right]=1\text{ cm}$$

Final image will form at 11 cm

If converging rays meet at the point of image, image is known as real. If diverging rays meet at the point of image, image is known as virtual.

16. (2), (3)

For the 1st refraction through the lens

$$\frac{1}{v}+\frac{1}{40}=\frac{1}{20}\Rightarrow\frac{1}{v}=\frac{1}{20}-\frac{1}{40}=\frac{1}{40}\Rightarrow v=40\text{ cm}$$

Distance of this image from the mirror is $d=70-40=30$, i.e. the centre of curvature of the mirror, therefore ray retraces its path and image coincides with the object & it will be inverted

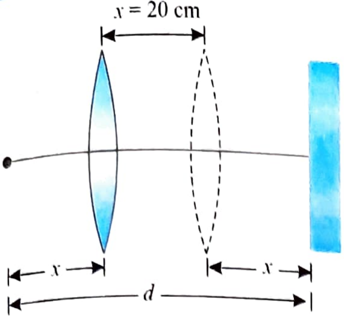
17. (1), (3)

In case of displacement method

$$m_1m_2=1\Rightarrow\left(\frac{h_{i_1}}{h_0}\right)\left(\frac{h_{i_2}}{h_0}\right)=1$$

$$\Rightarrow\quad h_0=\sqrt{h_{i_1}h_{i_2}}=6\text{ cm}$$

For first position of the lens



$$m = -2 = -\frac{x+20}{x}$$

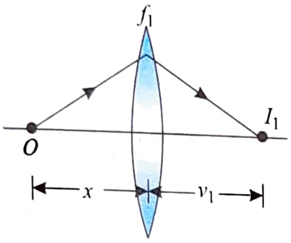
$$\Rightarrow \quad x = 20 \text{ cm} \Rightarrow D = 60 \text{cm}$$

From displacement method

Using $f = \frac{D^2 - x^2}{4D} = \frac{60^2 - 20^2}{4 \times 60} = \frac{40}{3}$cm

(2), (4)

The image form by 1st lens

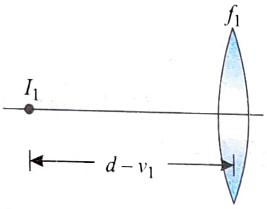


$$u_1 = -x$$

$$\frac{1}{v_1} - \frac{1}{-x} = \frac{1}{f_1} \Rightarrow v_1 = \frac{x f_1}{x - f_1}$$

$$m_1 = \frac{v_1}{u_1} = \frac{v_1}{-x} = -\left(\frac{f_1}{x - f_1}\right)$$

The image form by 2nd lens



$$u_2 = -(d - v_1)$$

$$\frac{1}{v_2} - \frac{1}{-(d - v_1)} = \frac{1}{f_2}$$

$$v_2 = \frac{(d - v_1) f_2}{d - v_1 - f_2}$$

$$m_2 = \frac{v_2}{-(d - v_1)} = -\left(\frac{f_2}{d - v_1 - f_2}\right)$$

$$m_1 m_2 = \left(\frac{f_1}{x - f_1}\right)\left(\frac{f_2}{d - \frac{x f_1}{(x - f_1)} - f_2}\right)$$

$$= \frac{f_1 f_2}{x(d - f_1 - f_2) - d f_1 + f_1 f_2}$$

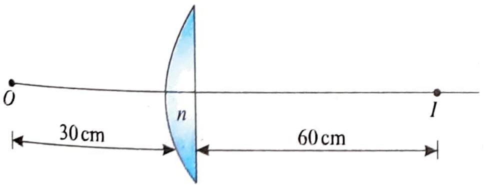
As m is independent of x

$$\Rightarrow \quad (d - f_1 - f_2) = 0 \Rightarrow d = f_1 + f_2$$

$$\therefore \quad m = -\frac{f_2}{f_1}$$

19. (1), (4)

Case-1: Image formation from lens.



For lens using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

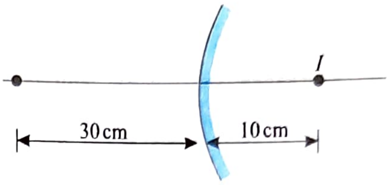
$$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f_1} \Rightarrow \frac{1}{f_1} = \frac{1}{60} + \frac{2}{60} \Rightarrow f_1 = 20 \text{ cm}$$

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\left(\frac{1}{(+R)} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad \frac{1}{f_1} = \frac{(n-1)}{R}$$

$$\text{or} \quad f_1 = \frac{R}{(n-1)} = 20 \text{ cm} \qquad \ldots\text{(i)}$$

Case-2: Image formation from reflection from convex surface of the lens using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.



$$\frac{1}{(+10)} + \frac{1}{(-30)} = \frac{1}{f_2}$$

$$f_2 = 15 \text{ cm} = \frac{R}{2} \Rightarrow R = 30 \text{ cm} \qquad \ldots\text{(ii)}$$

$$\frac{R}{n-1} = +20 \text{ cm} = \frac{30}{n-1}$$

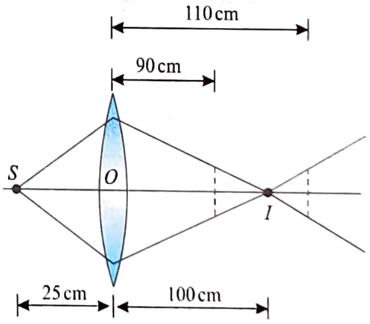
$$2n - 2 = 3 \Rightarrow n = \frac{5}{2} \qquad \ldots\text{(iii)}$$

From Eqs. (i), (ii) and (iii), $f_1 = +20$ cm

20. (2), (3), (4)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-25)} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{25} = \frac{1}{100} \Rightarrow v = 100 \text{ cm}$$



From O to I intensity increases and then decreases.

At $x = 90$ cm and 110 cm the intensity is same. Radius at $x = 200$ cm is equal to the radius of lens.

21. (1), (2)

Apply the refraction formula through the curved surface :

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{3}{2v} - \frac{1}{-120} = \frac{\frac{3}{2} - 1}{+60} \Rightarrow v = \infty$$

That means after refraction light falls on the plane mirror normally for any value of d_2.

This is the condition for object and image formed at the same point. When, $d_1 = 240$

$$\frac{3}{2v} - \frac{1}{-240} = \frac{\frac{3}{2} - 1}{+60} \Rightarrow v = 360 \text{ cm}$$

That means after refraction light falls on the pole of plane mirror if $d_2 = 360$ cm only. This condition is also for the object and the image formed at the same point.

22. (1), (3)

For real object:

$$\frac{\mu_2}{v}-\frac{\mu_1}{-u}=\frac{\mu_2-\mu_1}{R} \Rightarrow \frac{\mu_2}{v}=\frac{-\mu_1}{u}+\frac{\mu_1-\mu_2}{R}$$

- If $\mu_2 > \mu_1$ then, $v < 0$ (must be) that means (it forms virtual image.)
- $\mu_1 > \mu_2$ then (v) can be +ve or –ve both.

For virtual object:

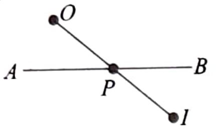
$$\frac{\mu_1}{v}-\frac{\mu_2}{u}=\frac{\mu_1-\mu_2}{R} \Rightarrow \frac{\mu_1}{v}=\frac{\mu_2}{v}+\frac{\mu_1-\mu_2}{R}$$

$$\frac{\mu_1}{v}-\frac{\mu_2}{-u}=\frac{\mu_1-\mu_2}{R} \Rightarrow \frac{\mu_1}{v}-\frac{\mu_2}{v}+\left(\frac{\mu_1-\mu_2}{R}\right)$$

- If $\mu_2 > \mu_1$ then (v) can be +ve or –ve both.
- If $\mu_1 > \mu_2$ then (v) > 0 (must be) that means real image is form.

23. (1), (4)

For the lens: Line joining *OI* intersects at pole of the lens with *AB* (optical axis)

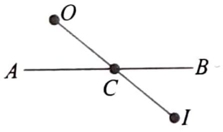


Then, lens must be converging, since ray must converge at point *I*. And the image is real. Hence, option (1) and (2) are correct.

For the mirror: Line *OI* intersects *AB* at centre of curvature, *C*.

Hence, option (2), is not correct.

- For the convex mirror, real object always forms virtual image.
- For the concave mirror, object on left side of *C* must form real image.



24. (1), (3)

$u = -40$ cm. Since, principal axis of lens is along x-axis. For the image position (apply the lens formula)

$$\Rightarrow \quad \frac{1}{v}-\frac{1}{-40}=\frac{1}{20} \Rightarrow v = 40 \text{ cm}$$

$$\text{Magnification, } m=\frac{v}{u}=\frac{40}{-40}=-1$$

Here, y and z coordinates of the object are

$$(h_i)_y = m(h_0)_y = (-1)(1) = -1$$

$$(h_i)_z = m(h_0)_z = (-1)(-1) = 1$$

Co-ordinates of image = (40 cm, – 1 cm, 1 cm)

25. (2), (3), (4)

From the displacement method: $h_0=\sqrt{h_1h_2}=\sqrt{9\times 4}=6$ cm

Hence, option (3) is correct

$$m_1=\frac{h_1}{h_0}=\frac{9}{6}=\frac{3}{2} \text{ and } m_2=\frac{h_2}{h_0}=\frac{4}{6}=\frac{2}{3}$$

From the displacement method

$$|m_1-m_2|=\frac{d}{f} \Rightarrow \frac{3}{2}-\frac{2}{3}=\frac{d}{f} \qquad \text{...(i)}$$

$$f=\frac{D^2-d^2}{4D}=\frac{90^2-d^2}{4\times 90} \qquad \text{...(ii)}$$

On solving Eqs. (i) and (ii) we get, $d = 18$ cm, $f = 21.6$ cm

Hence, option (4) is correct.

For the position of object,

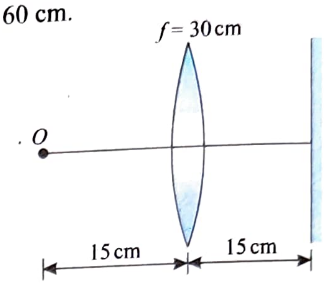
$$x_2 - x_1 = d = 18,\ x_2 + x_1 = D = 90$$

$$\therefore \quad x_1 = 36 \text{ cm},\ x_2 = 54 \text{ cm}$$

Here, x_1 and x_2 be the position of object for two positions of the lens. Hence, option (2) is correct.

26. (2), (3)

The image formed by lens will act as object for plane mirror, which form the image 45 cm from rightward of it. Again this image will act as object for the lens for which object distance should be 60 cm.



$$\text{Image formed by lens } \frac{1}{v}-\frac{1}{(-15)}=\frac{1}{30}$$

$$\frac{1}{v}=\frac{1}{30}-\frac{1}{15} \Rightarrow v=-30 \text{ cm}$$

$$\frac{1}{v}-\frac{1}{-60}=\frac{1}{30} \Rightarrow v = 60 \text{cm}$$

(2) Final image is at 60 cm from lens towards left of it.

(3) Final image is real

27. (2), (3), (4)

Height of object

$$=\sqrt{H \text{ of image 1} \times H \text{ of image 2}}=\sqrt{I_1I_2}=\sqrt{9\times 4}$$

Height of object = 6 cm

$$m_1=\frac{I_1}{O}=\frac{v_1}{u} \Rightarrow \frac{9}{6}=\frac{v_1}{u_1}$$

$$u_2=v_1=\frac{3}{2}v_1 \quad \therefore x=\frac{3}{2}u_1-u_1=\frac{u_1}{2}=x$$

x is the distance between two positions of the lens.

$$D=90=u_2+u_1=\frac{3}{2}u_1+u_1$$

$$u_1=36 \Rightarrow x=\frac{36}{2}=18$$

$\therefore$ Distance between two position of lens = x = 18

$$\text{Focal length, } f=\frac{D^2-x^2}{4D}=\frac{90^2-18^2}{4\times 90}=21.6 \text{ cm}$$

Linked Comprehension Type

For Problems 1–3

1. (3) $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

Taking refraction first at curved surface, $\frac{2}{v}+\frac{1}{x}=\frac{1}{R}$

$$\Rightarrow \qquad v_1=\frac{2Rx}{x-R}$$

For plane surface, $v' = v_1 - R$

$$\Rightarrow \qquad v'=\frac{xR+R^2}{x-R}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{2(x-R)}{R(x+R)} = 0$$

$$\therefore \quad \frac{1}{v} = \frac{2(x-R)}{R(x+R)}$$

For virtual image, $\frac{1}{v} < 0$

$$\Rightarrow \quad \frac{2(x-R)}{R(x+R)} < 0$$

$$x < R$$

(4) For $x = 2R$

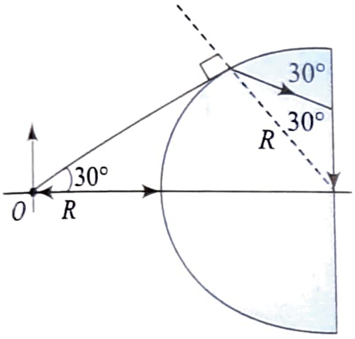
$$v_1 = \frac{4R^2}{R} = 4R \Rightarrow u = -2R$$

$$m_1 = \frac{\mu_1}{\mu_2} \cdot \frac{v}{u} = \frac{1}{2} \cdot \frac{4R}{(-2R)} = -1$$

$$m_2 = 1 \Rightarrow m_1 m_2 = -1$$

Image is real, inverted, and of same size.

(1)



Hence, correct answer is 90°

Problems 4–6

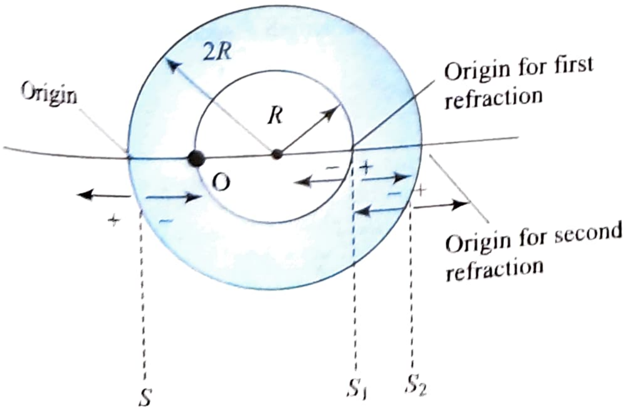
4), 5. (3), 6. (4)

From the formula of equivalent focal length of thin lenses in contact, we get

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} - \frac{1}{3f} \quad \text{or} \quad F = \frac{3f}{2}$$

Problems 7–8

(2) Viewer on the left of hollow sphere: Single refraction takes place at surface S. From the single surface refraction equation, we have



$$\frac{1}{v} - \frac{n}{(-R)} = \frac{(1-n)}{(-2R)}$$

which on solving for v yields $v = -\left(\frac{2R}{n+1}\right)$

Image is on the right of refracting surface S.

$$\text{Shift} = \text{Real depth} - \text{Apparent depth} = R - \left(\frac{2R}{n+1}\right) = \frac{(n-1)}{(n+1)}R$$

8. (2) When the viewer is on the right, two refractions take place at surfaces S_1 and S_2.

For refraction at surface S_1: $\frac{n}{v_1} - \frac{1}{(-2R)} = \frac{(n-1)}{(-R)}$

which on solving for v_1 yields $v_1 = -\frac{2nR}{2n-1}$

The first lies to the left of S_1 and acts as object for refraction at the second surface. We have to shift the origin of Cartesian coordinate system to the vertex of S_2. The object distance for the second surface is

$$u_2 = -\left[\frac{2nR}{2n-1} + R\right] = -\left(\frac{4n-1}{2n-1}\right)R$$

$$\frac{1}{v_2} = -\frac{n}{-\left[\frac{4n-1}{2n-1}\right]R} = \frac{1-n}{-2R}$$

On solving for v_2, we get $v_2 = -\frac{2(4n-1)}{(3n-1)}R$

The minus sign shows that image is virtual and lies to the left of S_2.

Shift = Real depth – Apparent depth

$$= 3R - \frac{2(4n-1)R}{(3n-1)} = \frac{(n-1)}{(3n-1)}R$$

For Problems 9–10

9. (1) The light retraces its path if it is incident normally on a mirror. The ray after refraction through the lens and the liquid are parallel. We will apply the general thin lens equation with parameters.

$$n_1 = 1, n_2 = 3/2, n_3 = 4/3, u = -15 \text{ cm, and } v = \infty$$

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R} + \left(\frac{n_3 - n_2}{R}\right)$$

$$\frac{n_3}{\infty} - \frac{1}{(-15)} = \frac{(3/2)-1}{R} - \frac{[(4/3)-(3/2)]}{R}$$

On solving for R, we get $R = 10$ cm.

10. (1) Similarly, when second liquid is filled, we have

$$\frac{n_3'}{\infty} - \frac{1}{(-25)} = \frac{(3/2)-1}{10} - \frac{n_3' - (3/2)}{10}$$

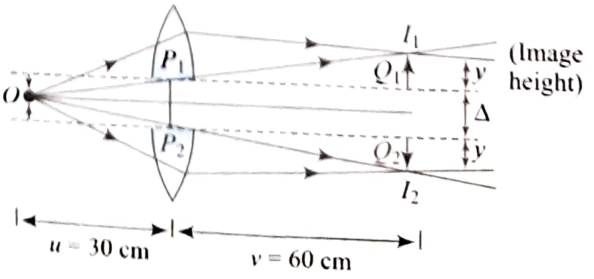
On solving for n_3', we get, $n'_3 = 1.6$

For Problems 11–13

11. (2) Each half lens will form an image in the same plane. The optic axes of the lenses are displaced,

$$\frac{1}{v} - \frac{1}{(-30)} = \frac{1}{20}, \; v = 60 \text{ cm}$$

12. (1) From similar triangles OI_1I_2 and OP_1P_2, we have



$$\frac{I_1I_2}{P_1P_2}=\frac{u+v}{u}\Rightarrow I_1I_2=\frac{90}{30}(2\times 0.05)=0.3\text{ cm}$$

Thus, the two images are 0.3 cm apart.

Alternatively, imagine two arrows(see figure) that act as objects for the lens.

Magnification, $m=\frac{v}{u}=\frac{(+60)}{(-30)}=-2$

Image of height of arrow is $y=2\times(0.05)=0.10$ cm

Thus, two inverted images are formed whose tips are at I_1 and I_2, respectively.

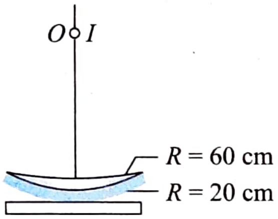
13. (3) $I_1I_2=2y+\Delta=(2\times 0.1)+0.1=0.3$ cm

For Problems 14–15

14. (3) Method 1: The optical arrangement is equivalent to the concave mirror of focal length F given by

$$\frac{1}{F}=\frac{1}{f_g}+\frac{1}{f_m}+\frac{1}{f_g}$$

where f_g is the focal length of the lens without silvering and f_m is the focal length of the mirror.



$$\frac{1}{f_g}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

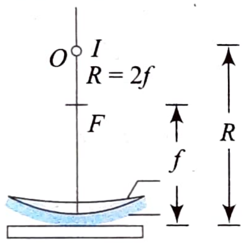
$$=(1.5-1)\left(\frac{1}{20}-\frac{1}{+60}\right)=\frac{1}{60}$$

$f_g=60$ cm

$f_g=R_1/2=20/2=10$ cm

$$\frac{1}{F}=\frac{1}{60}+\frac{1}{10}+\frac{1}{60}=\frac{8}{60}$$

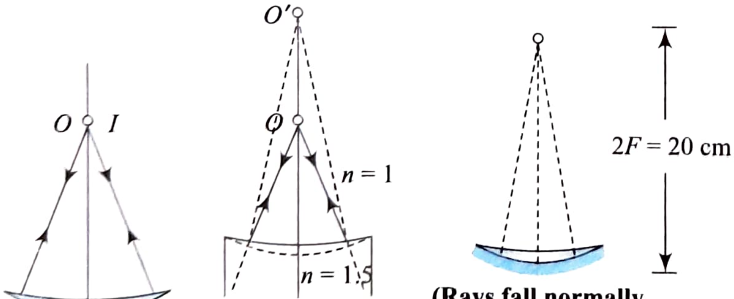
$$F=\frac{60}{8}=7.5\text{ cm}$$



For the image to be formed at the place of the object, $x=R=2f=7.5\times 2=15$ cm

Method 2: We use the relation $\frac{n_2}{x_2}-\frac{n_1}{x_1}=\frac{n_2-n_1}{R}$

For the object and the image to coincide, the rays fall normal on the reflecting surface, i.e., on the silvered face of the lens.



Let $x_2=20$ cm

(Rays fall normally on the mirror)

Then, the rays retrace backward and meet at the object point again (optical reversibility).

For the refraction at the upper surface of the lens,

$n_1=1.0$, $n_2=1.5$, $x_1=20$, $R=+60$

($x_2=+20$ ensures that the rays fall on the silvered face normally.)

$$\frac{1.5}{20}-\frac{1.0}{x_1}=\frac{1.5-1.0}{+60}$$

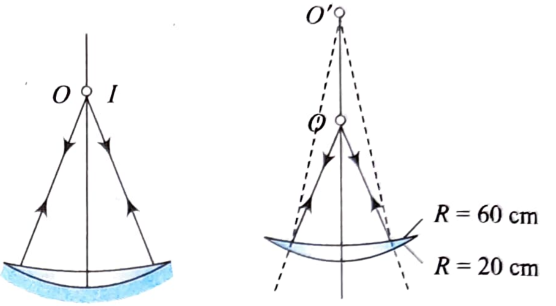
$$\frac{1.0}{x_1}=\frac{1.5}{20}-\frac{0.5}{60}=\frac{3.0}{60}$$

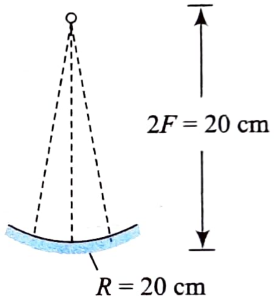
$x_1=15$ cm

Method 3: We use lensmaker's formula and the equation

$$\frac{1}{f}=\frac{1}{x_2}-\frac{1}{x_1}$$

The given optical arrangement can be visualised as a convex lens of focal length 60 cm and a concave mirror of focal length 10 cm kept in contact as shown in the figure.





If the rays fall normally on the mirror after the refraction through the lens, they will retrace backward and meet at the point of the pin again.

For the lens, $x_1=?$

$x_2=+20$ (for normal incidence on the mirror)

$f=-60$ (using cartesian–coordinate sign convention)

$$\frac{1}{-60}=\frac{1}{+20}-\frac{1}{x_1}$$

15. (1) Method 1: When the concave part is filled with water of refractive index 4/3, the optical arrangement is equivalent to concave mirror of focal length F such that

$$\frac{1}{F}=\frac{1}{f_w}+\frac{1}{f_g}+\frac{1}{f_m}+\frac{1}{f_g}+\frac{1}{f_w}$$

$$\frac{1}{f_w}=\left(\frac{4}{3}-1\right)\left(\frac{1}{60}-\frac{1}{\infty}\right)=\frac{1}{180}$$

$f_w=180$ cm

$f_g=60$ cm (calculated earlier)

$$\frac{1}{F}=\frac{1}{180}+\frac{1}{60}+\frac{1}{10}+\frac{1}{60}+\frac{1}{180}=\frac{26}{180}$$

$$F=\frac{180}{26}\text{ cm}$$

$$x_1=R=2F=\frac{180}{26}\times 2=\frac{180}{13}=13.85\text{ cm}$$

$\Delta x=15.0-13.85=1.15$ cm

ethod 2: We use the equation

$$\frac{n_2}{x_2}-\frac{n_1}{x_1}=\frac{n_2-n_1}{R}$$

or refraction at the interface '1' (air water),

$$\frac{4/3}{x_2}-\frac{1}{x_1}=\frac{4/3-1}{\infty} \quad \text{...(i)}$$

he image of interface '1' is the object for the interface '2'.

$$\frac{1.5}{+20}-\frac{1}{x_1}=\frac{1.5-4/3}{+60}$$

$$x_1=\frac{360}{26}=13.85$$

$$\Delta x=15.0-13.85=1.15\text{ cm}$$

ethod 3: Using lensmaker's formula and the relation

$$\frac{1}{F}=\frac{1}{x_2}-\frac{1}{x_1}$$

$f_m = 180$ cm (using lensmaker's formula)

$f_g = 60$ cm (using lensmaker's formula)

$$\text{For the water lens: } \frac{1}{-180}=\frac{1}{x_2}-\frac{1}{x_1} \quad \text{...(i)}$$

$$\text{For the glass lens: } \frac{1}{-60}=\frac{1}{+20}-\frac{1}{x_1} \quad \text{...(ii)}$$

(The image by the water lens is the object for the glass lens and if the image by the glass lens is at +20, then the rays will fall normally on the mirror.)

Adding Eqs. (i) and (ii),

$$\frac{1}{-180}+\frac{1}{-60}=\frac{1}{20}-\frac{1}{x_1}$$

$$x_1=\frac{180}{13}=13.85\text{ cm}$$

$$\Delta x=15.0-13.85=1.15\text{ cm}$$

Problems 16–17

(4), 17. (1)

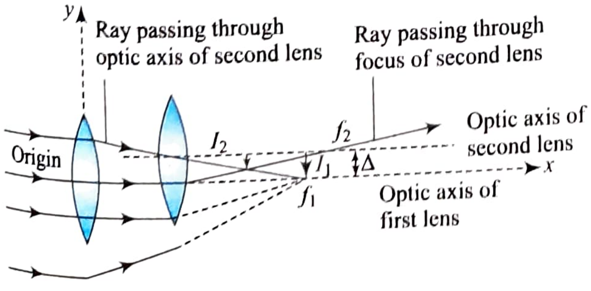
The parallel rays will be focussed at the focal point of the first lens. The first image lies at I, at a distance f_1 from the origin. This image I_1 will act as an object for refraction through the second lens. The object distance for the second lens, $u = (f_1 - d)$.

$$\text{From lens equation, } \frac{1}{v}-\frac{1}{+(f_1-d)}=\frac{1}{f_2}$$

$$v=\frac{f_2(f_1-d)}{(f_1+f_2-d)}$$

Hence, the x-coordinate of final image I_2 is

$$x=d+u=\frac{d+f_2(f_1-d)}{(f_1+f_2-d)}=\frac{d(f_1-d)+f_1f_2}{(f_1+f_2-d)}$$


y
Ray passing through optic axis of second lens
Ray passing through focus of second lens
Origin
I_2
f_2
I_1
Δ
f_1
Optic axis of second lens
x
Optic axis of first lens

Imagine an arrow tip is at f_1: its image from lens f_1 is the final image.

$$\text{Magnification, } m=\frac{v}{u}=\frac{I_2}{O}=\frac{I_2}{\Delta}$$

$$I_2=\left(\frac{v}{u}\right)\Delta=\frac{f_2}{(f_1+f_2-d)}\Delta$$

Thus, y–coordinate of tip of I_2 is

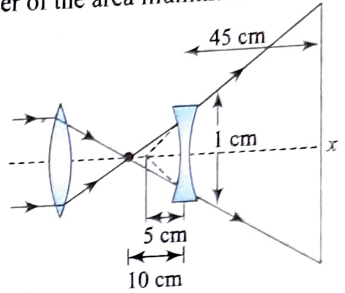
$$\Delta-I_2=\Delta\left[1-\frac{f_2}{(f_1+f_2-d)}\right] \quad \frac{(f-d)\Delta}{(f_1+f_2-d)}$$

For Problems 18–19

18. (2) In case (a), the incident parallel beam emerges as a parallel beam. So area illuminated,

$$A_1=\pi(1)^2=\pi\text{ cm}^2$$

In case (b), let x be the diameter of the area illuminated.



Then,

$$\frac{x}{45}=\frac{1}{5} \Rightarrow x=9\text{ cm}$$

$$A_2=\pi\left(\frac{9}{2}\right)^2=\frac{81}{4}\pi\text{ cm}^2$$

$$\frac{A_2}{A_1}=\frac{81}{4}$$

19. (3) When liquid of refractive index μ is filled to the right of this lens, the first surface of the lens (radius of curvature = 10 cm) forms the image at the object only. Considering the refraction at the second surface.

$$\frac{\mu}{\infty}-\frac{1.5}{-10}=\frac{\mu-1.5}{10} \quad \text{(therefore, same area} \Rightarrow v\to\infty)$$

$$\Rightarrow \quad \mu=3$$

For Problems 20–21

20. (2) Focal lengths of lenses L_1 and L_2 are, respectively, given by

$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R}-\frac{1}{\infty}\right) \Rightarrow f_1=50\text{ cm}$$

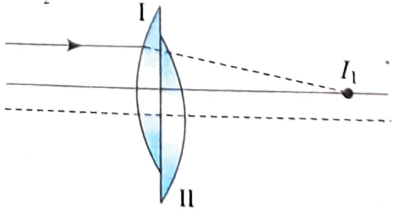
$$\frac{1}{f_2}=(\mu-1)\left[\frac{1}{\infty}-\left(-\frac{1}{R}\right)\right] \Rightarrow f_2=40\text{ cm}$$

The equivalent focal length f of the combination is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \quad \Rightarrow \quad f=\frac{200}{9}\text{ cm}$$

Hence, the image of the parallel beam is formed on the common principal axis at a distance of 22.22 cm from the combination on the right side.

21. (1) Image formed by L_1 is at a distance of 50 cm behind the lens. This image lies on the principal axis of L_1 and will act as an object for L_2.



For L_2, object distance, $u = +50$ cm

$f_2 = +40$ cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \Rightarrow \quad \mu=\frac{200}{9}\text{ cm}$$

Magnification caused by L_2, $m = \frac{v}{u} = \frac{4}{9}$

For L_2, object I_1 is at a distance of 4.5 mm above its principal axis.

Hence, distance of image I_2 of the object (virtual) I_1 is at a distance $(4/9) \times 4.5 = 2$ mm above the principal axis of L_2.

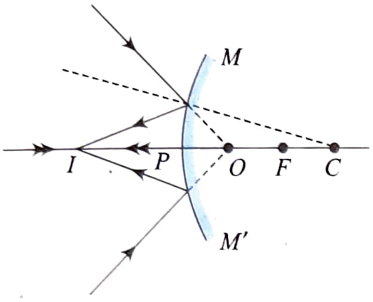
[$\because$ height of image $= m \times$ height of object]

Hence, final image is at a distance of 22.22 cm behind the combination at a distance of 2.5 mm below the principal axis of L_1.

For Problems 22–23

22. (4) For this situation, object will be virtual as shown in the figure.

Here, $u = +10$ cm and $f = +20$ cm.

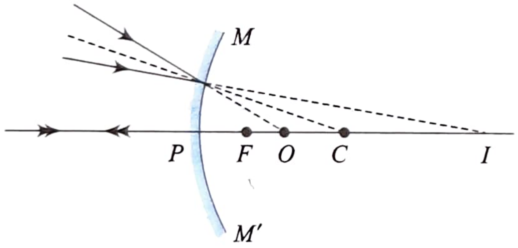


So, $$\frac{1}{v} + \frac{1}{+10} = \frac{1}{+20}$$

i.e., $v = -20$ cm

i.e., the image will be at a distance of 20 cm in front of the mirror and will be real, erect and enlarged with $m = -(20/10) = +2$.

23. (1) For this situation also, object will be virtual as shown in the figure.



Here, $u = +30$ cm and $f = +20$ cm.

$$\frac{1}{v} + \frac{1}{+30} = \frac{1}{+20} \quad \text{i.e.,} \quad v = +60 \text{ cm}$$

i.e., the image will be at a distance of 60 cm behind the mirror and will be virtual, inverted, and enlarged with $m = -(+60/30) = -2$.

For Problems 24–28

24. (4) $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Here $v = 2.5$ (distance of retina as position of image is fixed)

$u = -x$

$$\frac{1}{f} = \frac{1}{2.5} + \frac{1}{x}$$

For f_{min}: x is minimum $\frac{1}{f_{min}} = \frac{1}{2.5} + \frac{1}{25} = \frac{11}{25}$

$$f_{min} = \frac{25}{11} \text{ cm}$$

25. (2) For f_{max}: x is maximum $\frac{1}{f_{max}} = \frac{1}{2.5} + \frac{1}{\infty}$

$$f_{max} = 2.5 \text{ cm}$$

26. (2) For the near-sighted man, lens should make the image of th object within 100 cm range.

For lens, $u = -\infty$, $v = -100$ cm

$$\frac{1}{f_{lens}} = \frac{1}{-100} - \frac{1}{-\infty} = \frac{-1}{100}$$

$$P = \frac{1}{f(\text{in m})} = -1 \text{ D}$$

27. (3) For the far-sighted man, lens should make image of the nearb object at distances beyond 100 cm. For grown up person leas distance is 25 cm. For lens, $u = -25$, $v = -100$

$$\frac{1}{f} = \frac{1}{-100} - \frac{1}{(-25)}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{3}{100}$$

$P = +3$ so number of spectacle is +3

28. (4) Since he can see beyond the range of a normal eye [25 cm, ∞ therefore it is an exceptional eye having no defect.

For Problems 29–31

29. (2) $\frac{\mu}{y} - \frac{1}{\infty} = \frac{\mu - 1}{R} \Rightarrow y = \frac{\mu R}{\mu - 1}$

The above is valid for paraxial rays, hence beam should be thin.

30. (3) If beam is focussed at A, than $y = 2R$

$$\Rightarrow \quad 2R = \frac{\mu R}{\mu - 1} \Rightarrow \mu = 2$$

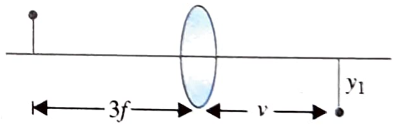
31. (4) If $\mu > 2$, then $y < 2R$, hence beam can be focussed before A.

For thin beam to be focussed at A, $y = R$.

$$\Rightarrow \quad R = \frac{\mu R}{\mu - 1} \Rightarrow \mu = \infty \qquad \text{(not possible)}$$

For Problems 32–34

32. (2), 33. (3), 34. (3)



$$\frac{1}{v} - \frac{1}{-3f} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{3f} \Rightarrow v = \frac{3f}{2}$$

$$\frac{y_1}{y} = \frac{v}{3f} \Rightarrow y_1 = y\frac{v}{3f} = \frac{y}{2}$$

If $y_{max} = A$, then $y_{1max} = A_1 = A/2$

Max velocity $= \omega A_1 = \omega A/2$

For Problems 35–37

35. (1) Since final image will coincide with the object if the ray retraces its path after reflection from the concave mirror, i.e., the ray strike the mirror normally. The normal rays at the mirror after being extended must pass through the center of curvature of the mirror for the refraction at the convex lens

$v = (2f - 12) = 24$ cm

$u = d$

$$\frac{1}{-24} - \frac{1}{-d} = \frac{1}{40}$$

$$\frac{1}{d} = \frac{1}{40} + \frac{1}{24} = \frac{3+5}{120}$$

$d = 15$ cm

(3) Here $f = -60$ cm

$$\frac{1}{-24} - \frac{1}{-d} = \frac{1}{-60}$$

$d = 40$ cm

(1) Shift produced by the glass plate,

$$\Delta x = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{2}{3}\right)6 = 2 \text{ cm}$$

For the mirror, the object is placed at a distance $[12 + 6 + d - \Delta x] = (16 + d)$

The image is coincident with the object itself if $(16 + d) = 36 \text{ cm} \Rightarrow d = 20$ cm

Problems 38–39

(2) $$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-105/2} = \frac{1.5-1}{-35}$$

$v = -45$ cm.

(1) From II$^{\text{nd}}$ surface, $\dfrac{n_2}{v} - \dfrac{n_1}{u} = \dfrac{n_2 - n_1}{R}$

$$\frac{1.5}{v} + \frac{2}{35} = \frac{1.5-1}{-35}$$

$v = -21$ cm

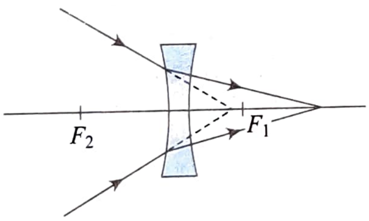
Hence separation $= 2 \times 35 - (45 + 21) = 4$ cm

atrix Match Type

1. i. → b., c.; ii. → b., c.; iii. → b., d.; iv → b., d.

(i) $\dfrac{1}{v} = \dfrac{1}{u} + \dfrac{1}{f}$

$m = \dfrac{f}{u+f}$

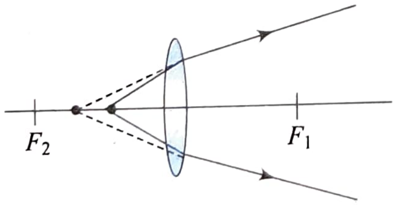


Here f is negative, u is positive and less than f.

So v will also be positive $m > 1$. So image is erect and its size is greater than object. Here object is virtual.

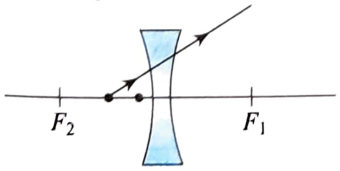
(ii) $\dfrac{1}{v} = \dfrac{1}{u} + \dfrac{1}{f}$

$m = \dfrac{f}{u+f}$



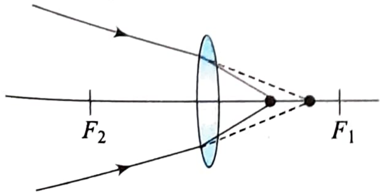
f is positive, u is negative and less than f. So v is negative. $m > 1$, so image is erect and its size is greater than object. Here, object is real.

(iii) u is negative, f is negative, and u is less than f.



$0 < m < 1$, object is real.

(iv) u is positive, f is positive.



2. i. → a., c.; ii. → b., c.; iii. → b., c.; iv. → b., c.

Initially, the image is formed at infinity.

(i) As m is increased, the focal length decreases. Hence, the object is at a distance larger than the focal length. Therefore, final image is real. Also, final image becomes smaller in size in comparison to the size of image before the change was made.

(ii) If the radius of curvature is doubled, the focal length decreases. Hence, the object is at a distance lesser than the focal length. Therefore, final image is virtual. Also, final image becomes smaller in size in comparison to the size of image before the change was made.

(iii) Due to insertion of slab, the effective object for lens shifts rightward. Hence, final image is virtual. Also, final image becomes smaller in size in comparison to the size of image before the change was made.

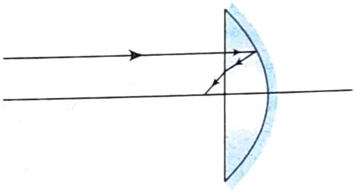
(iv) The object comes to center of curvature of right spherical surface. Hence, the final image is virtual. Also, final image becomes smaller in size in comparison to the size of image before the change was made.

3. i. → b.; ii. → b.; iii. → c.; iv. → d.

(i) A convex lens in a denser medium will behave like a concave lens or diverging lens.

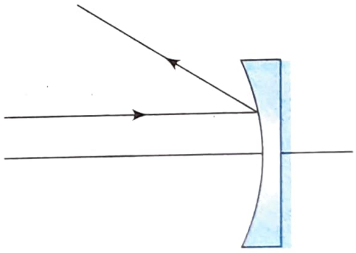
(ii) A concave lens in a rarer medium will behave like a concave lens or diverging lens.

(iii)



Behaves like concave mirror.

(iv)

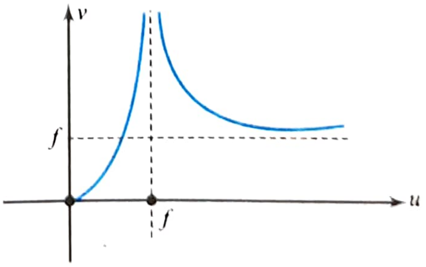


Behaves like convex mirror.

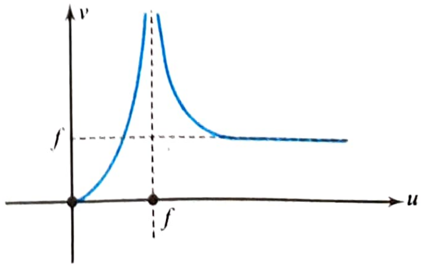
4. i. → a.; ii. → a.; iii. → b.; iv. → b.

(i) Converging lens or convex lens,

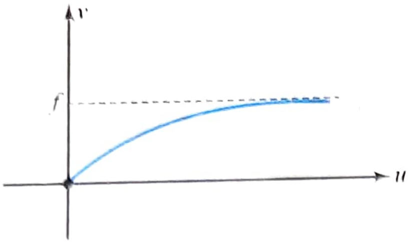
$v = \dfrac{uf}{u+f}$



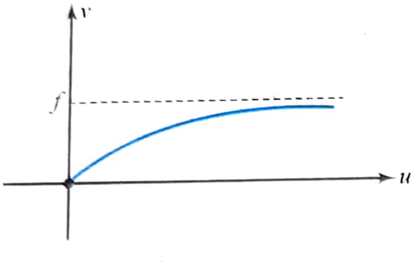
(ii) Converging mirror or concave mirror



(iii) Diverging lens or concave lens



(iv) Diverging mirror or convex mirror



5. **i. → a., ii. → b., iii. → a., iv. → c.**

(i) The focal length of mirror is independent of refractive index of the surrounding medium and hence from mirror formula, only one image can be confirmed.

(ii) Lens can be considered as two thin plano-convex lenses in contact. Since two media on other side of lens are present, two distinct focal lengths are possible and hence two images.

(iii) Same reasoning as for (ii), but since only one medium is present on the other side, only one image is formed.

(iv) Lens can be considered as two half lenses having different focal lengths and medium on the other side of upper lens is of two types while for lower lens is of only one type. So, a total of three images are possible.

6. **i. → b.; ii. → d.; iii. → c.; iv. → a.**

Real image is formed by convex lens only.

Let object distance x and image distance $= y$, then

$$x+y=40,\ \frac{y}{x}=\frac{1}{2}$$

Solve to get $x=\frac{80}{3},\ y=\frac{40}{3}$

$$\frac{1}{y}+\frac{1}{x}=\frac{1}{f} \Rightarrow \frac{3}{40}+\frac{3}{80}=\frac{1}{f} \Rightarrow f=\frac{80}{9}\text{cm}$$

If mirror were used, then concave mirror can form the real image.

7. **i. → b.; ii. → c.; iii. → a.**

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

When $u\to\infty, v\to f$; when $v\to\infty, u\to f$

$\frac{1}{v}$ vs $\frac{1}{u}$ graph will be straight line as shown in (c)

$$\frac{1}{u}=\frac{1}{f}-\frac{1}{v} \Rightarrow u=\frac{vf}{v-f}$$

$$|m|=\frac{v}{u}=\frac{v-f}{f};$$

hence, $|m|$ vs v graph will be straight line as shown in (a).

8. **i. → a., c.; ii. → b., d.; iii. → b., d.; iv. → a., c.**

For analyzing the situation, use following concepts:

The line joining object position and image position cuts the principle axis at center of curvature of mirror.

The line joining object position and image position cuts the principle axis at optical center of the lens.

Use the concept of magnification of image in case of lens and mirrors.

For (i): Convex mirror

For real object: image is virtual and erect. So (a) is possible, (d) is not possible.

For virtual object: u is positive, here f is also positive.

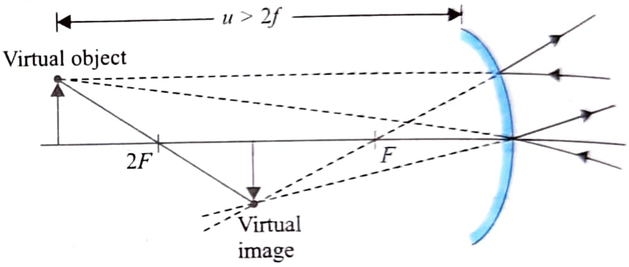
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow v=\frac{uf}{u-f}$$

$$m=-\frac{v}{u}=\frac{-f}{u-f}=\frac{f}{f-u}$$

For (b), m is positive. For this $u<f$, then for this m comes out to be >1.

But in Fig. (b), $m<1$. Hence (b) is not possible.

For (c), m is negative and $-1<m<0$, this is possible for $u>2f$. Ray diagram for this is shown below:



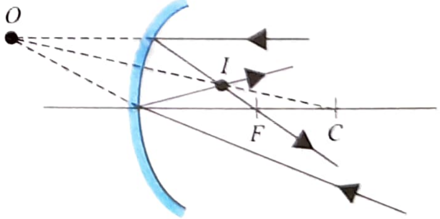
For (ii): Concave mirror

For real object: if the image is real it is inverted. Hence (d) is possible. But (a) is not possible, because here image is erect and it should be enlarged.

For virtual object: u is positive, here f is negative.

$$v=\frac{uf}{u-f};\ m=\frac{f}{f-u}$$

Here $0<m<1$, hence image is erect and diminished. This is possible for (b) and not possible for (c).



For (iii): Converging lens (Convex lens)

For real object: in (a), image is virtual, erect and diminished. But virtual image for convex lens is enlarged. Hence (a) is not possible.

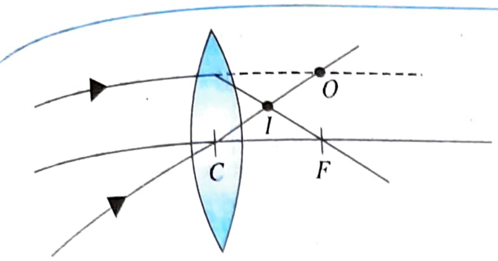
(d) is possible, because here image is inverted and diminished.

For virtual object: u (+ve), here f is positive.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow v=\frac{uf}{u+f}$$

$$m=\frac{v}{u}=\frac{f}{u+f} \Rightarrow 0<m<1$$

So image is always erect and diminished. Hence (b) is possible and (c) is not possible.



For (iv): Diverging (Concave lens)

For real object: image is always virtual, erect and diminished.

For this (a) is possible (d) is not possible.

For virtual object:

$$v = \frac{uf}{u+f},\ m = \frac{f}{u+f}$$

Here f is negative, u is positive.

Put $f = -f_0$

$$v = \frac{uf_0}{f_0 - u},\ m = \frac{f_0}{f_0 - u}$$

If $u < f_0$, then $m > 1$, then erect and enlarged image should be formed. So (b) is not possible.

If $u > f_0$, then m is negative, then inverted image will be formed, which can be enlarged or diminished. Hence, (c) is possible.

9. (2)

(i) Focal length of uncut lens.

$$\frac{1}{f_0} = (\mu_2 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) \Rightarrow f_0 = \frac{R}{2(\mu_2 - 1)}$$

Focal length of cut lens

$$\frac{1}{f'} = (\mu_2 - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) \Rightarrow f' = \frac{R}{(\mu_2 - 1)}$$

Hence focal length double and nature of lens is still converging.

(ii) The radius of curvature of both the sides will remain unchanged hence the nature and focal length will remain same.

(iii) In this case the light ray will pass the lens undeviated. Hence the lens will act as a glass slab.

(iv) The lens maker formula is

$$\frac{1}{f} = \left(\frac{\mu_2 - 1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If $\mu_1 > \mu_2$, the focal length of the lens will be negative. The nature of the lens will change. The lens will be diverging.

10. (1)

(i) $$\frac{1}{f} = \underbrace{\left(\frac{n_l}{n_s} - 1\right)}_{-}\underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{+}$$

$f = -\text{ve}$

$$P = \frac{1}{f} = -\text{ve}$$

Hence (i) – b., e.

(ii) $$\frac{1}{f} = \underbrace{\left(\frac{n_l}{n_s} - 1\right)}_{+}\underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{+}$$

$f = +\text{ve}$

$$P = \frac{1}{f} = +\text{ve}$$

Hence (ii) – a., c., d.

(iii) $$\frac{1}{f} = \underbrace{\left(\frac{n_l}{n_s} - 1\right)}_{-}\underbrace{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}_{-}$$

$$f = +\text{ve},\quad P = \frac{1}{f} = +\text{ve}$$

Hence (iii) –a., c., d.

(iv) $$f = -\frac{R}{2}$$

$$P = -\frac{1}{f} = +\text{ve}$$

Hence (iv) – a., c., e.

Numerical Value Type

1. (2) For magnification +2, $u = -x$, $v = -2x$ and $f = 2.0$ m

From $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have $\frac{1}{-2x} + \frac{1}{x} = \frac{1}{z}$

$x = 1.0$ m

For magnification of –2, $u = -y$, $v = +2y$, $f = 2.0$ m

we have $\frac{1}{2y} + \frac{1}{y} = \frac{1}{2} \Rightarrow y = 3.0$ m

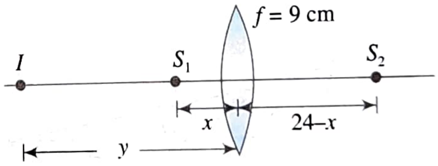
So distance to be moved $= y - x = 3 - 1 = 2$ m

2. (9) When object is placed at the focus of the lens, i.e., at 22 cm from the lens, image will be formed at infinity. Shift in the position of object:

$$25 - 22 = \left(1 - \frac{1}{\mu}\right)t$$

$$\Rightarrow\quad 3 = \left[1 - \frac{1}{1.5}\right]t \Rightarrow t = \frac{(3)(1.5)}{0.5} = 9 \text{ cm}$$

3. (6) Let both the images are formed at I. For S_1, the image is virtual and for S_2, the image is real.



For S_1: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-y} - \frac{1}{-x} = \frac{1}{9}$ ...(i)

For S_2: $\frac{1}{y} - \frac{1}{-(24-x)} = \frac{1}{9}$ ...(ii)

From (i) and (ii) $\frac{1}{x} + \frac{1}{24-x} = \frac{2}{9}$

$\Rightarrow\quad x^2 - 24x + 108 = 0$

$\Rightarrow\quad x = 6$ cm and 18 cm

But $x < f$. So answer is 6 cm

4. (8) $f = \frac{D^2 - d^2}{4D} = \frac{50^2 - 30^2}{4 \times 50} = 8$ cm

5. (8) When object is placed perpendicular to the principal axis, image size $= m \times$ (object size)

or $\quad 4 = m \times 2 \quad \Rightarrow \quad m = 2$

When object is placed along principal axis,

image size $= m^2$ (object size) $= 4 \times 2 = 8$ mm

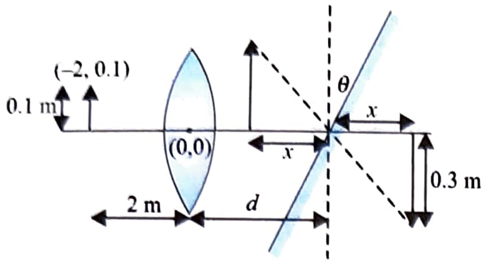
6. (3) Image is at ∞, so apparent position of object is at focus.

$d_{AC} = 25$ cm

Shift = 5 cm $= \left(\dfrac{\mu - 1}{\mu}\right)t \Rightarrow t = 15$ cm

$\therefore \quad K = 3$

7. (5) $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$



$$\frac{1}{v} - \frac{1}{-2} = \frac{1}{1.5}; \ v = 6 \text{ m}$$

$$m = \frac{v}{u} = -3$$

$$x = 6 - d;$$

$$\tan\theta = \frac{0.3}{x}$$

$$\Rightarrow \quad d = 5 \text{ m}$$

8. (2) Point '*B*' should be at the focus of curved surface $f = AB = 2R$

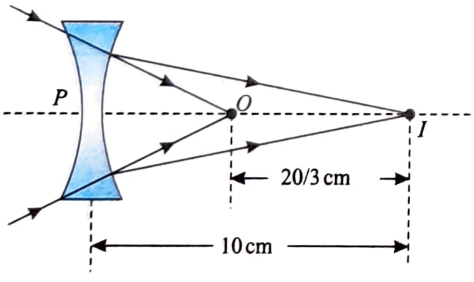
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu_2}{\infty} - \frac{\mu_1}{(+2R)} = \frac{\mu_2 - \mu_1}{+R}$$

$$\Rightarrow \quad -\frac{\mu_1}{2R} = \frac{\mu_2 - \mu_1}{R} \Rightarrow -\mu_1 = 2\mu_2 - 2\mu_1$$

$$\Rightarrow \quad \frac{\mu_1}{\mu_2} = 2$$

9. (5)



$$PO = 10 - \frac{20}{3} = \frac{10}{3} \text{ cm}; \ PI = 10 \text{ cm}$$

Using lens formula $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Here $\quad u = +\dfrac{10}{3}$ cm, $v = +10$ cm

Hence $\quad \dfrac{1}{(+10)} - \dfrac{1}{(+10/3)} = \dfrac{1}{f}$

$$f = -5 \text{ cm}$$

10. (15) To solve this problem first find the object for lens. This object is the image formed by the glass slab.

Applying lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{\infty} - \frac{1}{20} = \frac{1}{f} \Rightarrow u = -20 \text{ cm}$$

Here – ve sign indicates that object is at the left of the lense.

$\therefore$ Shift of image by the glass slab = 25 – 20 = 5 cm

Shift of image by the glass slab $= t\left(1 - \dfrac{1}{\mu}\right)$

$$\Rightarrow \quad 5 = t\left(1 - \frac{1}{1.5}\right) \Rightarrow t = 15 \text{ cm}$$

11. (11) Now according to the question, magnifications for the two are same in magnitude.

$$m = \frac{h_i}{h_0} = \left|\frac{v_1}{-u_1}\right| = \frac{v_1}{16} \Rightarrow m = \frac{h_i}{h_0} = \left|\frac{-v_2}{-u_2}\right| = \frac{v_2}{6}$$

v_1 is the distance of the real image for a real object. v_2 is the distance of the virtual image for a real object.

As, $\quad \dfrac{v_1}{16} = \dfrac{v_2}{6} \Rightarrow 6v_1 = 16v_2 \quad$ ...(i)

In case-I: $\quad \dfrac{1}{v_1} - \dfrac{1}{-16} = \dfrac{1}{f} \quad$ ...(ii)

In case-II: $\quad \dfrac{1}{-v_2} - \dfrac{1}{-6} = \dfrac{1}{f} \quad$ ...(iii)

[From the lens formula]

By solving above equations we get, $f = 11$ cm

+ve sign indicates that the lens is a convex (or converging) lens.

12. (5) Power of equivalent mirror $P'_m = 2P_{\text{lens}} + P_M$

$$\frac{1}{f_{\text{lens}}} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{1}{8} \times \frac{2}{R} = \frac{1}{4R}$$

and $\quad P_{\text{lens}} = \dfrac{1}{4R}$

$$P'_m = \frac{1}{-f'_m} = 2 \times \frac{1}{4R} + \frac{2}{R} = \frac{5}{2R} \Rightarrow f'_m = -\frac{2}{5}R$$

As image coincides with object hence $|h| = 2|f'_m|$

$$h = 2 \times \frac{2}{5}R = 4 \text{ m} \Rightarrow R = 5 \text{ m}$$

13. (12) Suppose that the object distance (between the object and the lens) be u and the image distance v. Then,

$$-\frac{1}{|u|} + \frac{1}{|v|} = \frac{1}{f} \quad \text{...(i)}$$

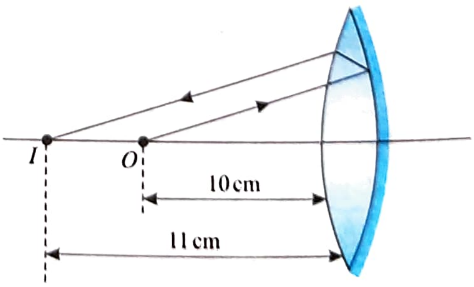
When water is poured into the vessel,

$$\frac{1}{(|v| + 6)} - \frac{1}{(|u| - 2)} = \frac{1}{f} \quad \text{...(ii)}$$

Solving the equation, we get

$$|u| = 20 \text{ cm} \Rightarrow |v| = 30 \text{ cm and } f = 12 \text{ cm}$$

14. (11) As radius of curvature of silvered surface is 22 cm,



so $\quad f_M = \dfrac{R}{2} = \dfrac{-22}{2} = -11 \text{ cm} = -0.11 \text{ m}$

and hence, $\quad M = -\dfrac{1}{f_M} = -\dfrac{1}{-0.11} = \dfrac{1}{0.11}$ D

Further as the focal length of lens is 20 cm, i.e., 0.20 m its power will be given by: $P_L=\frac{1}{f_L}=\frac{1}{0.20}\text{D}$

Now as in image formation, light after passing through the lens will be reflected back by the curved mirror through the lens again

$$P=P_L+P_M+P_L=2P_L+P_M$$

i.e., $P=\frac{2}{0.20}+\frac{1}{0.11}=\frac{210}{11}\text{D}$

So the focal length of equivalent mirror

$$F=-\frac{1}{P}=-\frac{11}{210}\text{m}=-\frac{110}{21}\text{cm}$$

i.e., the silvered lens behave as a concave mirror of focal length (110/21) cm. So for object at a distance 10 cm in front of it,

$\frac{1}{v}+\frac{1}{-10}=-\frac{21}{110}$ i.e., $v=-11$ cm i.e., image will be 11 cm in front of the silvered lens and will be real as shown in figure.

(1.5)

For case (a), Refraction from curved surface $\frac{\mu}{v_1}-\frac{1}{\infty}=\frac{\mu-1}{R}$

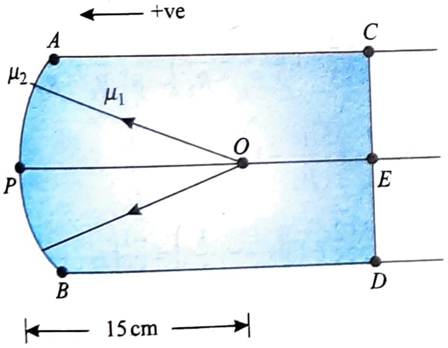
Refraction from plane surface, $\frac{1}{v_2}-\frac{\mu}{v_1-R}=\frac{1-\mu}{\infty}$

and $v_2=\frac{2R}{m}\Rightarrow m=2(\mu-1)\mu$

For case (b) $\frac{1}{v_2}-\frac{\mu}{\infty}=\frac{1-\mu}{-R}$ and $v_2=\frac{R}{m-1}\Rightarrow\mu=m$

Therefore $\mu=2(\mu-1)\mu\Rightarrow\mu-1=\frac{1}{2}\Rightarrow\mu=1.5$

(30) Rays starting from O will suffer single refraction from spherical surface APB. Therefore, applying



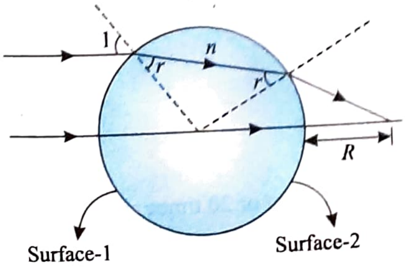
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R};\frac{1.0}{v}-\frac{2.0}{-15}=\frac{1.0-2.0}{-10}$$

$$\frac{1}{v}=\frac{1}{10}-\frac{1}{7.5}$$

or $v=-30$ cm

Therefore, image of O will be formed at 30 cm to the right of P. Note that image will be virtual. There will be no effect of the surface CED.

17. (1.33)



From surface-1: $\frac{n}{v}-\frac{1}{\infty}=\frac{n-1}{R};v=\frac{nR}{n-1}$

From surface-2: $\frac{1}{R}-\frac{n}{v-2R}=\frac{1-n}{-R};\frac{2}{R}=\frac{n}{R}+\frac{n}{v-2R}$

$\Rightarrow\quad\frac{2}{R}=\frac{n(v-2R+R)}{R(v-2R)}\Rightarrow 2v-4R=nv-nR$

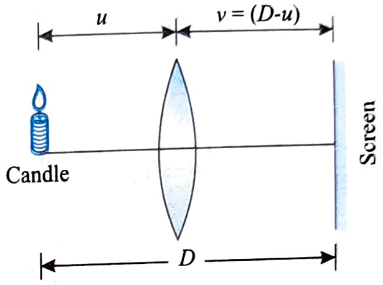
On putting the value of v

$$2\frac{nR}{n-1}-4R=\frac{n\times nR}{(n-1)}=-nR$$

$$2nR-4nR+4R=n^2R-n^2R+nR,\ n=\frac{4}{3}$$

18. (30) Given: distance between two positions of the lens for which image is formed,

$$x=u_2-u_1=0.8\text{m}$$



$D=u_1+v_1$ ...(ii)

$m_1=\frac{v_1}{u_1}=3\Rightarrow v_1=3u_1$ ...(iii)

$\frac{v_2}{u_2}=\frac{1}{3}\Rightarrow v_2=\frac{u_2}{3}$ ...(iv)

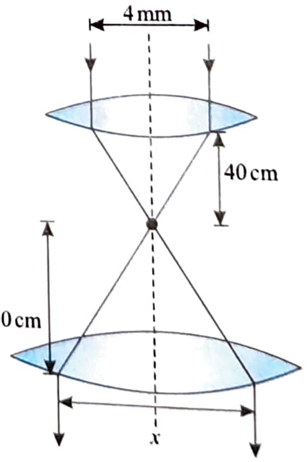
From Eqs. (ii) and (iii), $D=4u_1$ or $u_1=\frac{D}{4}$

From Eqs. (ii) and (iv), $D=\frac{4}{3}u_2$ or $u_2=\frac{3}{4}D$

Substituting values is Eq. (i), we get $D=2x=1.6$ m

$$f=\frac{D^2-x^2}{4D}=\frac{(1.6)^2-(0.8)^2}{4\times1.6}=0.3\text{ m}=30\text{ cm}$$

19. (1)

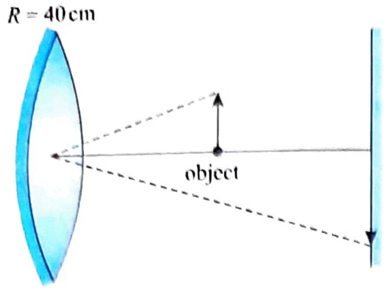


From similar triangles $\frac{0.4}{40}=\frac{x}{100}$; $x=1$ cm

20. (15) Initially the system will act as simple concave mirror given $v=-60$ cm, $m=-2\Rightarrow u=-30$ cm

$\Rightarrow\quad\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{-60}-\frac{1}{30}=\frac{1}{f}$

$\Rightarrow\quad f=-20\text{ cm}\Rightarrow R=40\text{ cm}$



In second case: lens + mirror system

Then $P_{\text{meq}} = P_L + P_L + P_M$

P_{meq} = equivalent power of the mirror

P_L = power of the lens

P_M = power of the mirror

$$P_{\text{meq}} = \frac{1}{f_L} + \frac{1}{f_L} + \frac{-1}{f_M} \qquad \left[\because \frac{1}{f_L} = \left(\frac{1}{3}\right)\frac{2}{R}\right]$$

$$= \frac{2\times 2}{3R} + \left(\frac{-2}{-R}\right)\left(\frac{3}{3}\right)$$

$$P_{\text{meq}} = \frac{10}{3R} = \frac{-1}{f_{\text{meq}}} \Rightarrow f_{\text{meq}} = \frac{-3R}{10}$$

$$f_{\text{meq}} = -12 \text{ cm}$$

Now if object is placed at distance = x then

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_{\text{eq}}} \quad (\because v = -60 \text{ cm})$$

$$\frac{1}{-60} + \frac{1}{u} = \frac{-1}{12} \Rightarrow \frac{1}{u} = \frac{1}{60} - \frac{1}{12} = \frac{-4}{60}$$

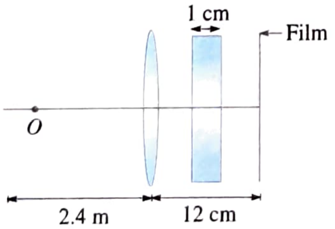
$$\Rightarrow \qquad u = -15 \text{ cm}$$

Archives

JEE Main

Single Correct Answer Type

1. (4) $\frac{1}{f} = \frac{1}{12} + \frac{1}{240} = \frac{20+1}{240}$



$$f = \frac{240}{21} \text{m}$$

$$\text{shift} = 1\left(1 - \frac{2}{3}\right) = \frac{1}{3}$$

Now $v' = 12 - \frac{1}{3} = \frac{35}{3}$ cm

$$\therefore \qquad \frac{21}{240} = \frac{3}{35} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left(\frac{3}{7} - \frac{21}{48}\right)$$

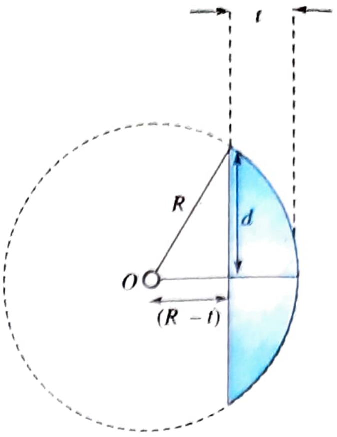
$$\frac{5}{u} = \left|\frac{144 - 147}{48 \times 7}\right|$$

$$u = 560 \text{ cm} = 5.6 \text{ m}$$

2. (2) $R^2 = d^2 + (R - t)^2$

$$R^2 - d^2 = R^2\left\{1 - \frac{t}{R}\right\}^2$$

$$1 - \frac{d^2}{R^2} = 1 - \frac{2t}{R}$$



$$R = \frac{(3)^2}{2\times(0.3)} = \frac{90}{6} = 15 \text{cm}$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$f = 30$ cm

3. (4) Given $\mu = \frac{3}{2}$ (crown glass) and focal length $= f$

Focal length f_1 when lens is placed in liquid of refractive index $\mu_1 = \frac{4}{3}$

Focal length $= f_2$ when lens is placed in liquid of refractive index $\mu_2 = \frac{5}{3}$

Using lens maker's formula $\frac{1}{f_1} = \left(\frac{\mu}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \quad \frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{8}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Similarly, $\frac{1}{f_2} = \left(\frac{\mu}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \quad \frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{-1}{10}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Hence, $f_1 = 4f$ and $f_2 = -5f$.

4. (3) Magnifying power $= \left(\frac{\beta}{\alpha}\right) = 20$

The object is of height 10 m which is very far away and when image also very far away, then height of image h.

$$\beta = \frac{h}{x} \text{ and } \alpha = \frac{10 \text{ m}}{x}$$

Hence $\qquad \frac{\beta}{\alpha} = \frac{h}{10}$

or $\qquad \frac{h}{10} = 20$

Hence $h = 20 \times 10$ m or 20 times taller.

Advanced

Correct Answer Type

(2) First image will be formed by lens at I' at shown. As object is placed at $2F$ of lens, so I is 30 cm from lens and 20 cm from mirror. Due to reflection from mirror image I'' will be formed at 10 cm from lens.

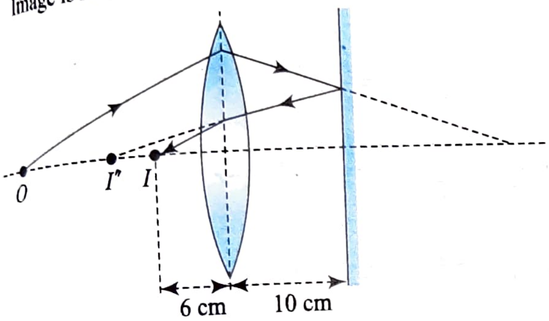
Final image is formed by lens.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{10}=\frac{1}{15}$$

$v = 6$ cm from lens

Image is real.



(2) $\frac{1}{f_1}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.5-1)\left[\frac{1}{14}-\frac{1}{\infty}\right]=\frac{0.5}{14}$

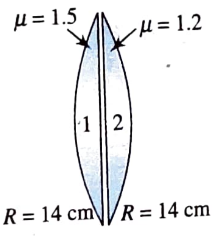
$$\frac{1}{f_2}=(1.2-1)\left[\frac{1}{\infty}-\frac{1}{-14}\right]$$

$$\frac{1}{f_2}=\frac{0.2}{14}$$

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{0.5}{14}+\frac{0.2}{14}=\frac{0.7}{14}$$

$$\frac{1}{v}=\frac{7}{140}-\frac{1}{40}=\frac{1}{20}-\frac{1}{40}\Rightarrow\frac{1}{v}=\frac{2-1}{40}$$

$v = 40$ cm



3. (3) $\mu=\frac{\lambda_0}{\lambda_m}=\frac{3}{2}$

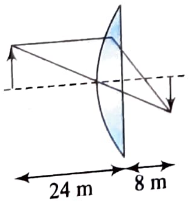
$$\Rightarrow \frac{1}{f}=\frac{\mu-1}{R}=\frac{1}{2R}$$

$$\Rightarrow \frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$\Rightarrow \frac{1}{8}-\frac{1}{-24}=\frac{1}{2R}$$

$$\Rightarrow \frac{3+1}{24}=\frac{1}{2R}$$

$\Rightarrow R = 3$ m



4. (2) $\frac{\mu_2}{V}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

$$\frac{1}{V}-\frac{1.5}{-50}=\frac{-0.5}{-10}=\frac{1}{20}$$

$$\frac{1}{V}=\frac{1}{20}-\frac{3}{100}$$

$V = 50$ cm

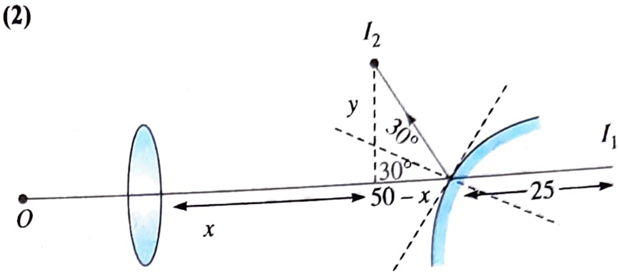
$$\frac{\mu_2}{\infty}-\frac{1.0}{u}=\frac{0.5}{10}$$

$$-\frac{1}{u}=\frac{1}{20}$$

$u = -20$ cm

Hence, $d = 50 + 20 = 70$

5. (2)



For I_1, $u = -50, f = +30, v = +75$ cm

final image is I_2

$$\frac{y}{50-x}=\tan 60^\circ=\sqrt{3}\Rightarrow y=50\sqrt{3}-\sqrt{3}x$$

$$y+\sqrt{3}x=50\sqrt{3}$$

$$\sqrt{y^2+(50-x)^2}\sin 30^\circ>25\sin 30^\circ$$

$$y^2+(50-x)^2>625$$

for option (1): $\frac{625}{3}+\frac{625}{9}<625$

for option (2): $625(3)+625>625$

So (2) is correct.

Multiple Correct Answers Type

1. (3), (4)

$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$ (mirror formula)

$f = -24$ cm

2. (1), (3)

Method 1:

$$\frac{1.4}{v}-\frac{1}{\infty}=\frac{1.4-1}{R}$$

$$\frac{1.4}{v}=\frac{0.4}{R};\ v=\frac{7R}{2}$$

Film to glass: $\frac{1.5}{v'}-\frac{1.4}{\frac{7R}{2}}=\frac{1.5-1.4}{+R}$

$$\frac{1.5}{v'}-\frac{0.4}{R}=\frac{0.1}{R}$$

$$\frac{1.5}{v'}=\frac{0.5}{R};\quad v'=3R\Rightarrow f_1=3R$$

Glass to film: $\frac{1.4}{v}-\frac{1.5}{\infty}=\frac{1.4-1.5}{-R}$

$$\frac{1.4}{v}=\frac{-0.1}{-R}$$

$v = -14R$

Film to air: $\frac{1}{v'}-\frac{1.4}{14R}=\frac{1-1.4}{-R}$

$$\frac{1}{v'}-\frac{1}{10R}=\frac{+0.4}{+R}$$

$$\frac{1}{v'}=\frac{0.4}{R}+\frac{0.1}{R}=\frac{0.5}{R}$$

$$v'=\frac{R}{0.5}=2R \Rightarrow f_2=2R$$

Method 2: For air to glass

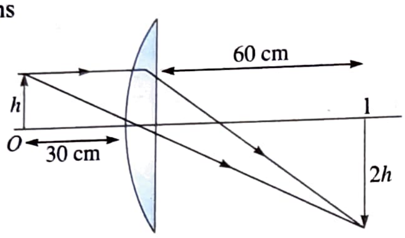
$$\frac{1.5}{f_1}=\frac{1.4-1}{R}+\frac{1.5-1.4}{R} \quad \therefore \quad f_1=3R$$

For glass to air.

$$\frac{1}{f_2}=\frac{1.4-1.5}{-R}+\frac{1-1.4}{-R} \quad \therefore \quad f_2=2R$$

3. (1), (4)

For lens

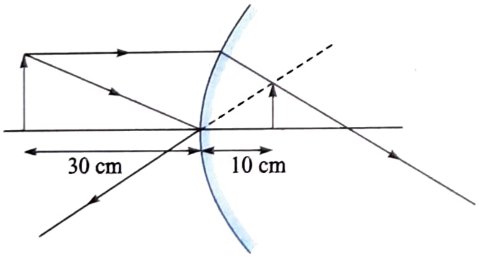


$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{60}-\frac{1}{(-30)}=\frac{1}{f}\Rightarrow f=20\text{cm} \qquad \dots\text{(i)}$$

Also $\frac{1}{f}=(n-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\left(\frac{n-1}{R}\right)$ ...(ii)

For reflection from convex mirror (curved surface)



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$$

$$\frac{1}{+10}+\frac{1}{-30}=\frac{1}{f}=\frac{2}{R} \qquad \dots\text{(iii)}$$

$R = 30$ cm

from (i), (ii) and (iii), $n = 2.5$, faint image erect and virtual.

Matrix Match Type

1. (i) → (a), (c); (ii) → (b), (d), (e); (iii) → (a), (c), (e); (iv) → (b), (d)

(a) $\mu_2=\mu_3$, as there is no deviation
$\mu_1<\mu_2$, as ray bends towards normal

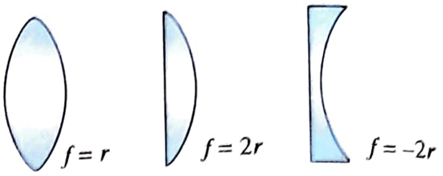
(b) $\mu_1>\mu_2$,as ray bends away from normal
$\mu_2>\mu_3$, as ray bends away from normal

(c) $\mu_2=\mu_3$, as there is no deviation
$\mu_1<\mu_2$,as ray bends

(d) $\mu_1>\mu_2$, as ray bends away from normal
$\mu_2>\mu_3$, as ray bends away from normal

(e) $\mu_1>\mu_2$, as ray bends away from normal
$\mu_2=\mu_3$, as there is no deviation

2. (2) $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$



Use $\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$

(i) $\frac{1}{f_{eq}}=\frac{1}{r}+\frac{1}{r}=\frac{2}{r}; f_{eq}=\frac{r}{2}$

(ii) $\frac{1}{f_{eq}}=\frac{1}{2r}+\frac{1}{2r}=\frac{1}{r}; f_{eq}=r$

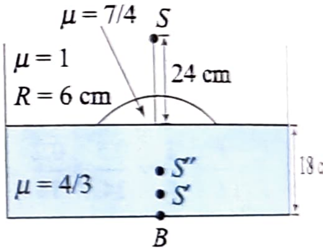
(iii) $\frac{1}{f_{eq}}=-\frac{1}{2r}-\frac{1}{2r}=-\frac{1}{r}; f_{eq}=-r$

(iv) $\frac{1}{f_{eq}}=\frac{1}{r}-\frac{1}{2r}=\frac{1}{2r}; f_{eq}=2r$

Numerical Value Type

1. (6) $m=\frac{f}{f+u}$

2. (2) $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$



$$\frac{7}{4v}-\frac{1}{-24}=\frac{\frac{7}{4}-1}{6}$$

$$\frac{7}{4v}=\frac{3}{24}-\frac{1}{24}=\frac{2}{24}=\frac{1}{12}$$

$$\frac{7\times 12}{4}=v=21\text{ cm}$$

$$\frac{21}{OS''}=\frac{7/4}{4/3}\Rightarrow OS''=16$$

$\therefore$ $BS''=2$ cm

3. (7) In air: For mirror $\frac{1}{V}+\frac{1}{-15}=\frac{-1}{10}$

$$\frac{1}{V}=\frac{1}{15}-\frac{1}{10}=\frac{-1}{30}$$

$\therefore$ $V=-30$

$$|m_1|=\frac{30}{15}=2$$

For lens: $u=-20$

$$\frac{1}{V_2}+\frac{1}{20}=\frac{1}{10}$$

$$\frac{1}{V_2}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$$

$V_2=20$ cm

$|m_2|=1$

$\therefore$ $|M_1|=|m_1|\,|m_2|=2$

After the medium is changed, by lensmaker's formula:

$$\frac{1}{F}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{10}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{F}=\left(\frac{3\times 6}{2\times 7}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Taking ratio, we get F as: $F=\frac{70}{4}=\frac{35}{2}$

Image formation by mirror remains unaffected by changing medium. Applying distance formula with new focal length of lens.

$$\frac{1}{V'}+\frac{1}{20}=\frac{2}{35} \quad \therefore \quad \frac{1}{V'}=\frac{2}{35}-\frac{1}{20}=\frac{5}{20\times 25}$$

$$V'=140$$

$$m_2=\frac{140}{20}=7$$

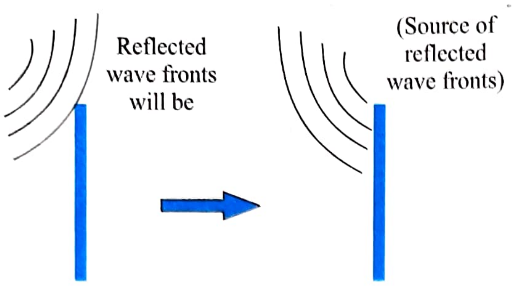
$\therefore$ $$M_2=7\times 2=14$$

So, $$\frac{M_2}{M_1}=\frac{14}{2}=7$$

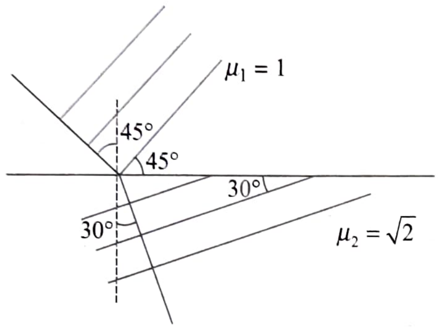
Concept Application Exercises

Exercise 3.1

1.


Reflected wave fronts will be
(Source of reflected wave fronts)

2.



$$\mu_1 \sin 45° = \mu_2 \sin\theta \Rightarrow \theta = 30°$$

3. (d) Path difference for points A and C is zero so constructive interference takes place.

Path difference for B and $D = 5\mu\text{m} = \frac{5\lambda}{2}$

So distructive interference take place.

4. Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

Resultant intensity at point A is

$$I_A = I + 4I + 2\sqrt{I}\sqrt{4I}\cos\frac{\pi}{2} = 5I$$

Resultant intensity at point B,

$$I_B = I + 4I + 2\sqrt{I}\sqrt{4I}\cos 2\pi = 9I$$

$$(\because \cos 2\pi = 1)$$

$$\therefore \quad I_B - I_A = 9I - 5I = 4I$$

5. $\phi_1 = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \times \lambda = 2\pi$

and $\quad \phi_2 = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

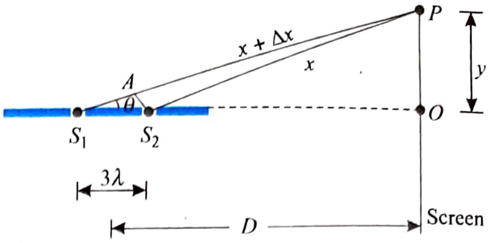
$$I_1 = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos 2\pi = 4I_0$$

and $\quad I_2 = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos\left(\frac{\pi}{2}\right) = 2I_0$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{4I_0}{2I_0} = 2 \Rightarrow I_2 = \frac{I_1}{2} = \frac{K}{2}\text{unit}$$

$$[\because I_1 = K\text{ unit}]$$

6.



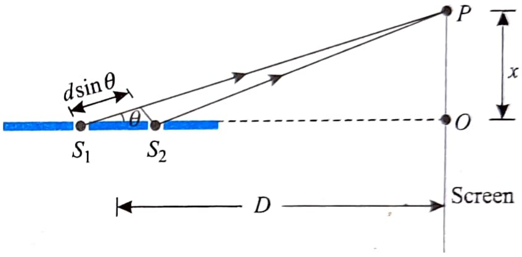
In this case path difference is $d\cos\theta$

So $\quad n\lambda = \lambda = 3\lambda\cos\theta \Rightarrow \cos\theta = \frac{1}{3}$

$$\Rightarrow \quad \frac{D}{\sqrt{D^2 + y^2}} = \frac{1}{3} \Rightarrow 3D = \sqrt{D^2 + y^2}$$

$$\Rightarrow \quad y = \sqrt{8}D = 2\sqrt{2}D$$

7. Path difference at O, $S_1O - S_2O = 2\lambda$, i.e., maximum intensity is obtained at O. Next maxima will be obtained at point P, where



$$S_1P - S_2P = \lambda$$

or $\quad d\cos\theta = \lambda$

or $\quad (2\lambda)\cos\theta = \lambda$

or $\quad \cos\theta = \frac{1}{2}$

$\therefore \quad \theta = 60°$

Now in ΔS_1PO, $\frac{PO}{S_1O} = \tan\theta$

or $\quad \frac{x}{D} = \tan 60° = \sqrt{3}$

$\therefore \quad x = \sqrt{3}\,D$

Exercise 3.2

1. (b) There is a phase change of π when the ray enters from rarer medium into denser medium, the boundary is called rigid boundary and also no change in phase when it enters into rarer medium.

2. Condition for maximum interference in the reflected light, in case of thin film interference, can be expressed as

$$2\mu t\cos r = (2n - 1)\,\lambda/2,\ n = 1, 2, \ldots$$

[for plane parallel films]

where μ is the refractive index of film relative to the surrounding; t is the thickness of film; and r is the angle of refraction.

For normal incidence, $r = 0$

$$\therefore \quad 2\mu t = (2n - 1)\lambda/2,\ n = 1, 2, 3, \ldots$$

For minimum thickness, $n = 1$

$$\therefore \quad 2\mu t_{\min} = \lambda/2 \quad \text{or} \quad t_{\min} = \frac{\lambda}{4\mu}$$

Given: $\lambda = 5360$ Å and $\mu = 1.34$

$$\therefore \quad t_{\min} = \frac{5360}{4 \times 1.34} \quad \text{or} \quad t_{\min} = 1000 \text{ Å} = 0.1\ \mu\text{m}$$

Conditions for maximum and minimum intensity in reflected light, in case of thin film interference, are

Maxima: $2\mu t = (2n - 1)\lambda/2,\ n = 1, 2, \ldots$

Minima: $2\mu t = n\lambda,\ n = 0, 1, 2, 3, \ldots$

Given: $\mu = 1.5$, $t = 3000$ Å $= 3000 \times 10^{-10}$ m (normal incidence)

Maxima: $2\mu t = (2n - 1)\lambda/2$

$$\therefore \quad 2 \times 1.5 \times (3000 \times 10^{-10}) = (2n - 1)\lambda/2$$

$$\text{or} \quad \lambda = \frac{4 \times 1.5 \times 3000 \times 10^{-10}}{2n - 1}$$

$$\therefore \quad \lambda = \frac{18000}{2n - 1} \times 10^{-10} \text{ m}$$

For $n = 1$, $\lambda = 18000$ Å

$n = 2$, $\lambda = 6000$ Å

$n = 3$, $\lambda = 3600$ Å

and so on

Only $\lambda = 6000$ Å for $n = 2$ lies in the visible range. Intensity will be maximum for $\lambda = 6000$ Å which belongs to the oranges–red part of the visible spectrum.

4. We must find which colours in the visible region, having vacuum wavelengths from 400 nm (violet) to 700 nm (red), will interfere constructively and which destructively. From the problem, with $n_1 = 1$ (air) maximum constructive interference occurs for

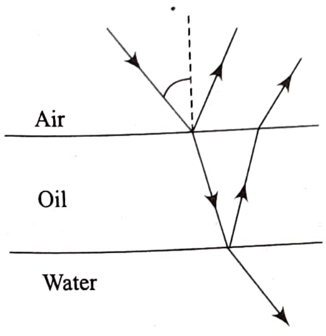
$$\lambda_1 = \frac{2(n_2/n_1)t}{m + \frac{1}{2}} = \frac{(900 \text{ nm})}{m + \frac{1}{2}}$$

$(m = 0, 2, \ldots)$

Only the value corresponding to $m = 1$, that is, $\lambda_1 = 450$ nm (violet) is in the visible range.

We infer that the red–orange–yellow end of the spectrum will be strongly reflected, while the violet–blue end will be greatly diminished in intensity as compared with the illuminating white light.

5. (a) The light reflected from the top of the oil film undergoes phase reversal. Since $1.45 > 1.33$, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have



$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

$$\text{or} \quad \lambda_m = \frac{2nt}{m + \frac{1}{2}} = \frac{2(1.45)(280 \text{ nm})}{m + \frac{1}{2}}$$

Substituting for m gives:

$m = 0$, $\lambda_0 = 1620$ nm (infrared)

$m = 1$, $\lambda_1 = 541$ nm (green)

$m = 2$, $\lambda_2 = 325$ nm (ultraviolet)

Both infrared and ultraviolet lights are invisible to the human eye, so the dominant color in reflected light is green.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2nt = m\lambda$$

$$\text{or} \quad \lambda = \frac{2nt}{m} = \frac{812 \text{ nm}}{m}$$

Substituting for m gives

$m = 1$, $\lambda_1 = 812$ nm (near infrared)

$m = 2$, $\lambda_2 = 406$ nm (violet)

$m = 3$, $\lambda_3 = 271$ nm (ultraviolet)

Of these, the only wavelength visible to the human eye (and hence the dominant wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is violet.

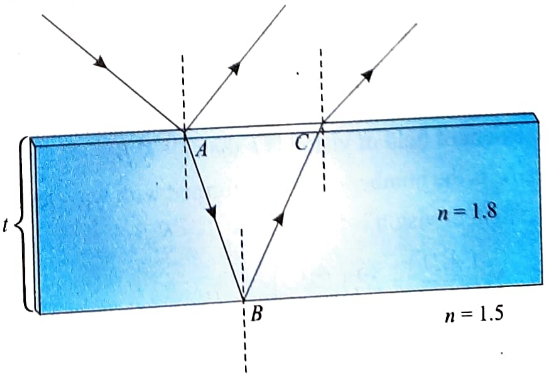
6. Treating the anti-reflectance coating like a camera lens coating,

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

$$\text{Let} \quad m = 0\text{: } t = \frac{\lambda}{4n} = \frac{3.00 \text{ cm}}{4(1.50)} = 0.500 \text{ cm}$$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar to 1.50 cm—now creating maximum reflection!

7. At point A, path difference of $\lambda/2$ will be created but at B, no path difference will exist. Thus, writing the condition of constructive interference,



$$2\mu r + \frac{\lambda}{2} = n\lambda$$

$$\text{or} \quad 2\mu t = \left[n - \frac{1}{2}\right]\lambda$$

For minimum t put $n = 1$

$$2\mu t = \frac{\lambda}{2}$$

or $$t = \frac{\lambda}{4\mu} = \frac{\lambda}{4} \times 1.8 = \frac{648}{7.2}\text{ nm}$$

which gives, $t = 90$ nm

Exercise 3.3

1. (d) For maxima, $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$

When $I_1 = I_2$ then $I_{\max} = 4I$

For minima $I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \Rightarrow I_{\min} = 0$

When $I_2 = \frac{I_1}{2}$, then $I'_{\max} = \left(\sqrt{\frac{I_1}{2}} + \sqrt{I_1}\right)^2 < 4I_1$

$$I'_{\min} = \left(\sqrt{\frac{I_1}{2}} - I_1\right)^2 > I_{\min}$$

2. Angular fringe width is given by

$$\beta_\theta = \frac{\lambda}{d}$$

$$\beta_\theta^{\text{air}} = \frac{\lambda_{\text{air}}}{d},\ \beta_\theta^{\text{water}} = \frac{\lambda_{\text{water}}}{d}$$

$$\frac{\beta_\theta^{\text{air}}}{\beta_\theta^{\text{water}}} = \frac{\lambda_{\text{air}}}{\lambda_{\text{water}}} = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{4}{3}$$

$$\therefore\ \beta_\theta^{\text{water}} = \frac{3}{4}\beta_\theta^{\text{air}} = 0.40° \times \frac{3}{4} = 0.30°$$

3. The angular position of maxima is given by

$$d \sin\theta = n\lambda$$

$$d\theta = n\lambda$$

[For small θ, $\sin\theta = \theta$]

The angular separation of two adjacent maxima is

$$\Delta\theta = \frac{\lambda}{d}$$

Let angular separation be 10% greater for wavelength λ'.

$$\therefore\ \frac{1.10\lambda}{d} = \frac{\lambda'}{d}$$

Then, $\lambda' = 1.10\lambda = (1.10 \times 589) = 648$ nm

4. The fringe width is given by $\beta = \frac{\lambda D}{d}$

If the extent of field of vision is l, then $l = N_1\beta_1$ where N_1 is the number of fringes formed with wavelength l.

As the field of vision is fixed, $l = N_1\beta_1 = N_2\beta_2$

$$N_1\left(\frac{\lambda_1 D}{d}\right) = N_2\left(\frac{\lambda_2 D}{d}\right)$$

$$N_1\lambda_1 = N_2\lambda_2$$

Thus, $$N_2 = \frac{N_1\lambda_1}{\lambda_2} = \frac{60 \times 5890}{5460} = 64.72 \approx 65$$

5. $$\beta_1 = \frac{\lambda_1 D}{d} = \frac{6000 \times 10^{-10} \times 1}{10^{-3}} = 0.6 \text{ mm}$$

$$\beta_2 = \frac{\lambda(2D)}{d} = 0.45 \text{ mm}$$

Let n_1th maxima of λ_1 and n_2th maxima of λ_2 coincide at a position y.

Then, $y = n_1P_1 = n_2P_2 = \text{LCM of } \beta_1 \text{ and } \beta_2$

$\Rightarrow$ y = LCM of 0.6 mm and 0.45 mm

or $y = 1.8$ mm

At this point, 3rd maxima for 6000 Å and 4th maxima for 4500 Å coincide.

6. $S_2L = d\sin\theta$

or $\Delta x = d\sin\theta$

For intensity to be minimum,

$$\Delta x = d\sin\theta$$

$$\Delta x = (2n - 1)\lambda/2$$

$$n = 1, 2, 3, \ldots$$

For third minimum, $n = 3$

$$d\sin\theta = \frac{5\lambda}{2}$$

Given: $\lambda = 420 \text{ nm} = 420 \times 10^{-9}$ m

$$\theta = 30°$$

$$\therefore\ d\sin 30° = \frac{5}{2} \times 420 \times 10^{-9} \Rightarrow d = 2.1 \times 10^{-6} \text{ m}$$

7. In an interference arrangement similar to double-slit experiment, S_1 and S_2 are illuminated with coherent source microwave source each of frequency 1 MHz.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^6} = 300 \text{ m}$$

$$\Delta x = d\sin\theta;\ \phi = \frac{2\pi}{\lambda} d\sin\theta = \pi\sin\theta$$

$$I_R = 4I\cos^2\phi/2 = 4I\cos^2\left(\frac{\pi\sin\phi}{2}\right)$$

For $I(\theta)$ to be maximum, $I_0 = 4I$

$$\Rightarrow\ I = \frac{I_0}{4} \quad\Rightarrow\quad I_0 = 4I$$

$$I_R = I_0\cos^2\left(\frac{\pi\sin\phi}{2}\right)$$

(a) $\theta = 0° \Rightarrow I_R = I_0$ (b) $\theta = 30° \Rightarrow I_R = \frac{I_0}{2}$

(c) $\theta = 90° \Rightarrow I_R = 0$

8. $$y_{\text{bright}} = \frac{\lambda L}{d} m$$

For $m = 1$, $$\lambda = \frac{yd}{L} = \frac{(3.40 \times 10^{-3}\text{ m})(5.00 \times 10^{-4})\text{ m}}{3.30\text{ m}} = 515 \text{ nm}$$

9. In equation $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$,

The first minimum is described by $m = 0$

And the tenth by $m = 9$ $\sin\theta = \frac{\lambda}{d}\left(9 + \frac{1}{2}\right)$

Also, $\tan\theta = y/D$

But for small θ, $\sin\theta \approx \tan\theta$

Thus, $$d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda D}{y}$$

$$D = \frac{9.5(6000 \times 10^{-10}\text{ m})(2.00\text{ m})}{7.5 \times 10^{-3}\text{ m}}$$

$$= 1.52 \times 10^{-3} \text{ m} = 1.52 \text{ mm}$$

At 30.0°, $d \sin\theta = m\lambda$

$$(3.20 \times 10^{-4}\ \text{m}) \sin 30.0^\circ = m(500 \times 10^{-9}\ \text{m})$$

So, $m = 320.$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead.

Therefore, there are 641 maxima.

(a) $$\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right) \quad \text{...(i)}$$

Therefore, $$\phi = 2\cos^{-1}\sqrt{\frac{I}{I_{max}}} = 2\cos^{-1}\sqrt{0.25}$$

$$= 2\cos^{-1}\left(\frac{1}{2}\right) = 2 \times \frac{\pi}{3} = \frac{2\pi}{3}\ \text{rad}$$

(b) $$\delta = \frac{\lambda\phi}{2\pi} = \frac{(486\ \text{nm})\left(\frac{2}{3}\pi\ \text{rad}\right)}{2\pi} = 162\ \text{nm}$$

$$\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right]^2$$

As $I_1 = 2I_2$

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{2}+1}{\sqrt{2}-1}\right]^2 = 34$$

Amplitudes from the slits are A and $2A$.

$$A_{max} = A + 2A = 3A$$

and $$A_{min} = 2A - A = A$$

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \frac{A^2_{max}}{A^2_{min}} = 9$$

The expression for resultant intensity at any point is

$$I = k(A_1^2 + A_2^2 + 2A_1A_2 \cos\phi)$$

where k is constant of proportionality, ϕ is the path difference between interfering waves, A_1 and A_2 are amplitudes of superposing waves. As $A_1 = 2A_2 = 2A$, we have the intensity at the central position.

$$I_0 = k(A_1A_2)^2 = k(2A + A)^2 = 9kA^2$$

So, $$k = \frac{I_0}{9A^2} \quad \text{...(i)}$$

The path difference at any point P is $\Delta x = d \sin\theta$

The phase difference, $$\phi = \frac{2\pi}{\lambda} d \sin\theta \quad \text{...(ii)}$$

Thus, Eq. (i) becomes

$$I = \frac{I_0}{9A^2}\left[(2A)^2 + A^2 + 2(2A)(A)\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right]$$

$$= \frac{I_0}{9A^2}\left[5A^2 + 4A^2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right]$$

$$= \frac{I_0}{9}\left[1 + 8\cos^2\frac{\pi d \sin\theta}{\lambda}\right] \quad \left[\text{as } \cos\theta = \left(2\cos^2\frac{\theta}{2} - 1\right)\right]$$

xercise 3.4

1. (a) $I' = \frac{3}{4}(4I) = 3I$

So $3I = I + I + 2I\cos\phi \Rightarrow \cos\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$

But value should lie between 3π and 6π

So it cannot be $\frac{\pi}{3}$.

For second minima $\phi = 3\pi$

For third maxima $\phi = 6\pi$

2. (a) Here the shift produced by mica sheet of thickness t is $(\mu t - t) = t(\mu - 1)$. It should be equal to extra path traveled by the ray SS_2O i.e. $\Delta = SS_2 - d = \sqrt{2}d - d$

$$\Rightarrow \quad \frac{t}{2} = d(\sqrt{2} - 1) \Rightarrow t = 2d\left(\sqrt{2} - 1\right) \text{ in front of } S_1$$

3. $$\Delta x = \frac{yd}{D} = 9 \times 10^{-4} \times 1 \times 10^{-3}\ \text{m} = 900\ \text{nm}$$

For minima, $\Delta x = (2n - 1)\lambda/2$

$$\Rightarrow \quad \lambda = \frac{2\Delta x}{(2n-1)} = \frac{1800}{(2n-1)} = \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7}, \ldots$$

Of these, 600 nm and 360 nm lie in the visible range. Hence, these will be the missing lines in the visible spectrum.

4. (a) $\theta = \theta_0 = 0.57^\circ$

$\Rightarrow \quad y = -D\tan\theta \simeq D\theta = -1\ \text{m} \times \left(\frac{0.57}{57}\ \text{rad}\right)$

$\Rightarrow \quad y = -1$ cm.

(b) For point O, $\theta = 0$

Hence, $\Delta x = d \sin\theta_0 = d\theta_0$

$$= 1\ \text{mm} \times (10^{-2}\ \text{rad}) = 10{,}000\ \text{nm} = 20 \times (500\ \text{nm})$$

$\Rightarrow \quad \Delta x = 20\lambda$

Hence, point O corresponds to 20th maxima

$\Rightarrow \quad$ Intensity at $O = I_0$

(c) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

5. Ray 1 has a longer path than that of ray 2 by a distance $d \sin 45^\circ$, before reaching the slits. Afterward ray 2 has a path longer than ray 1 by a distance $d \sin\theta$. The net path difference is therefore,

$$d \sin\theta - d \sin 45^\circ$$

(a) Central maximum is obtained where net path difference is zero

or $\quad d \sin\theta - d \sin 45^\circ = 0$

or $\quad \theta = 45^\circ$

(b) Third order maxima is obtained where net path difference is 3λ,

or $\quad d \sin\theta - d \sin 45^\circ = 3\lambda$

$$\sin\theta = \sin 45^\circ + \frac{3\lambda}{d}$$

Putting $d = 20\lambda$, we have

$$\sin\theta = \sin 45^\circ + \frac{3\lambda}{20\lambda}$$

or $$\theta = \sin^{-1}\left[\frac{1}{\sqrt{2}} + \frac{3}{20}\right]$$

6. As derived earlier, the total fringe shift $= \frac{w}{\lambda}(\mu - 1)t$

As each fringe width $= w$,

The number of fringes that will shift is

$$\frac{\text{Total fringe shift}}{\text{Fringe width}} = \frac{\frac{w}{\lambda}(\mu - 1)t}{w}$$

$$= \frac{(\mu - 1)t}{\lambda} = \frac{(1.6 - 1) \times 1.8 \times 10^{-5}\ \text{m}}{600 \times 10^{-9}\ \text{m}} = 18$$

7. Path difference due to the glass slab, $\Delta x = (\mu - 1)t$. Thirty fringes are displaced due to the slab.

Hence, $\Delta x = 30\lambda \Rightarrow (\mu - 1)t = 30\lambda$

or $$t = \frac{30\lambda}{\mu - 1} = \frac{30 \times 6 \times 10^{-5}}{1.5 - 1} = 3.6 \times 10^{-3} \text{ cm}$$

8. Optical path for light coming from upper slit S_1 is

$$S_1P + 1\ \mu\text{m} \times (3 - 1) = S_1P + 2\ \mu\text{m}$$

Similarly, optical path for light coming from S_2 is

$$S_2P + 0.5\ \mu\text{m} \times (2 - 1) = S_2P + 0.5\ \mu\text{m}$$

Path difference:

$$\Delta x = (S_2P + 0.5\ \mu\text{m}) - (S_1P + 2\ \mu\text{m})$$
$$= (S_2P - S_1P) - 1.5\ \mu\text{m}$$
$$= \frac{yd}{D} - 1.5\ \mu\text{m}$$

For central bright fringe, $\Delta x = 0$

$$\Rightarrow \quad y = \frac{1.5\ \mu\text{m}}{1\ \text{mm}} \times 1\ \text{m} = 1.5\ \text{mm}$$

The whole pattern is shifted by 1.5 mm upward.

9. Shift in fringe pattern $\Delta x = \frac{(\mu - 1)D}{d}$

and new fringe width $\beta = \frac{\lambda(2D)}{d}$

$\therefore \quad \Delta x = \beta$

$$\therefore \quad \frac{(\mu - 1)tD}{d} = \frac{2\lambda D}{d}$$

Hence $$\lambda = \frac{(\mu - 1)t}{2} = \frac{(1.6 - 1)(2.0 \times 10^{-6})}{2} = 6000\text{Å}$$

10. For variable refractive index, optical path $= \int \mu dx$

Path difference at point O,

$$\Delta x = \left[L + S_2O\right] - \left[\int_0^L (1 + ax)\,dx + S_1O\right]$$

$$\Delta x = L - \int_0^L (1 + ax)\,dx \qquad \left[\because S_2O - S_1O = 0\right]$$

At point 'O', $$\Delta x = L - \left(L + \frac{aL^2}{2}\right) = -\frac{aL^2}{2}$$

Then, for maxima 'a', $$\Delta x = -\frac{\lambda}{2} \Rightarrow \frac{aL^2}{2} = \frac{\lambda}{2} \Rightarrow a = \frac{\lambda}{L^2}$$

11. Let t be the thickness so corresponding $\Delta x = \mu t = \frac{3}{2}t$

Also $I_{max} = 4I$ so $I' = 2I$

We know $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$\Rightarrow \quad 2I = I + I + 2\sqrt{I^2}\cos\left(\frac{3\pi t}{\lambda}\right)$$

$$\Rightarrow \quad \cos\left(\frac{3\pi t}{\lambda}\right) = 0$$

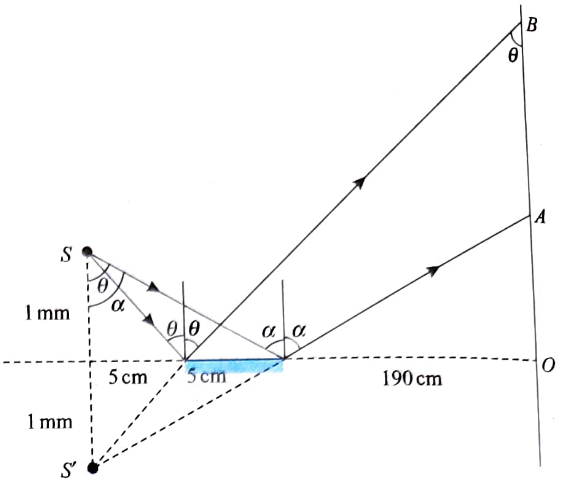
$$\Rightarrow \quad \cos\left(\frac{3\pi t}{\lambda}\right) = \cos\frac{\pi}{2} \text{ or } \cos\frac{3\pi}{2} \text{ or } \cos\frac{5\pi}{2}$$

$$\Rightarrow \quad \frac{3\pi t}{\lambda} = \frac{\pi}{2} \Rightarrow t = \frac{\lambda}{6};\ \frac{3\pi t}{\lambda} = \frac{3\pi}{2} \Rightarrow t = \frac{\lambda}{2}$$

and $$\frac{3\pi t}{\lambda} = \frac{5\pi}{2} \Rightarrow t = \frac{5\lambda}{6}$$

Exercise 3.5

1. (a) Here, point source S and its image S' behave as coherent source. SS' is parallel to screen, so shape will be hyperbolic.



(b) $$\beta = \frac{D}{d}\lambda = \frac{2}{2 \times 10^{-2}} \times 500 \times 10^{-9} = 0.5 \times 10^{-3}\text{m} = 0.5\text{mm}$$

(c) Length of fringe pattern $= AB = OB - OA = 2\text{cm}$

(d) $$N = \left[\frac{AB}{\beta}\right] = \frac{2 \times 10^{-2}}{0.5 \times 10^{-3}} = 40$$

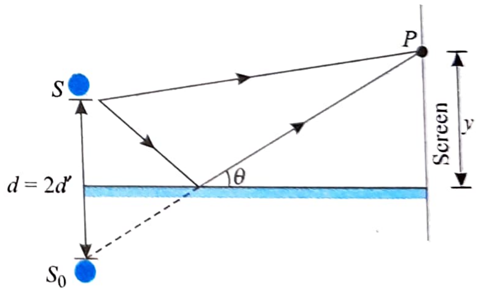
(e) $$\Delta x = S'O + \frac{\lambda}{2} - SO = \frac{\lambda}{2}$$

2. The image S' behaves as another source then distance between two sources is

$$d = 2d' = 2 \times 5\,\text{mm} = 1\,\text{cm}$$
$$\lambda = 5000 \times 10^{-8}\ \text{cm}$$

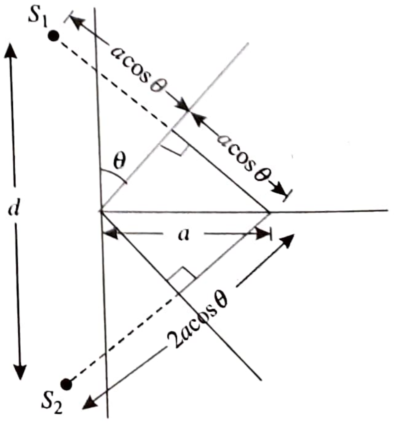
and $D = 200$ cm

Hence, the fringe width is $$\beta = \frac{\lambda D}{d} = \frac{5 \times 10^{-5} \times 200}{1} = 10^{-2}\ \text{cm}$$



3. Distance between two sources S_1 and S_2

$$d = 2 \times 2a\cos\theta\sin\theta = 2a\sin 2\theta$$



Screen distance $D = b + 2a\cos^2\theta$

$$\beta = \frac{\lambda D}{d} = \frac{\lambda(b+2a\cos^2\theta)}{2a\sin 2\theta} = \frac{\lambda(b+2a)}{4a\theta}$$

(if θ is very much small)

The deviated rays 1 and 2 can be used for interference. Hence the images, S_1 and S_2 can act as the coherent sources S of monochromatic light. The distance between the sources is

$$d = 2d' = 2a\delta$$

where δ = Deviation of light rays = $(n-1)\theta$

$$d = 2a(n-1)\theta \qquad \ldots\text{(i)}$$

Since the fringe width is $\beta = \frac{\lambda D}{d}$, using Eq. (i),

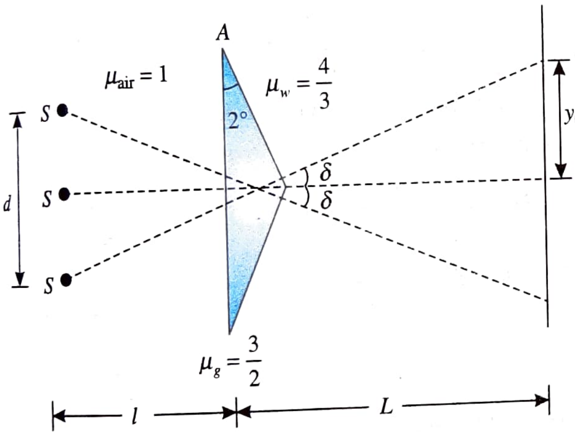
$$d = \frac{\lambda D}{\beta} = 2a(n-1)\theta$$

or $$\lambda = \frac{2a(n-1)\theta\beta}{D}$$

$$= \frac{2\times 50(1.5-1)\left(\frac{180°-179°}{2}\right)\frac{\pi}{180}\times\frac{360}{\pi}}{100} = 5000\text{Å}$$

Angle of deviation of the prism with water on one slab is

$$\delta = A\left[\frac{\mu_g}{\mu_w}-1\right] = \frac{2}{180}(314)\left[\frac{15}{133}-1\right] = 0.0043\,\text{rad}$$



$$d = 2\delta l = 2(0.0043)(575) = 5\,\text{mm}$$

$$D = l + L = 150\,\text{cm}$$

$\therefore$ Fringe width, $\beta = \frac{\lambda D}{\mu_w d} = \frac{(0.5\times10^{-6})(1.5)}{(1.33)(5\times10^{-3})} = 0.1125\,\text{mm}$

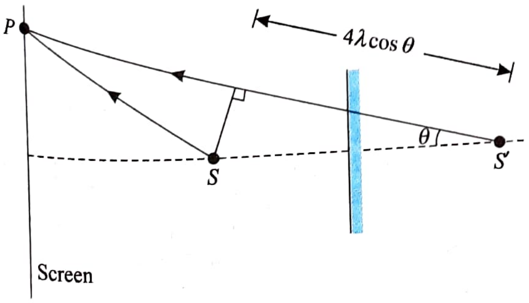
$$\tan\delta = \delta = \frac{y_0}{L}$$

$\therefore$ $y_0 = L\delta = 3.975$

$\therefore$ Number of bright fringe $= 2\left[\frac{y_0}{\beta}\right]+1 = 2[35.35]+1$

$$= 2\times 35+1 = 71$$

6. Here, path difference is $\Delta x = 2\lambda\cos\theta + \frac{\lambda}{2}$



For minima, $\Delta x = (2m+1)\frac{\lambda}{2}$

$$4\lambda\cos\theta + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}$$

or $$4\lambda\cos\theta = (2m+1)\frac{\lambda}{2} - \frac{\lambda}{2}$$

or $$4\lambda\cos\theta = m\lambda$$

or $$\cos\theta = \frac{m}{4}$$

$$-1 < \cos\theta < 1 \Rightarrow \quad -1 < \frac{m}{4} < 1$$

or $$-4 < m < 4$$

or $$m = -3, -2, -1, 0, 1, 2, 3$$

Thus, the number of minima is 7.

Exercises

Single Correct Answer Type

1. (4) $\beta_w = \frac{\lambda D}{\mu d}$

We need to increase $\beta \Rightarrow D$ increases; d decreases.

2. (1) Path difference, $\Delta x = \frac{yd}{D}$

Here, $y = \frac{5\lambda}{2}$

and $D = 10d = 50\lambda$ (as $d = 5\lambda$)

So, $\Delta x = \left(\frac{5\lambda}{2}\right)\left(\frac{5\lambda}{50\lambda}\right) = \frac{\lambda}{4}$

Corresponding phase difference will be

$$\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x) = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$$

or $$\frac{\phi}{2} = \frac{\pi}{4}$$

$\therefore$ $$I = I_0\cos^2\left(\frac{\phi}{2}\right) = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

3. (1) $\mu = \frac{c}{v} = \frac{v\lambda}{v\lambda'}$

$$\frac{3}{2} = \frac{\lambda}{\lambda'} \quad \text{or} \quad \lambda' = \frac{2\lambda}{3}$$

Note that the frequency remains unchanged.

4. (1) $I_{min} \propto (A_1 - A_2)^2$

$I_{min} \propto (2a - a)^2$

Clearly, the intensity of minima increases. Again,

$I_{max} \propto (A_1 + A_2)^2$

$I_{max} \propto (2a + a)^2$

Clearly, the intensity of maxima increases.

5. (4) $\beta' = \frac{D\lambda}{3d} = \frac{\beta}{3}$

6. (1) $\beta = \frac{D\lambda}{d} = \frac{f\lambda}{d} = \frac{1\times4890\times10^{-10}}{0.2\times10^{-3}}$

$= 0.29\times10^{-2}\,\text{m} = 2.9\,\text{mm} \approx 3\,\text{mm}$

7. (4) $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$

Here $I_1 = I_0$, $I_2 = I_0/2$

For maximum intensity, $\cos\Delta\phi = 1$

For minimum intensity, $\cos\Delta\phi = -1$

Before painting, $I = I_2 = I_0$, $I_{\max} = 4I_0$, $I_{\min} = 0$

After painting, $I_1 = I_0$, $I_{\max} = \left(\dfrac{3+2\sqrt{2}}{2}\right)I_0 < 4I_0$

$$I_2 = \frac{I_0}{2},\ I_{\min} = \left(\frac{3-2\sqrt{2}}{2}\right)I_0 > 0$$

8. (3) A slit would give divergent; a biprism would give double; a glass slab would give a parallel wavefront. Edge is downward.

9. (2) When slits are of unequal width, then intensity of sources S_1 and S_2 is not equal. So, position of minimum intensity will not be completely dark.

10. (4) Because white light will give a general illumination at the central maxima.

11. (1) When slits of equal width are taken, then intensity at maxima is $4a^2$ and at minima it is zero ($I \propto w$).

When one slit is doubled, then intensity at maxima will increase whereas intensity at minima will not be equal to zero and will be finite.

12. (1) White fringe is formed at the center of screen. Position of central fringe will remain unchanged on moving the screen.

13. (1) Optical path difference at $O = \left(\dfrac{\mu_2}{\mu_1} - 1\right)t$

14. (4) Fringe width, $\beta = \dfrac{\lambda D}{d}$

β becomes (1/4th) when λ is halved and d is doubled. The separation between successive dark fringes reduces. It does not remain unchanged.

15. (2) $I(\phi) = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$

Here, $I_1 = I$ and $I_2 = 4I$

At point A, $\phi = \dfrac{\pi}{2}$

$\therefore\quad I_A = I + 4I = 5I$

At point B, $\phi = \pi$

$\therefore\quad I_B = I + 4I - 4I = I$

$\therefore\quad I_A - I_B = 4I$

16. (2) Δx = path difference between two light waves

$= [n_1L_1 + L_2 - L_1] - [n_2L_2]$

$\Delta\phi$ = phase difference between two waves

For longest wavelengths, $\Delta\phi$ is the smallest.

For constructive interference, $\Delta\phi = 2\pi$

$$\begin{aligned} l_{\max} &= n_1L_1 - n_2L_2 + (L_2 - L_1) \\ &= (1.7)(3.5\times10^{-6}) - (1.25)(5.0\times10^{-6}) + (5.0-3.5)\times10^{-6} \\ &= (5.95 - 6.25 + 1.5)\times10^{-6} = 1.25\ 10^{-6}\ \text{m} \\ &= 1.2\ \mu\text{m} \end{aligned}$$

17. (3) For 100th maximum, $d\sin\theta = 100\lambda$

$$\Rightarrow\quad \sin\theta = \frac{100\times500\times10^{-9}}{1\times10^{-3}} = \frac{5\times10^{-4}}{10^{-3}} = 0.5 = \frac{1}{2}$$

$$\therefore\quad y = D\tan\theta = 1\times\tan 30^\circ = \frac{1}{\sqrt{3}}\ \text{m}$$

18. (3) At path difference $\dfrac{\lambda}{6}$, phase difference is $\dfrac{\pi}{3}$.

$$I = I_0 + I_0 + 2I_0\cos\frac{\pi}{3} = 3I_0$$

So, the required ratio is $\dfrac{3I_0}{4I_0} = 0.75$.

19. (3) There can be three minima from central point to ∞ corresponding to $\dfrac{\lambda}{3}, \dfrac{3\lambda}{2}, \dfrac{5\lambda}{2}$ path differences.

Therefore, total number of minima $= 2n_{\max} = 6$.

20. (2) For dark fringe, path difference $= (2n-1)\dfrac{\lambda}{2}$

21. (1) $y_1 = 3\left(\dfrac{\lambda_1 D}{d}\right)$ and $y_2 = 4\left(\dfrac{\lambda_2 D}{d}\right)$

Since both fall at the same location, so $y_1 = y_2$

$$\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

22. (1) The detector receives direct as well as reflected waves. The distance moved between two consecutive position of maxima is $\lambda/2$.

$$14\times\frac{\lambda}{2} = 7\lambda = 0.14\ \text{m}$$

$$\Rightarrow\quad \lambda = 0.02\ \text{m}$$

$$c = n\lambda$$

Putting $c = 3\times10^8$ m/s, we have $n = \dfrac{c}{\lambda} = \dfrac{3\times10^8}{0.02} = 1.5\times10^{10}$ Hz

23. (2) $y_n = \left(n + \dfrac{1}{2}\right)\dfrac{D\lambda}{d}$, $n = 1, 2, 3$

$$\left(3+\frac{1}{2}\right)\frac{D\lambda_1}{d} = 2\times\frac{D\lambda_2}{d}$$

$$\lambda_2 = \frac{7}{4}\times400 = 700\ \text{nm}$$

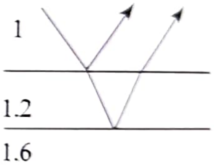
24. (4) Path difference $= d\sin\phi + d\sin\theta$

For maxima, $\Delta x = m\lambda$

$$\Rightarrow\quad \sin\phi + \sin\theta = \frac{m\lambda}{d}$$

25. (2) $2\mu t = n\lambda \Rightarrow t = \dfrac{n\lambda}{2\mu}$

$$\Rightarrow\quad t = \frac{\lambda}{2\mu} = 200\ \text{nm}$$



26. (4) Let intensity of one slit be I.

For maxima, $\Delta\phi = 0$

$$\Rightarrow\quad I_0 = I + I + 2\sqrt{II}\ \cos(0)$$

$$\Rightarrow\quad I = \frac{I_0}{4}$$

27. (1) Path difference at P is $\Delta x = 2\left(\dfrac{x}{2}\cos\theta\right) = x\cos\theta$

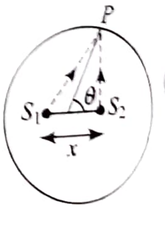
For intensity to be maximum,

$\Delta x = n\lambda \qquad (n = 0, 1, 2, 3, \ldots)$

or $\quad x\cos\theta = n\lambda$

or $\quad \cos\theta = \dfrac{n\lambda}{x} \geq 1$

$\therefore\quad n \geq \dfrac{x}{\lambda}$



Substituting $x = 5\lambda$, we get $n \geq 5$

or $\quad n = 1, 2, 3, 4, 5, \ldots$

Therefore, in all four quadrants there can be 20 maxima. There are more maxima at $\theta = 0^\circ$ and $\theta = 180^\circ$. But $n = 5$ corresponds to $\theta = 90^\circ$ and $\theta = 270^\circ$ which are coming only twice while we have multiplied it four times. Therefore, total number of maxima are still 20, i.e., $n = 1$ to 4 in four quadrants (total 16) plus more at $\theta = 0^\circ, 90^\circ, 180^\circ$ and 270°.

28. (1) $I = 4I_0 \cos^2\left(\dfrac{\phi}{2}\right)$

$$\Rightarrow \quad I_0 = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{1}{2}$$

$$\text{or} \quad \frac{\phi}{2} = \frac{\pi}{3}$$

$$\text{or} \quad \phi = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right)\Delta x$$

$$\text{or} \quad \frac{1}{3} = \left(\frac{1}{\lambda}\right) y \frac{d}{D} \quad \left(\Delta x = \frac{yd}{D}\right)$$

$$\therefore \quad y = \frac{\lambda}{3 \times \dfrac{d}{D}} = \frac{6 \times 10^{-7}}{3 \times 10^{-4}} = 2 \times 10^{-3}\ \text{m} = 2\ \text{mm}$$

29. (1) Condition for observing bright fringe is $2nd = \left(m + \dfrac{1}{2}\right)\lambda$

$$\therefore \quad \lambda = \frac{2nd}{\left(m + \dfrac{1}{2}\right)} = \frac{2 \times 1.5 \times 4 \times 10^{-5}}{m + \dfrac{1}{2}} = \frac{12 \times 10^{-5}}{m + \dfrac{1}{2}}$$

The integer m that gives the wavelength in the visible region (4000 Å to 7000 Å) is $m = 2$. In that case,

$$\lambda = \frac{12 \times 10^{-5}}{2 + \dfrac{1}{2}} = 4.8 \times 10^{-5} = 4800\ \text{Å}$$

30. (1) In this case, both the rays suffer a phase change of 180° and the conditions for destructive interference is

$$2nd = \left(m + \frac{1}{2}\right)\lambda_1$$

$$2nd = \left(m + \frac{3}{2}\right)\lambda_2$$

$$\therefore \quad \frac{m + \dfrac{1}{2}}{m + \dfrac{3}{2}} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{7000} = \frac{5}{7}$$

$$\text{and} \quad d = \frac{\left(m + \dfrac{1}{2}\right)\lambda_1}{2n} = \frac{2.5 \times 7000}{2 \times 1.3} = 6738\ \text{Å} = 6.738 \times 10^{-5}\ \text{cm}$$

31. (2) Effective path difference is $\mu_1 L_1 - \mu_2 L_2$.

Therefore, phase difference $= \dfrac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$

32. (3) In the first case, $I = I_0 + I_0 + 2I_0 \cos 0°$

or $\quad I = 4I_0$

In the second case, $I' = I_0 + I_0 = 2I_0$

$$\therefore \quad \frac{I}{I'} = \frac{4I_0}{2I_0} = \frac{2}{1}$$

33. (3) Fringe width, $\beta \propto \lambda$. Therefore, λ and hence β will decrease 1.5 times when immersed in the liquid. The distance between central maxima and 10th maxima is 3 cm in vacuum. When immersed in the liquid, it will reduce to 2 cm. Position of central maxima will not change while 10th maxima will be obtained at $y = 4$ cm.

34. (4) If $d \sin\theta = (\mu - 1)t$, central fringe is obtained at O.

If $d \sin\theta > (\mu - 1)t$, central fringe is obtained above O.

If $d \sin\theta < (\mu - 1)t$, central fringe is obtained below O.

35. (3) Intensity at the centre will be zero if path difference is $\lambda/2$. That is,

$$(\mu - 1)t = \frac{\lambda}{2}$$

$$\text{or} \quad t = \frac{\lambda}{2(\mu - 1)}$$

36. (2) For maximum intensity on the screen, $d \sin\theta = n\lambda$

$$\text{or} \quad \sin\theta = \frac{n\lambda}{d} = \frac{(n)(2000)}{(7000)} = \frac{n}{3.5}$$

$\sin\theta \geq 1 \quad \Rightarrow \quad n = 0, 1, 2, 3$ only

Thus, only seven maxima can be obtained on both sides of the screen.

37. (1) Fringe width, $\beta = \lambda D/d$. When the apparatus is immersed in a liquid, λ and hence β is reduced μ (refractive index) times.

$$10\beta' = (5.5)\beta$$

$$\text{or} \quad 10\lambda'\left(\frac{D}{d}\right) = (5.5)\frac{\lambda D}{d}$$

$$\text{or} \quad \frac{\lambda}{\lambda'} = \frac{10}{5.5} = \mu$$

$$\text{or} \quad \mu = 1.8$$

38. (2) $S_1 = \dfrac{\Delta_1 D}{d} = 11 \times 10^{-3}$

$$S_2 = \frac{\Delta_2 D}{d} = 12 \times 10^{-3}$$

$$\Rightarrow \quad \Delta_1/\Delta_2 = 11/12$$

$$\Rightarrow \quad 12\Delta_1 = 11\Delta_2$$

39. (1) $y = \dfrac{d}{2} = \dfrac{(2n - 1)D\lambda}{2d}$

$$\Rightarrow \quad \lambda = \frac{d^2}{D(2n - 1)}, \quad n = 1, 2, 3$$

40. (3) $I_p = \dfrac{I_{\max}}{2}[1 + \cos\phi] = \dfrac{I_{\max}}{2}\left(1 + \cos\dfrac{2\pi y}{\beta}\right)$

where $\beta = D\lambda/d$.

First maxima is observed at P, i.e., $\cos\dfrac{2\pi y}{\beta} = 1$. As D increases β will increase and the value of $\cos\dfrac{2\pi y}{\beta}$ should be negative. Hence, the ratio $I_p/I_{\max}$ starts decreasing but starts increasing again as $\cos\dfrac{2\pi y}{\beta}$ again starts becoming positive.

41. (2) The wavelength missing from the reflected spectrum must satisfy the condition, $2\mu t = n\lambda$, where t is thickness of air film.

$$2\mu t = n\lambda_1 = (n + 1)\lambda_2$$

$$\text{or} \quad n \times (7200) = (n + 1)\,5400$$

$$\therefore \quad n = 3$$

The next wavelengths must satisfy the condition,

$$n\lambda_1 = (n + 2)\lambda_2$$

$$\text{or} \quad 7200 \times 3 = (3 + 2)\lambda_2 = 5\lambda_2$$

$$\Rightarrow \quad \lambda_2 = 4320\ \text{Å}$$

42. (2) Fringe width, $\beta = \dfrac{\lambda D}{d}$, i.e., $\beta \propto \lambda$

When the wavelength is decreased from 600 nm to 400 nm, fringe width will also decrease by a factor of 4/6 or 2/3 or the number of fringes in the same segment will increase by a factor of 3/2. Therefore, number of fringes observed in the same segment is $12 \times \frac{3}{2} = 18$.

43. (1) Path difference due to slab should be integral multiple of λ.

Hence, $\Delta x = n\lambda$

or $\quad (\mu - 1)t = n\lambda, \quad n = 1, 2, \ldots$

or $\quad t = \dfrac{n\lambda}{\mu - 1}$

For minimum value of t, $n = 1$.

$$\therefore \quad t = \frac{n\lambda}{\mu - 1} = \frac{\lambda}{1.5 - 1} = 2\lambda$$

44. (2) Angular separation is λ/d.

For angular separation to be 10% greater, λ should be 10% greater.

$$\therefore \quad \text{New wavelength} = \left(589 + \frac{589}{10}\right)\text{nm}$$

$$= (589 + 58.9)\ \text{nm}$$

$$= 647.9\ \text{nm}\ (\approx 648\ \text{nm})$$

45. (1) $I \propto 4a^2 \cos^2 \dfrac{\phi}{2}$

In the first case, $\phi = 2\pi$

$\therefore \quad I' \propto 4a^2$

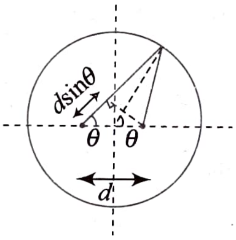
In the second case, $\phi = \dfrac{2\pi}{3}$

$\therefore \quad I' \propto 4a^2 \cos^2 \dfrac{2\pi}{3} \quad$ or $\quad I' \propto a^2$

$\dfrac{I'}{I} = \dfrac{1}{4} \quad$ or $\quad I' = \dfrac{I}{4}$

46. (1) Condition for maxima is $d \sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{d} = n\left(\frac{0.50}{2.0}\right) = 0.25n$$



As $\sin\theta$ lies between -1 and 1, so we wish to find all values of n for which $|0.25n| \le 1$

These values are $-4, -3, -2, -1, 0, +1, +2, +3, +4$. For each of these, there are two different values of θ except for -4 and $+4$. A single value of θ, $-90°$ and $+90°$, is associated with $n = -4$ and $n = +4$, respectively. Thus, there are 16 different angles in all and therefore 16 maxima.

47. (4) The path difference introduced due to introduction of transparent sheet is given by $\Delta x = (\mu - 1)t$.

If the central maxima occupies position of nth fringe, then

$$(\mu - 1)t = n\lambda = d\sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{(\mu - 1)t}{d} = \frac{(1.17 - 1) \times 1.5 \times 10^{-7}}{3 \times 10^{-7}} = 0.085$$

Hence, the angular position of central maxima is

$$\theta = \sin^{-1}(0.085) = 4.88°$$

For small angles, $\sin\theta = \theta = \tan\theta$

$$\Rightarrow \quad \tan\theta = \frac{y}{D}$$

$$\therefore \quad \frac{y}{D} = \frac{(\mu - 1)t}{d}$$

Shift of central maxima is $y = \dfrac{D(\mu - 1)t}{d}$

This formula can be used if D is given.

48. (1) The experimental set-up is in a liquid, therefore the wavelength of light will change

$$\lambda_{\text{liquid}} = \frac{\lambda_{\text{air}}}{\mu} = \frac{6300\text{Å}}{1.33} = \frac{6300 \times 10^{-10}}{1.33}\text{m}$$

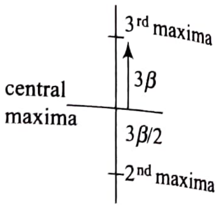
$$\text{Fringe width, } \beta = \frac{\lambda_{\text{liquid}} D}{d} = \frac{\lambda_{\text{air}} D}{\mu d} = \frac{6300 \times 10^{-10}}{1.33} \times \frac{1.33}{10^{-3}}$$

$$= 6.3 \times 10^{-4}\ \text{m}$$

49. (4) $d = 0.5$ mm and $D = 0.5$ m

Separation $= 3\beta + 1.5\beta = 4.5\beta$

$$= 4.5 \times \frac{\lambda D}{d} = 2.25\ \text{mm}$$



50. (3) Change in path difference for any point on screen is $|\mu - 1.8|t$.

For central maxima, phase difference = 0.

Hence, $d\sin\theta - |\mu - 1.8|t = 0$

$\Rightarrow \quad d\theta = |\mu - 1.8|t \qquad$ [θ is very small and is in radian]

$$\Rightarrow \quad |\mu - 1.8| = \frac{10^{-3} \times 0.1}{0.5 \times 10^{-3}} = 0.2$$

$$\Rightarrow \quad \mu = 2 \quad \text{or} \quad 1.6$$

51. (1) $\beta = 0.03$ cm, $D = 1$ m $= 100$ cm

Distance between images of the source = 0.8 cm

Distance of image from lens, $v = 80$ cm

Distance of slit from lens $= u$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{60} + \frac{1}{u} = \frac{1}{16} \quad \Rightarrow \quad u = 20\ \text{cm}$$

$$\text{Magnification} = \frac{v}{u} = \frac{80}{20} = 4$$

$$\text{Magnification} = \frac{\text{Distances between images of slits}}{\text{Distance between slits}}$$

$$= \frac{0.8}{d} = 4$$

$$\Rightarrow \quad d = 0.2\ \text{cm} \quad \Rightarrow \quad \beta = \frac{D\lambda}{d} = \frac{100\lambda}{2} = 0.03$$

$$\Rightarrow \quad \lambda = 6000\ \text{Å}$$

52. (3) In Young's double slit experiment, $\sin\theta = \theta = y/D$

So, $\Delta y/D$ and hence angular fringe width

$\theta_0 = \Delta\theta$ (with $\Delta y = \beta$) will be

$$\theta_0 = \frac{\beta}{D} = \frac{D\lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d}$$

$$\Rightarrow \quad \theta_0 = 1° = \left(\frac{\pi}{180}\right)\text{rad, and } \lambda = 6 \times 10^{-7}\ \text{m}$$

or $\quad d = \dfrac{\lambda}{\theta_0} = \dfrac{180}{\pi} \times (6 \times 10^{-7}) = 3.44 \times 10^{-5}$ m

or $\quad d = 0.0344$ mm

53. (1) As a bright fringe is formed in front of slit, therefore

$d/2$ = integral multiple of fringe width

$$\frac{d}{2} = \frac{n\lambda D}{d} \quad \Rightarrow \quad n = \frac{d^2}{2\lambda D}$$

54. (1) When mica sheet of thickness t and refractive index μ is introduced in the path of one of the interfering beams, optical path increases by $(\mu - 1)t$. Therefore, the shift on the screen is given by

$$y_0 = \frac{D}{d}(\mu - 1)t \qquad \text{...(i)}$$

When the distance between the plane of slits and screen is changed from D to $2D$, then

$$\beta = \frac{2D}{d}\lambda \qquad \text{...(ii)}$$

$$\therefore \quad \frac{D}{d}(\mu - 1)t = \frac{2D(\lambda)}{d} \Rightarrow \lambda = \frac{1}{2}(\mu - 1)t$$

(1) According to the question, shift = 5 fringe widths

$$\Rightarrow \quad \frac{(\mu - 1)tD}{d} = \frac{5\lambda D}{d}$$

$$\therefore \quad t = \frac{5\lambda}{\mu - 1} = \frac{25000}{1.5 - 1} = 50{,}000\ \text{Å} = 5 \times 10^{-6}\ \text{m}$$

(1) y_9 = Position of 9th bright fringe $= 9\left(\frac{\lambda D}{d}\right)$

y_2 = Position of 2nd dark fringe $= \left(2 - \frac{1}{2}\right)\frac{\lambda D}{d} = \frac{3}{2}\frac{\lambda D}{d}$

$$y_9 - y_2 = 7.5\ \text{mm} \Rightarrow \frac{\lambda D}{d}\left(9 - \frac{3}{2}\right) = 7.5 \times 10^{-3}$$

$$\therefore \quad \lambda = (7.5 \times 10^{-3})\left(\frac{2}{15}\right)\left(\frac{0.5 \times 10^{-3}}{100 \times 10^{-2}}\right)$$

$$= (75)\left(\frac{2}{15}\right)(5)(10^{-4-4}) = 50 \times 10^{-8}\ \text{m}$$

$$= 5000\ \text{Å}$$

(1) δ = phase difference between the waves from S_1 and S_2 at P

$$= \frac{\pi}{2} - \frac{2\pi}{2}(d \sin\theta)$$

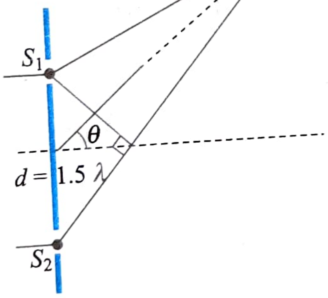
For maximum intensity at P, $\delta = n\pi$,

where $n = 0, \pm1, \pm2, \ldots$

$$\therefore \quad \frac{2\pi}{2}(1.5\lambda \sin\theta)\frac{\pi}{2} = n\pi$$

$$\Rightarrow \quad n - \frac{1}{2} = 3\sin\theta$$

$$\Rightarrow \quad \sin\theta = \left(\frac{n - \frac{1}{2}}{3}\right)$$



For $n = 0$, $\sin\theta = -\frac{1}{6}$

For $n = \pm1$, $\sin\theta = \frac{1}{6}, -\frac{1}{2}$

For $n = \pm2$, $\sin\theta = \frac{1}{2}, -\frac{5}{6}$

58. (3) The maxima in this case is obtained whenever $x_{\text{cm}} = \frac{n\lambda D}{d}$

We can write for first wavelength, $(x_4)\lambda_1 = \frac{4\lambda_1 D}{d}$

This must be equal to $(x_3)\lambda_2 = \frac{3\lambda_2 D}{d}$, since $(x_4)\lambda_1 = (x_3)\lambda_2$.

$$\therefore \quad \frac{4\lambda_1 D}{d} = 3\lambda_2\frac{D}{d} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{3}{4}$$

Hence, when both wavelengths are incident simultaneously, the maxima due to two will coincide at a point where the fourth maxima due to λ_1 occurred. This point will have maximum intensity and intensity will not be zero at any point.

59. (3) Here, direction of light is given by normal vector $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ Angle made by the $\vec{n}$ with y-axis is given by

$$\cos\beta = \frac{2}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{14}}$$

60. (4) In cases I, II, III and IV, the path differences are respectively, $\frac{\lambda}{2}, \lambda, \frac{\lambda}{4}$ and $\frac{3\lambda}{2}$.

Phase differences are, respectively, $\pi, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}$, and

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

Therefore, the intensities in the four cases are $0, I_0, \frac{I_0}{2}, \frac{I_0}{2}$, respectively.

61. (1) There are three and a half fringes from first maxima to fifth minima as shown.

$$\beta = \frac{7\ \text{mm}}{3.5} = 2\ \text{mm} \Rightarrow \lambda = \frac{\beta D}{d} = 600\ \text{nm}$$

62. (1) In ΔS_1PO, $\tan\frac{\theta}{2} = \frac{d/2}{D}$

As $D >> d$, θ is very small.

$$\therefore \quad \tan\frac{\theta}{2} = \frac{\theta}{2} \Rightarrow \frac{\theta}{2} = \frac{d}{2D} \Rightarrow \frac{D}{d} = \frac{1}{\theta}$$

$$\therefore \quad \text{Fringe width} = \frac{\lambda D}{d} = \frac{\lambda}{\theta}$$

63. (2) Angular width $= \frac{\lambda}{d} = 10^{-3}$ (given)

No. of fringes within 0.12° will be $n = \frac{0.12 \times 2\pi}{360 \times 10^{-3}} \cong 2.09$

The number of bright spots will be three.

64. (1) Optical path difference between the waves $= (n_3 - n_2)t$

$$\therefore \quad \text{Phase difference} = 2\pi\frac{(n_3 - n_2)t}{\lambda_{(\text{vacuum})}} = 2\pi\frac{(n_3 - n_2)t}{n_1\lambda_1}$$

65. (4) The nearest white spot will be at P, the central maxima.

$$y = \frac{2d}{3} - \frac{d}{2} = \frac{d}{6}$$

66. (2)

$$\frac{\lambda_1 D}{d} = 4.84\ \text{mm} \qquad \text{...(i)}$$

Let required wavelength be λ_2. Then according to given information,

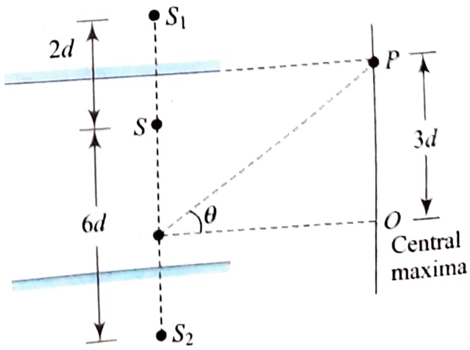
$$\frac{\lambda_2 D}{2d} = 4.84\ \text{mm} \qquad \text{...(ii)}$$

From (i) and (ii), $\frac{\lambda_1}{\frac{\lambda_2}{2}} = 1 \Rightarrow \lambda_2 = 1200\ \text{nm}$

67. (1) At P, $\Delta x = \frac{(8d) \times 3d}{D}$

For 2nd maxima, $\Delta x = 2\lambda$

$$\Rightarrow \quad \frac{24d^2}{D} = 2\lambda$$



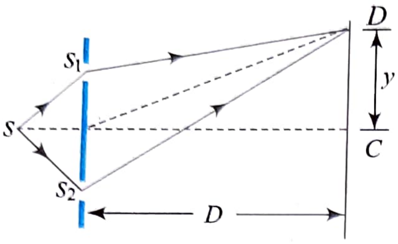
$$\Rightarrow \quad \lambda = \frac{12d^2}{D}$$

68. (3) Let n_1^{th} maxima corresponding to λ_1 be overlapping with n_2^{th} maxima corresponding to λ_2. Then, the required distance,

$$y=\frac{n_1\lambda_1 D}{d}=\frac{n_2\lambda_2 D}{d}=\text{LCM of }\frac{\lambda_1 D}{d}\text{ and }\frac{\lambda_2 D}{d}$$

69. (1) Path difference $\Delta x=\frac{dy}{D}$

Here $d=2\times10^{-3}$ m, $y=\frac{1}{6}\times10^{-3}$ m, $D=2$ m



$$\Delta x=\frac{2\times10^{-3}\times1\times10^{-3}}{2\times6}=\frac{10^{-6}}{5}\text{m}$$

Phase difference $\Delta\phi=\frac{2\pi}{\lambda}\times\Delta x=\frac{2\pi}{500\times10^{-9}}\times\frac{10^{-9}}{6}=-\frac{2\pi}{3}$

Resultant intensity at P

$$I=I_1+I_2+2\sqrt{I_1I_2}\,\cos\Delta\phi$$
$$=I_0+9I_0+2\times3I_0\cos\frac{2\pi}{3}$$
$$=10I_0-3I_0=7I_0$$

70. (4) $\Delta x=(\mu_A-1)t_A-(\mu_B-1)t_B=\mu_At_A-\mu_Bt_B-t_B+t_B=t_B-t_A$

If $\Delta x>0$, the fringe pattern will shift upward.

If $\Delta x<0$, the fringe pattern will shift downward.

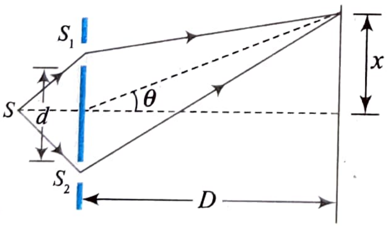
71. (3) At point P, $\Delta\phi=\frac{2\pi}{\lambda}\Delta x$ and $\Delta x=\frac{dx}{D}$

$$\Delta\phi=\frac{2\pi}{\lambda}\times\frac{dx}{D}=\frac{2\pi x}{\beta}$$

$$I_P=I_1+I_2+2\sqrt{I_1I_2}\,\cos\Delta\phi;\ I_1=I_2=I$$

$$I_P=4I\cos^2\frac{\Delta\phi}{2}$$

Given $4I=I_0$

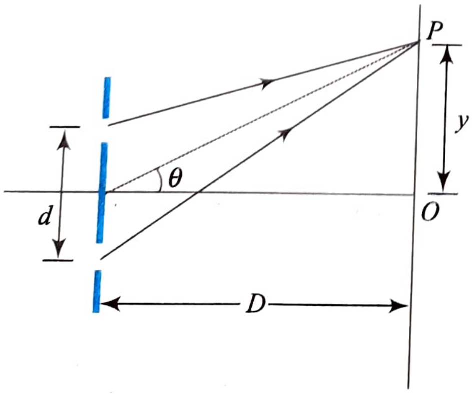


$\Rightarrow\quad I_1=I_2=\frac{I_0}{4}$

$$I_P=\frac{I_0}{2}+\frac{I_0}{2}\cos\Delta\phi$$

$$I_P=I_0\cos^2\frac{\Delta\phi}{2}=I_0\cos^2\frac{\pi x}{\beta}$$

72. (2) Let the intensity of individual waves be I. Then



$$I_0=4I\Rightarrow I=\frac{I_0}{4}$$

At P, $\Delta x=d\sin\theta$

$\Rightarrow\quad \Delta x=d\sin\theta=\frac{dy}{D}$

$\Rightarrow\quad \Delta x=\frac{d}{D}\times\frac{\beta}{4}=\frac{d}{D}\times\frac{\lambda D}{4d}=\frac{\lambda}{4}$

$\therefore\quad \Delta\phi=\frac{2\pi}{\lambda}\times\frac{\pi}{4}=\frac{\pi}{2}\qquad[\Delta\phi=k\,\Delta x]$

$$I'=I+I+2\sqrt{I^2}\,\cos\frac{\pi}{2}=2I=\frac{I_0}{2}$$

73. (3) $y_n=n\left(\frac{D\lambda}{d}\right)$ and $y'_n=n'\left(\frac{D\lambda'}{d}\right)$

Equating y_n and y'_n, we get $\frac{n}{n'}=\frac{\lambda'}{\lambda}=\frac{900}{750}=\frac{6}{5}$

Hence, the first position at which overlapping occurs is

$$y_6=y'_5=\frac{6(2)(750\times10^{-9})}{2\times10^{-3}}=4.5\text{ mm}$$

74. (3) (Shift) = 7 (Position of central maxima)

$\Rightarrow\quad (\mu-1)t\frac{D}{d}=7\left(\frac{\lambda D}{d}\right)$

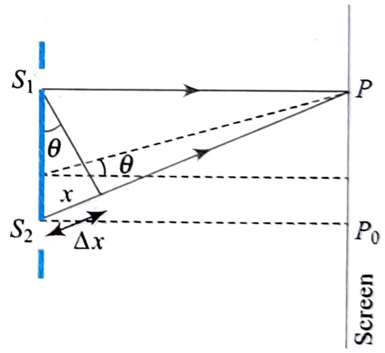
$\therefore\quad t=\frac{7\lambda}{\mu-1}$

$\Rightarrow\quad t=\frac{7(600)}{0.5}$

$\Rightarrow\quad t=8400$ nm

75. (3) $\Delta x=d\sin\theta=\frac{\lambda}{8}\sin\theta$

$\Delta\phi=$ Phase difference at P



$$\Delta\phi=\frac{2\pi}{\lambda}(\Delta x)+\Delta\phi'=\frac{2\pi}{\lambda}\left(\frac{\lambda}{8}\sin\theta\right)+\frac{\pi}{4}=\frac{\pi}{4}(1+\sin\theta)$$

$$I(\theta)=4I_0\cos^2\left(\frac{\Delta\phi}{2}\right)=4I_0\cos^2\left[\frac{\pi}{8}(1+\sin\theta)\right]$$

76. (1) There is air on both sides of the soap film. Therefore, the reflections of the light produces a net 180° phase shift.

The condition for bright fringes is $2t=\left(m+\frac{1}{2}\right)\lambda_{\text{film}}$

$$t=\frac{\left(m+\frac{1}{2}\right)\lambda_{\text{film}}}{2}=\frac{\left(m+\frac{1}{2}\right)\lambda}{2n}$$

$$=\frac{\left(\frac{1}{2}\right)(650\times10^{-9}\text{m})}{2(1.41)}=1.2\times10^{-7}\text{ m}$$

(3) When mica sheet of thickness t and refractive index μ is introduced in the path of one of the interfering beams, optical path increases by $(\mu - 1)t$. Hence, the shift on the screen,

$$y_0 = \frac{D}{d}(\mu - 1)t \qquad \text{...(i)}$$

When the distance between the plane of slits and screen is changed from F to $2D$, then

$$\beta = \frac{2D}{d}\lambda \qquad \text{...(ii)}$$

$$\frac{D}{d}(\mu - 1)t = \frac{2D(\lambda)}{d}$$

$$\Rightarrow \quad \lambda = \frac{1}{2}(\mu - 1)t = \frac{1}{2}(1.6 - 1) \times 2.0 \times 10^{-6}\text{ m} = 6000\text{ Å}$$

(1) If after placing the plate, intensity at the position of central maxima position remains unchanged, then it means first maxima takes position of central maxima. In case of minimum thickness of plate 2, path difference created by the plate should be equal to λ.

i.e., $t(\mu - 1) = \lambda$

$$t\left(\frac{3}{2} - 1\right) = \lambda \Rightarrow t = 2\lambda$$

(3) Path difference $= (\mu_2 - \mu_1)t = 12480$ Å

For maxima, $n\lambda = 12480$ Å

$\lambda_1 = 12480$ Å

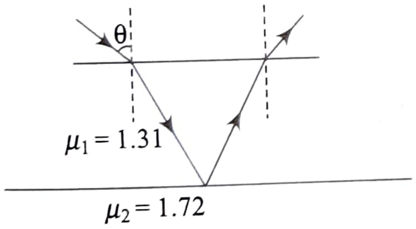
$\lambda_2 = 6240$ Å

$\lambda_3 = 4160$ Å

$\lambda_4 = 3120$ Å

Therefore, only 6240 Å and 4160 Å exist in the spectrum.

(3) There is no net change in phase produced by the two reflections. Hence,



$$\frac{\lambda}{2} = 2\mu_1 d\cos\theta$$

For normal incidence, $\cos\theta = 1$.

$$\therefore \quad d = \frac{\lambda}{4\mu_1} = \frac{5.3 \times 10^{-7}}{4 \times 1.31} = 10^{-7}\text{ m} = 0.1\ \mu\text{m}$$

(3) Intensity of direct ray $= I_0 = kA_0^2$

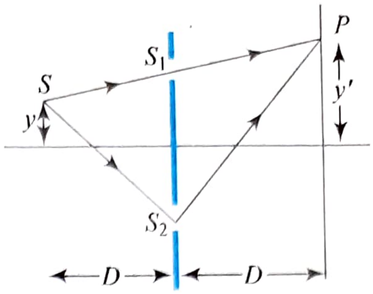
Intensity of reflected ray $= \frac{64}{100}I_0 = k\left(\frac{8A_0}{10}\right)^2$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{(A_0 + 0.8A_0)^2}{(A_0 - 0.8A_0)^2} = \left(\frac{1.8}{0.2}\right)^2 = \frac{81}{1}$$

(1) $y' = \frac{d}{2}$, at point P exactly in front of S_1

$$\Delta x = \frac{yd}{D} + \frac{d^2}{2D}$$

For minimum intensity, $\Delta x = (2n - 1)\frac{\lambda}{2} \quad (n = 1)$



Putting the values, we get

$$(0.5\sin\pi t) \times 10^{-6} + 0.25 \times 10^{-6} = \frac{500}{2} \times 10^{-9}$$

$$0.5\sin\pi t + 0.25 = \frac{0.5}{2}$$

$$\sin\pi t = 0 \quad \Rightarrow \quad \pi t = 0, \pi, 2\pi, \ldots$$

$$\Rightarrow \quad t = 1\text{ s}$$

83. (3) $I = I_0\sin^2\theta$

Differentiating I with respect to θ,

$$\frac{dI}{d\theta} = 2I_0\sin\theta\cos\theta$$

$$\frac{dI}{I} = \frac{2I_0\sin\theta\cos\theta\, d\theta}{I_0\sin^2\theta} = 2\cos\theta\, d\theta$$

Percentage error in angle is

$$\frac{d\theta}{\theta} \times 100 = \left(\frac{dI}{2I\cos\theta}\right)\frac{1}{\theta} \times 100$$

$$= \frac{0.002}{2 \times 5\cos 30°} \times \frac{6 \times 100}{\pi} = \frac{4}{\pi}\sqrt{3} \times 10^{-2}\,\%$$

84. (2) For destructive reflection:

$$\text{At} \quad \theta_1 = 20°\text{C}, \frac{2\mu_1}{\mu_g}t_1 = n\lambda_1 \qquad \text{...(i)}$$

$$\text{At} \quad \theta_2 = 170°\text{C}, \frac{2\mu_1}{\mu_g}t_2 = n\lambda_2 \qquad \text{...(ii)}$$

$$\frac{t_2}{t_1} = \frac{\lambda_2}{\lambda_1}$$

$$\frac{t_1[1 + \alpha(\theta_2 - \theta_1)]}{t_1} = \frac{\lambda_2}{\lambda_1}$$

(α is the coefficient of linear expansion of the film)

$$\Rightarrow \quad [1 + \alpha(170 - 20)] = \frac{606}{600}$$

$$\text{or} \quad \alpha = \frac{6}{600 \times 150} = 6.6 \times 10^{-5}\ °\text{C}^{-1}$$

85. (4) $I = I_0\cos^2\frac{(\lambda d\sin\theta)}{\lambda} = \frac{I_0}{2}$

$$\cos\left(\frac{\pi d\sin\theta}{\lambda}\right) = \frac{1}{\sqrt{2}}$$

$$x = \frac{\lambda D}{4d} = \frac{650 \times 10^{-9} \times 0.75}{4 \times 0.25 \times 10^{-3}}$$

$$= 487.5 \times 10^{-6}\text{ m} = 0.4875\text{ mm} \approx 0.5\text{ mm}$$

86. (2) For constructive interference, $d\sin\theta = n\lambda$

$$(\sin\theta)_{\max} = 1$$

$$n_{\max} = \frac{d}{\lambda} = \frac{0.012 \times 10^{-3}}{650 \times 10^{-9}} = 18.46$$

Therefore, total number of bright fringes including the central fringe is 18 + 18 + 1 = 37.

87. (2) Path difference = $\mu t - \mu_0 t_0 = n\lambda_0$

where n is the number of fringes that shift on the screen

$$\Rightarrow \quad \frac{\mu_0(1+\alpha_1\theta)t_0(1+\alpha_2\theta)-\mu_0 t_0}{\lambda_0} = n$$

$$\frac{\mu_0 t_0(\alpha_1+\alpha_2)\theta}{\lambda_0} = n$$

Given, $\mu_0 = 1.48$, $t_0 = 3 \times 10^{-2}$ m

$\alpha_1 = 2.5 \times 10^{-5}\ {}^\circ\text{C}^{-1}$

$\alpha_2 = 0.5 \times 10^{-8}\ {}^\circ\text{C}^{-1}$, $\theta = 5^\circ\ \text{min}^{-1}$

$\lambda_0 = 589$ nm

$$\therefore \quad n = \frac{1.48 \times 3 \times 10^{-2}(3 \times 10^{-5}) \times 5}{589 \times 10^{-9}} = 11$$

88. (1) $d \sin\theta_1 = n\lambda_1$, $d\sin\theta_2 = n\lambda_2$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \mu$$

$$\mu = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}$$

89. (1) Optical path difference,

$$\Delta x = 2\int_0^{L/2} \mu dx - L = 2\mu_0 \int_0^{L/2}\left(1+\frac{2x}{L}\right)dx - L$$

$$= \frac{3}{2}\mu_0 L - L = \left[(1.5)\mu_0 - 1\right]L$$

Now, $\Delta x = N\lambda$

$$\Rightarrow \quad N = \frac{\Delta x}{\lambda} = \frac{(1.5\mu_0 - 1)L}{\lambda}$$

Multiple Correct Answers Type

1. (1), (3)

If the screen is perpendicular to y-axis (line joining the sources), i.e., xz plane, the fringes will be circular (i.e., concentric circles with their centers on the point of intersection of the screen with y-axis). In this situation, central fringe will be bright if $S_1S_2 = n\lambda$ and dark if $S_1S_2 = (2n-1)\lambda/2$. From all this, it is clear that shape of fringes depends on the nature of sources and direction of observation, i.e., position of screen.

2. (1), (3)

For a certain point P on the screen at a distance x from the center of the screen, path difference $\Delta = \frac{xd}{D}$.

Path difference introduced due to sheet = $(\mu - 1)t$.

For central maximum at P, $\frac{xd}{D} = t(\mu - 1)$

or $\quad x = t(\mu - 1)\frac{D}{d}$

Now, $\quad \beta = \frac{D\lambda}{d}$

$\therefore \quad \frac{D}{d} = \frac{\beta}{\lambda}$

Hence, $x = t(\mu - 1)\frac{\beta}{\lambda}$

3. (1), (3)

Path difference = $\sqrt{D^2 + d^2} - D = 1$ m

Also, $\left[\sqrt{D^2+d^2} - D\right] = (2n-1)\frac{\lambda}{2}$

$$\lambda = \frac{2(1)}{2n-1}$$

For $n = 1, 2, 3, \ldots$

$$\lambda = 2 \text{ cm}, \frac{2}{3}\text{cm}, \frac{2}{5}\text{ cm}, \ldots$$

4. (1), (3), (4)

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = \left(\sqrt{I_1} + \sqrt{\frac{I}{2}}\right)^2 < 4I$$

$$I_{\min} = \left(\sqrt{I_1} - \sqrt{\frac{I}{2}}\right)^2 > 0$$

5. (1), (3)

Path difference at point $O = d \sin\alpha = 0.5$ mm

Corresponding phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi(0.5\times 10^{-3})}{5000 \times 10^{-10}} = 2000\pi = 2\pi \times 1000$$

O is a point corresponding to a maxima with the point at 1 m, below O corresponding to central maxima.

6. (1), (2), (3), (4)

Δx at $O = d$ [path difference is maximum at O]

So, if $d = \frac{7\lambda}{2}$, O will be a minima. If $d = \lambda$, O will be a maxima.

If $d = \frac{5\lambda}{2}$, O will be a minima and hence intensity is minimum.

If $d = 4.8\lambda$, then total 10 minima can be observed on the screen. 5 above O and 5 below O, which correspond to

$$\Delta x = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \pm\frac{7\lambda}{2}, \pm\frac{9\lambda}{2}$$

7. (2), (4)

It is better to make a chart for all the four cases which show the phase difference due to reflection at top and bottom surfaces of film.

	Top surfaces	Bottom surfaces	Due to 2L	Total
Case I	0	0	6π	6π
Case II	0	π	6π	7π
Case III	0	π	6π	7π
Case IV	π	π	6π	8π

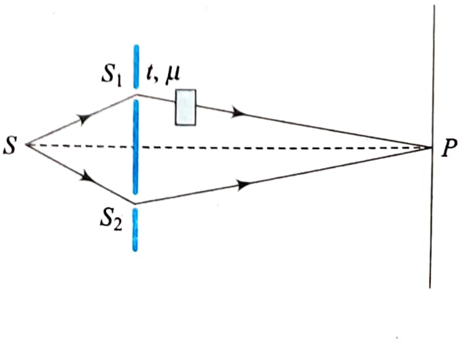
8. (1), (2), (3)

Due to absorption of light by the slab, intensity of two sources would be different and hence intensity at central maxima changes. If no absorption takes place, then it will not change. Intensity of dark fringes will not be absolutely zero, if some absorption is there.

9. (1), (3), (4)

Δx (at P) after insertion of slab

$= (S_1P - t)_{\text{air}} + t_{\text{medium}} - S_2P_{\text{air}} = (\mu - 1)t$



Earlier, Δx (at P) $= S_1P - S_2P = 0$

So, change in optical path due to insertion of slab is $(\mu - 1)\,t$.

For intensity to be zero at P, we have

$$\Delta x = \frac{(2n-1)\lambda}{2} \quad (n = 1, 2, \ldots)$$

$$(\mu - 1)t = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

(3), (4)

$$\phi_f = \frac{2\pi}{\lambda_0} l$$

$$\phi_f = \frac{2\pi}{\lambda} l$$

$$\Delta\phi = 2\pi l\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Further, by Snell's law, $n\lambda = (1)\lambda_0 \Rightarrow \lambda = \dfrac{\lambda_0}{n}$

$$\Rightarrow \quad \Delta\phi = \frac{2\pi l}{\lambda_0}(n-1)$$

11. (2), (4)

Whenever a ray goes from one medium to another, its frequency remains same. Its wavelength changes in accordance with $\mu\lambda$ = constant, i.e.,

$$\mu_0\lambda_0 = \mu\lambda$$

$$\lambda_0 = \mu\lambda \qquad (\because \mu_0 = \mu_{\text{air}} = 1)$$

$$\lambda = \frac{\lambda_0}{\mu} (<\lambda)$$

So, λ decreases when it goes from rarer to denser medium and vice versa.

$$\Delta\lambda = \lambda - \lambda_0$$

$$\Delta\lambda = \lambda_0\left(\frac{1}{\mu} - 1\right)$$

12. (2), (4)

We know that $\dfrac{I_{\text{max}}}{I_{\text{min}}} = \dfrac{(a+b)^2}{(a-b)^2}$

where $I_1 \propto a^2$ (a is amplitude of wave 1) and $I_2 \propto b^2$.

Hence, $\dfrac{I_{\text{max}}}{I_{\text{min}}} = \dfrac{9}{1}$

$$\frac{a+b}{a-b} = \frac{3}{1} \Rightarrow \frac{a}{b} = \frac{1}{2}$$

$$\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{1}{4}$$

13. (2), (3)

$$\beta_2 = \frac{\lambda_2 D}{d} = 2\frac{\lambda_1}{d} D$$

as $\quad \beta_1 = \dfrac{\lambda_1}{d} D$

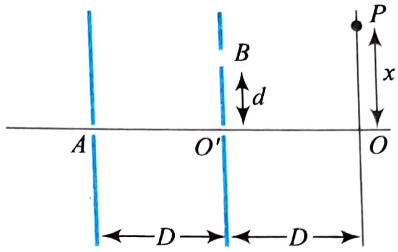
n^{th} order maxima of λ_2 coincides with $2n^{\text{th}}$ order maxima of λ_1.

n^{th} order minima of λ_2 does not coincide with $2n^{\text{th}}$ order maxima of λ_1.

14. (2), (4)

There is a dark fringe at O if the path difference

$$\delta = ABO - AO'O = \frac{\lambda}{2}$$



$$\Rightarrow \quad 2\sqrt{D^2 - d^2} - 2D = \frac{2d^2}{2D} = \frac{d^2}{D} = \frac{\lambda}{2}$$

$$d_{\text{min}} = \sqrt{\frac{\lambda D}{2}}$$

The bright fringe is formed at P if the path difference

$$\delta = AO'P - ABP = \lambda$$

$$= D + \sqrt{D^2 + x^2} - \sqrt{D^2 + d^2} - \sqrt{D^2 + (x-d)^2} = \lambda$$

$$= \frac{x^2}{2D} - \frac{d^2}{2D} - \frac{(s^2 + d^2 - 2xd)}{2D} = \lambda$$

Given $d = d_{\text{min}}$

On solving, $x = d_{\text{min}} = \sqrt{\dfrac{\lambda D}{2}}$

15. (1), (4)

If the amplitudes due to two individual sources at point P are A_0 and $3A_0$, then the resultant amplitude at P will be

$$A = \sqrt{A_0^2 + (3A_0)^2 + 2(A_0)(3A_0)\cos\pi/3} = A_0\sqrt{13}$$

Resultant intensity, $I \propto 13\,A_0^2$

16. (1), (2), (3)

Path difference, $\delta = BP - AP$

$$\sqrt{x^2 + 9^2} - x = n\lambda$$

$$\Rightarrow \quad x^2 + 9^2 = n^2\lambda^2 + x^2 + 2n\lambda x$$

$$\Rightarrow \quad x = \frac{9^2 - n^2\lambda^2}{2n\lambda}$$

$$\lambda = \frac{c}{v} = \frac{3 \times 10^8}{120 \times 10^6} = 2.5 \text{ m}$$

$n = 1, x = 14.95$ m; $n = 3, x = 1.65$ m;

$n = 2, x = 5.6$ m; $n = 4$ not possible

17. (2), (3), (4)

$$I = I_0 \cos^2\frac{(xd\sin\theta)}{\lambda}$$

(1) Path difference, $\delta = d\sin\theta = 5 \times 10^{-7}$ m

$$I_1 = I_0\cos^2\frac{(\pi \times 5 \times 10^{-7})}{(5 \times 10^{-7}\text{ m})} = I_0\cos^2(\pi) = I_0$$

(2) $I_2 = I_0\cos^2\left(\dfrac{4\pi}{2\times 3}\text{rad}\right) = I_0\cos^2\left(\dfrac{4\pi}{6}\right) = 0.25\,I_0$

(3) $I_3 = I_0 \cos^2\left(\dfrac{\pi \times 7.5 \times 10^{-7}\,\text{m}}{5 \times 10^{-7}\,\text{m}}\right)$

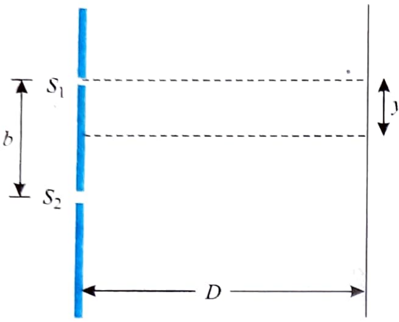
$= I_0 \cos^2(1.5\pi\,\text{rad}) = I_0 \cos^2(270°) = 0$

(4) $I_4 = I_0 \cos^2\left(\dfrac{\pi}{2 \times 3}\right)\text{rad} = 0.75\, I_0$

18. (1), (3)

$$y = (2n-1)\frac{\lambda}{2}\frac{D}{d} = (2n-1)\frac{\lambda}{2}\frac{D}{b} \qquad (\because d = b)$$

But $y = b/2$



$$\therefore\ \frac{b}{2} = (2n-1)\frac{\lambda}{2}\frac{D}{b} \Rightarrow \lambda = \frac{b^2}{(2n-1)D}$$

When $n = 1, 2$: $\lambda = \dfrac{b^2}{D}, \dfrac{b^2}{3D}$

19. (2), (3)

For central maxima path difference is zero

$$\frac{yd}{1} + y_1\frac{d}{4} = 0$$

$$y_1 = y(-4/1) = -4\sin \pi t$$

For maximum intensity, $\dfrac{yd}{1} + y_p\dfrac{d}{4} = n\lambda$

For first time $(n = 1)$,

$$yd + \frac{d^2}{8} = \lambda$$

Solving $t = \dfrac{1}{\pi}\sin^{-1}\left(\dfrac{54}{80}\right)$

For minimum intensity, $\dfrac{yd}{1} + y_p\dfrac{d}{4} = \left(n - \dfrac{1}{2}\right)\lambda$

For first time $n = 1$

$$yd + \frac{d^2}{8} = \frac{\lambda}{2}$$

Solving $t = \dfrac{1}{\pi}\sin^{-1}\left(\dfrac{27}{80}\right)$

20. (2), (3)

Optical path difference at P,

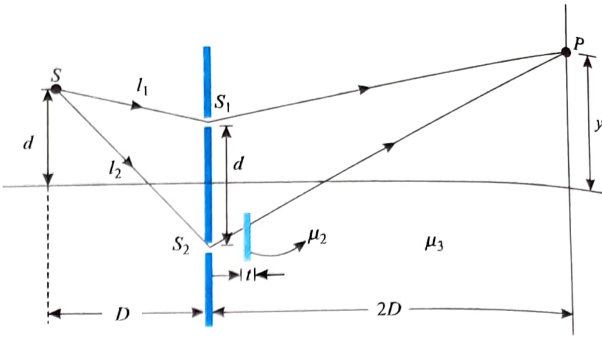
$$\Delta x = \left[SS_2 + (S_2P - t)\mu_3 + \mu_2 t\right] - \left[SS_1 + (S_1P)\mu_3\right]$$

$$= (SS_2 - SS_1) + (S_1Q - S_2Q)\mu_3 + (\mu_2 - \mu_3)t$$

$$= \frac{d^2}{D} + \left[\mu_3\frac{yd}{D} + (\mu_2 - \mu_3)t\right]$$

$$\Delta x = \frac{d^2}{D} + (\mu_2 - \mu_3)t + \frac{\mu_3 yd}{2D}$$

For position of central maxima $\Delta x = 0 \Rightarrow y = \dfrac{2}{9}$mm



Thickness of the slab so that central maxima forms at point P

$$\Rightarrow t(\mu_2 - \mu_3) + \frac{d^2}{D} = 0 \Rightarrow t = \frac{(2\times10^{-3})^2}{0.6} = \frac{20}{3}\times10^{-6}\,\text{m}$$

21. (2), (3)

When wavelength of light is changed from red to blue, it means wavelength of light is decreased then, $\beta = \dfrac{\lambda D}{d}$ also decrease.

Then consecutive fringes will come close.

When fringe width (β) is decreased, the number of maxima on the screen increases.

22. (1), (3)

$$\Delta x_0 = d\sin\theta = (10^{-3})\left(\frac{1}{2}\right) = 5\times10^{-4}\,\text{m} = (10^3)\lambda$$

Since, Δx_0 is integer multiple of λ, it will produce maximum intensity or $4I_0$ at O.

$$\omega = \frac{\lambda D}{d} = \frac{(5\times10^{-7})(2)}{(10^{-3})} = 10^{-3}\,\text{m} = 1\,\text{mm}$$

At 4 mm, we will get 4th order maxima.

23. (1), (3)

For overlapping of maxima $\dfrac{n_1\lambda_1 D}{d} = \dfrac{n_2\lambda_2 D}{d}$

or $\quad \dfrac{n_1}{n_2} = \dfrac{\lambda_2}{\lambda_1} = \dfrac{7}{5}, \dfrac{14}{10}\ldots$

$\Rightarrow$ 14th order maxima of λ_1 will coincide with 10th order maxima of λ_2

For overlapping of minima $\dfrac{(2n_1-1)\lambda_1 D}{2d} = \dfrac{(2n_2-1)\lambda_2 D}{2d}$

$$\therefore\ \frac{2n_1 - 1}{2n_2 - 1} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$$

Option (3) with $n_1 = 11$ and $n_2 = 8$ gives this ratio.

24. (1), (3), (4)

At P any point on the screen, optical path difference between SS_2P and SS_1P is zero.

$$(SS_2P - SS_1P)_{\text{optical}} = 0$$

$$y = -SS_2(\mu - 1)\frac{D}{d} = -2.5\left(\frac{1}{5}\right)\times\frac{10^3}{3} = \frac{-1}{6}\,\text{m}(\text{below } O)$$

Order of fringe at O is $\dfrac{y}{\lambda D/d} = \dfrac{1\times3\times10^{-3}}{6\times500\times1\times10^{-9}} = 750$

If a film of refractive index $\mu = 1.8$ is to be placed in front of S_1, then the path difference at the position is 200th maxima.

Then, $t(\mu - 1) = 200\lambda \Rightarrow t = \dfrac{200\lambda}{(\mu - 1)}$

$$t = \frac{200\times500\times10^{-9}}{(1.8-1)} = 1.25\times10^{-3}\,\text{m}$$

$\Rightarrow \quad t = 1.25$ mm

ed Comprehension Type

roblems 1–3

1) Optical path difference = $2n_1t$

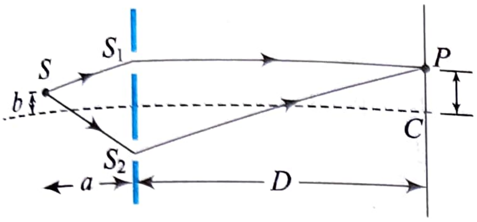
3) For destructive interference, $2n_1t_{\min} = \dfrac{\lambda}{2}$ $(\because n_1 < n_2)$

$$\therefore \quad t_{\min} = \frac{500}{4\times 1.25} = 100 \text{ nm}$$

1) $2n_1t = \dfrac{3\lambda}{2} \Rightarrow t = 3t_{\min} \cong 300$ nm

roblems 4–6

3)



Path difference at any point on the screen $\Delta x = \dfrac{bd}{a} + \dfrac{y'd}{D}$

For central maxima. $\Delta x = 0$

$$\Rightarrow \quad y' = -\frac{bD}{a} = -\frac{(1+\cos\pi t)2}{1}$$

$$y' = -2(1+\cos \pi t)$$

Position of central maxima at any time is

$$y' = -2\,(1+\cos \pi t) \text{ mm}$$

4) At $t = 2$ s, $y' = -4$ mm.

1) At $t = 1$ s, central maxima is at $y' = 0$.

But when a plate is inserted central maxima is at

$$y_1 = \frac{D}{d}(\mu-1)t = 1 \text{ mm}$$

roblems 7–9

3) Introduction of a glass plate shifts the central maxima by

$$\Delta x = \frac{D(\mu-1)t}{d} = \frac{\beta(\mu-1)t}{\lambda}$$

As the central maxima shifts by five fringes, therefore

$$\frac{\beta_R(\mu_R-1)t}{\lambda_R} = 5\beta_R \qquad \text{...(i)}$$

$$t = \frac{5\lambda_R}{(\mu_R-1)t} = \frac{5\times 7\times 10^{-6}}{(1.5-1)} = 7\ \mu\text{m}$$

2) Similarly, when green light is used, the central maxima shifts by six fringes; therefore

$$\frac{\beta_G(\mu_G-1)t}{\lambda_G} = 6\beta_G \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{(\mu_R-1)}{(\mu_G-1)} = \frac{5}{6} \quad \Rightarrow \quad \mu_G - 1 = \frac{6}{5}(1.5-1)$$

$$\Rightarrow \quad \mu_G = 1.6$$

(1) As given in the problem, $5\beta_R = 10^{-3} \Rightarrow \beta_R = 2\times 10^{-4}$

$$\therefore \quad \frac{\beta_G}{\beta_R} = \frac{(\lambda_G D/d)}{(\lambda_R D/d)} = \frac{\lambda_G}{\lambda_R} = \frac{5}{7}$$

$$\Rightarrow \quad \frac{\beta_G}{\beta_R} - 1 = \frac{5}{7} - 1 = -\frac{2}{7}$$

$\beta_G - \beta_R$ $\quad 2$

$$\therefore \quad \Delta\beta = \beta_G - \beta_R = -\frac{2}{7}\times\beta_R = -0.57\times 10^{-4} \text{ m}$$

The minus sign denotes that fringe width will decrease when green light is used.

For Problems 10–12

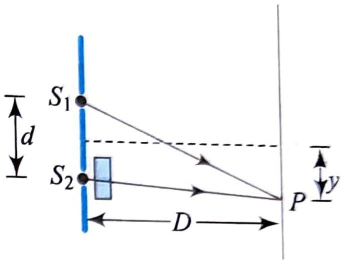
10. (2) Path difference at point P on the screen, $\Delta x = \dfrac{yd}{D}$

At the position of central maxima, the optical path lengths S_2P and S_1P are equal.

$$\frac{(S_2P - t)}{c/\mu_1} + \frac{t}{c/\mu_2} = \frac{S_1P}{c/\mu_1}$$

where $\mu_1 = 4/3$, $\mu_2 = 3/2$.

$$\mu_1(S_1P - S_2P) = (\mu_2 - \mu)t$$



$$\mu_1\left(\frac{yd}{D}\right) = (\mu_2 - \mu_1)t$$

$$y = \frac{(\mu_2-\mu_1)tD}{\mu_1 d} = \frac{[(3/2)-(4/3)]\times 10.4\times 1.5}{(4/3)\times 0.45\times 10^{-3}} = 4.33 \text{ mm}$$

11. (4) At point O, net path difference, $\Delta x = \left(\dfrac{\mu_2}{\mu_1} - 1\right)t$

Net phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{6\times 10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4\times 10^{-6}) = \left(\frac{13}{3}\right)\pi$$

Thus, intensity $I = I_{\max}\cos^2(\phi/2) = I_{\max}\cos^2\left(\dfrac{13\pi}{6}\right) = \dfrac{3}{4}I_{\max}$

12. (1) For maximum intensity at point O,

$$\Delta x = n\lambda \qquad \text{(where } n = 1, 2, 3, \ldots)$$

Path difference at point O,

$$\Delta x = \left(\frac{1.5}{4/3} - 1\right)(10.4\times 10^{-6}) = 1300 \text{ nm}$$

Thus, maximum intensity will correspond to $\dfrac{1300}{2}$ nm, $\dfrac{1300}{3}$ nm, … .

In the given range, required values are 650 nm and 433.33 nm.

For Problems 13–15

13. (2) The angular fringe width, i.e., the angle subtended by a fringe at the center of slits is given by

$$\beta_\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

According to the given condition,

$$\beta_\theta = \frac{\lambda}{d} \geq \frac{\pi}{180\times 60}$$

$$d \leq \frac{6\times 10^{-7}\times 180\times 60}{\pi}$$

$$d \leq 2.06\times 10^{-3} \text{ m}$$

Thus, d $= 2.06\times 10^{-3}$ m = 2.06 mm

14. (1) Position of 3rd bright fringe,

$$y_3 = \frac{3\lambda D}{d} = \frac{3\times 6\times 10^{-7}\times 1}{2.06\times 10^{-3}} = 8.74\times 10^{-4}\text{ m}$$

15. (3) Position of 5th dark fringe,

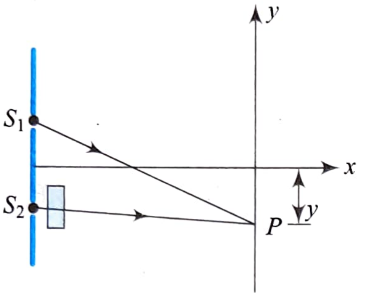
$$y_5 = \frac{\left(n-\frac{1}{2}\right)\lambda D}{d} = \frac{9\lambda D}{2d}$$

$$= \frac{9\times 6\times 10^{-7}\times 1}{2\times 2.06\times 10^{-3}} = 13.1\times 10^{-4}\text{ m} = 1.31\text{ mm}$$

For Problems 16–17

16. (2), 17. (1)

We consider a point P on the screen.



Optical path length,

$$[S_1P]_{\text{liquid}} = [\mu_1 S_1 P]_{\text{air}}$$

$$[S_1P - t]_{\text{liquid}} + t_{\text{glass}} = \mu_1[S_2P - t]_{\text{air}} + [\mu_g t]_{\text{air}}$$

$$= [\mu_1 S_2 P + (\mu_g - \mu_1)t]_{\text{air}}$$

Hence, optical path difference at P,

$$\Delta x = [\mu_1 S_2 P + (\mu_g - \mu_1)t] - \mu_1 S_1 P = \mu_1(S_2P - S_1P) + (\mu_g + \mu_1)t$$

For a point P at the screen in the absence of liquid,

$$S_2P - S_1P = \frac{yd}{D}$$

If a liquid is filled, $[S_2P - S_1P]_{\text{liquid}} = \dfrac{\mu_1 yd}{D}$

Thus, $\Delta x = \dfrac{\mu_1 yd}{D} + [\mu_g - \mu_1]t$

(i) For central maxima, $\Delta x = 0$

$$y = \left[\frac{\mu_g - \mu_1}{\mu_1}\right]\frac{tD}{d} = \frac{(4-T)tD}{(10-T)d}$$

At point O, $y = 0$

Thus, $T = 4$ s

(ii) Speed of central maxima, $v = \left|\dfrac{dy}{dt}\right| = \dfrac{6Dt}{(10-T)^2 d}$

Central maxima is at O at time $t = 4$ s.

$$\therefore \quad v = \frac{6Dt}{36d} = \frac{1\times 36\times 10^{-6}}{6\times 2\times 10^{-3}} = 3\times 10^{-3}\text{ m s}^{-1}$$

For Problems 18–19

18. (2) From lens equation, we have

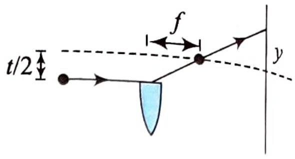
$$v = \frac{uf}{f-u} \qquad (v \text{ lies on the left side})$$

$$d = 2\left(\frac{1}{2}\right)\left[\frac{v}{u} - 1\right] = t\left[\frac{u}{f-u}\right]$$

$$D = L + v = L + \frac{uf}{f-u}$$

Fringe width,

$$\beta = \frac{\lambda D}{d} = \left[\frac{\lambda\left[L + \frac{\mu f}{f-u}\right]}{t\left[\frac{u}{f-u}\right]}\right]$$



$$\beta = \frac{\lambda}{t}\left[f + \frac{L(f-u)}{u}\right]$$

When $u \rightarrow f$, $\beta = \dfrac{\lambda f}{t}$

19. (4) From the figure, $\dfrac{y}{L-f} = \dfrac{t/2}{f} \Rightarrow y = \dfrac{t/2}{f}(L-f)$

Distance of the point up to which interference occurs,

$$y_0 = \frac{t/2}{f}(L-f) + \frac{t}{2} = \frac{tL}{2f}$$

$$\text{Number of visible maxima} = \frac{tLt}{2f\times\lambda f} = \frac{Lt^2}{2\lambda f^2}$$

For Problems 20–22

20. (1) For dark spot at O,

$$\Delta x = (AB+BO) - AO = 2(D^2+d^2)^{1/2} - 2D = 2D\left[\left(1+\frac{d^2}{D^2}\right)^{1/2} - 1\right]$$

$$\Rightarrow \quad \Delta x = \frac{d^2}{D}$$

$$\frac{d^2}{D} = \frac{\lambda}{2} \quad \Rightarrow \quad d = \sqrt{\frac{D\lambda}{4}}$$

21. (3) Δx at P: $\Delta x = (AB+BP) - (AC+PC) = (AB-AC) + (BP-PC)$

Δx at O: $\Delta x = \dfrac{\lambda}{2}$

The path difference at P will be zero if $x = d$. Hence next maxima will form at P.

22. (2) Fringe width = distance between two consecutive dark or bright fringes = 2 × distance between two consecutive bright dark fringes = $2\times d$

For Problems 23–25

23. (1) $z = \dfrac{\lambda D}{2d}$

At S_4: $\dfrac{\Delta x}{d} = \dfrac{z}{D}$

$$\Rightarrow \quad \Delta x = \frac{\lambda D}{2d}\frac{d}{D} = \frac{\lambda}{2}$$

Hence, minima at S_4 and maxima at S_3 (intensity $4I_0$).

Hence, $\dfrac{I_{\max}}{I_{\min}} = 1$

24. (3) $z = \dfrac{\lambda D}{d}$

Δx at S_4: $\Delta x = \dfrac{\lambda D}{d}\dfrac{d}{D} = \lambda$

Hence, maxima at S_4 as well as S_3.

Resultant intensity at S_4, $I = 4I_0$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{[(4I_0)^{1/2} + (4I_0)^{1/2}]^2}{[(4I_0)^{1/2} - (4I_0)^{1/2}]^2} = \infty$$

25. (4) $z = \dfrac{\lambda D}{4d}$

$$\Delta x = y\frac{d}{D} = \frac{\lambda D}{4d}\frac{d}{D} = \frac{\lambda}{4}$$

$$\phi = \frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = \frac{\pi}{4}$$

Intensity at S_4: $I_4 = 2I_0\left(1+\cos\frac{\pi}{2}\right) = 2I_0$

Intensity at S_3: $I_3 = 4I_0$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \frac{[(4I_0)^{1/2}+(2I_0)^{1/2}]^2}{[(4I_0)^{1/2}-(2I_0)^{1/2}]^2} = \left[\frac{(2+\sqrt{2})}{(2-\sqrt{2})}\right]^2$$

$$= \left[\frac{(2+\sqrt{2})^2}{4-2}\right]^2 = \frac{1}{4}[4+2+4\sqrt{2}]^2$$

$$= \frac{1}{4}[6+4\sqrt{2}]^2 = [3+2\sqrt{2}]^2$$

Problems 26–28

(4) $I_{av} = \frac{I_{\max}+I_{\min}}{2} = \frac{(A_1+A_2)^2+(A_1-A_2)^2}{2} = A_1^2 + A_2^2 = I_1 + I_2$

$I_{av} = I_1 + I_2$

So, choice (4) is correct.

Neither loss nor gain of energy is observed, but only redistribution of energy takes place.

(1), (3), (4)

If $A_1 = A = a$, $I_{\max} = 4a^2$, $I_{\min} = 0$. So, visibility is the best. Choice (1) is correct.

Since $I_{\min} = 0$, choice (3) is also correct.

Width decides intensity and thereby the amplitude. So, choice (4) is correct making (2) wrong.

(4) Sources are independent. They cannot form a coherent source since ϕ cannot be constant with time. So, choices (1), (2) and (3) are wrong, i.e., choice (4) is answer.

Problems 29–31

(2), (3)

Broad sources provide wide angular incidence of light. The thickness should be small, since the path difference should be comparable with the wavelength. Thick slabs cannot bring wavelength comparable path differences. So, choices (1) and (4) are wrong. Choices (2) and (3) are correct.

(1) Position of nth bright fringe $= Y = \frac{D\Delta x}{d}$, where Δx is the path difference.

When a slab of refractive index 'μ' and thickness 't' is introduced, the new position will be

$$Y' = \frac{D}{d}[\Delta x + (\mu+1)t]$$

$$\therefore \quad \text{Shift} = \frac{D}{d}(\mu-1)t$$

Shift is given by $5\lambda D/d$.

$$\therefore \quad \frac{5\lambda D}{d} = \frac{D}{d}(\mu-1)t$$

$$\Rightarrow \quad t = \frac{5\lambda}{\mu-1}$$

Choices (3) and (4) are wrong dimensionally. Choice (2) is wrong numerically. So, choice (1) is correct.

31. (2) In Lloyd's single mirror experiment, the interference is between the light reflected by the mirror and the direct light from the source. The reflected light undergoes a phase difference of π radians or path difference of $\lambda/2$.

So, path difference $+ \frac{\lambda}{2} = N\lambda$ for bright fringe. Hence,

Path difference $= (2N-1)\frac{\lambda}{2}$

So, choice (2) is correct and rest are wrong.

For Problems 32–34

32. (2) $I \propto \frac{1}{(\text{distance})^2}$

$$\therefore \quad \frac{\Delta I}{I_0} = \frac{L^2-(L-x)^2}{L^2} \approx \frac{2A\sin\omega t}{L}$$

33. (4) Intensity of the fringes will increase.

34. (4) $\beta = \frac{\lambda D}{d} = \text{constant}$

For Problems 35–37

35. (2) Intensity at O is proportional to intensity at S_3 and at S_4

$I = k\cos^2\frac{\delta}{2}$, where k is constant and δ is the phase difference.

$\delta = \frac{2\pi}{\beta} \times \frac{Z}{2} = \frac{\pi Z}{\beta}$, where β is fringe width

$$\beta = \frac{\lambda D}{d}$$

When $Z = \lambda D/2d$:

$$\delta = \frac{\pi d}{\lambda D} \times \frac{\lambda D}{2d} = \frac{\pi}{2}$$

$$I = I_0 = k\cos^2\left(\frac{\pi}{4}\right)$$

$$k = 2I_0$$

When $Z' = \frac{2\lambda D}{d}$:

Path difference $\delta' = \frac{2\pi}{\beta}\frac{Z'}{2} = \frac{2\pi d}{\lambda D} \times \frac{2\lambda D}{d\times 2} = 2\pi$

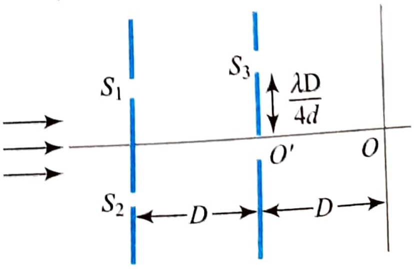
Required intensity at O is $I' = k\cos^2\frac{\delta}{2} = 2I_0\cos^2(\pi) = 2I_0$

36. (4) Intensity at O is zero if $\delta/2 = \pi/2$. That is,

$$\delta = \frac{2\pi}{\beta}\frac{Z}{2} = \pi \quad \Rightarrow \quad Z = \beta = \frac{\lambda D}{d}$$

37. (3) Intensity at $S_3 = A_0^2 + A_0^2 + 2A_0^2\cos\phi$

where $\phi = \frac{2\pi}{\beta}\frac{\lambda D}{4d} = \frac{2\pi d}{\lambda D} \times \frac{\lambda D}{4d} = \frac{\pi}{2}$



$$I_{S_3} = A_0^2\left(2+2\cos\frac{\pi}{2}\right) = 2A_0^2 \times 2\cos^2\frac{\pi}{4} = \frac{4}{2}A_0^2 = 2A_0^2$$

Amplitude of wave at S_3, $A_{S_3} = \sqrt{2}A_0$

Maximum intensity at $O' = (A_0 + \sqrt{2}A_0)^2$

Maximum intensity at $O = (\sqrt{2}A_0 - A_0)^2$

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = (\sqrt{2}+1)^4 = 34$$

For Problems 38–40

38. (1) Wavefronts are parallel in both media. Therefore, light which is perpendicular to the wavefront travels as a parallel beam in each medium.

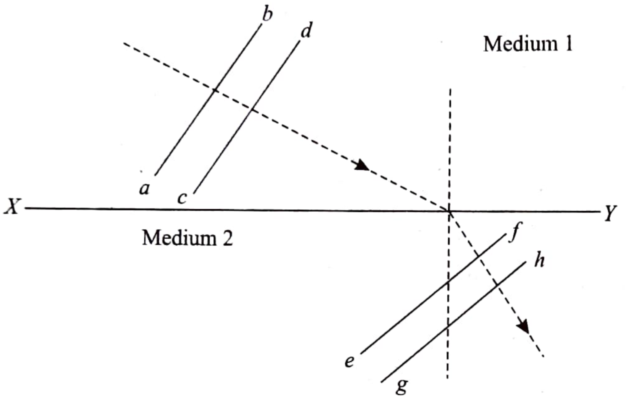
39. (3) All points on a wavefront are at the same phase.

$$\phi_d = \phi_c \quad \text{and} \quad \phi_f = \phi_e$$

$$\phi_d - \phi_f = \phi_c - \phi_e$$

Hence, the correct option is (3).

40. (2) In medium 2, wavefront bends away from the normal after refraction. Therefore, ray of light which is perpendicular to wavefront bends toward the normal in medium 2 during refraction. So, medium 2 is denser or its speed in medium 1 is more.



Hence, option (2) is correct.

Matrix Match Type

1. i. → c.; ii. → c.; iii. → d.; iv. → a.

By using $(\mu - 1)\,t = n\lambda$, we can find value of n, that is order of fringe produced at P, if that particular strip has been placed over any of the slit. If two strips are used in conjuction (over each other), path difference due to each is added to get net path difference created. If two strips are used over different slits, their path differences are subtracted to get net path difference. Now,

$$n_1 = \frac{(\mu_1 - 1)t_1}{\lambda} = 5$$

$$n_2 = 4.5$$

$$n_3 = 0.5$$

For (i), order of the fringe is 4.5, i.e., fifth dark.

For (ii), net order is 5 – 0.5 = 4.5, i.e., fifth dark.

For (iii), net order is 5 – (0.5 + 4.5) = 0, i.e., it is central bright again at P.

For (iv), net order is (5 + 0.5) – (4.5) = 1, i.e., first bright.

2. i. → b., c., d.; ii. → a., b., c., d.; iii. → b., c., d.; iv. → a., b., c., d.

Intensity at a distance x from central maxima on screen is

$$I = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{2\pi x}{\beta}$$

where $\beta = \dfrac{D\lambda}{d}$

$$I_{\max} = 9I_0 \quad \text{and} \quad I_{\min} = I_0$$

At points where intensity is (1/9)th of maximum intensity, minima is formed. Therefore, distance between such points is β, $2\beta, 3\beta, 4\beta, \ldots$

At points where intensity is (3/9)th of maximum intensity,

$$\cos\frac{2\pi x}{\beta} = -\frac{1}{2} \text{ or } x = \frac{\beta}{3}$$

Hence, distance between such points is

$$\frac{\beta}{3}, \frac{2\beta}{3}, \beta, \beta + \frac{\beta}{3}, \beta + \frac{2\beta}{3}, 2\beta, \ldots.$$

If $\quad \cos\dfrac{2\pi x}{\beta} = 0 \quad$ or $\quad x = \dfrac{\beta}{4}$

Hence, distance between such points is $\dfrac{\beta}{2}, \beta, \beta + \dfrac{\beta}{2}, 2\beta, \ldots$

If $\cos\dfrac{2\pi x}{\beta} = \dfrac{1}{2} \quad$ or $\quad x = \dfrac{\beta}{6}$

Hence, distance between such points is $\dfrac{\beta}{3}, \dfrac{2\beta}{3}, \beta, \beta + \dfrac{\beta}{3}, \beta + \dfrac{2\beta}{3}, 2\beta, \ldots.$

3. i. → b., c.; ii. → b, d.; iii. → a., d.; iv. → a., b., c., d.

For $\mu_1 = \mu_2$, the two waves in the reflected system differ by optical phase difference of π and hence due to destructive interference, the film appears to be dark. In transmitted system, the two waves are in phase and hence film appears to be shiny. Similarly, we can go for the other conditions.

4. i. → a.; ii. → b.; iii. → c.; iv. → d.

Depending on the value of refractive index, the path travelled by light would be decided, which in turn decides the shapes of wavefronts.

For more refractive index, speed of light is less and hence wavefront location would be somewhat located back side.

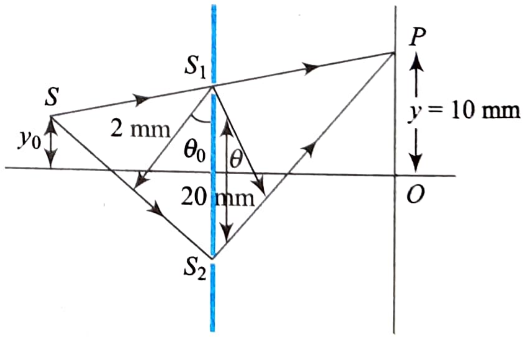
5. i. → d.; ii. → a.; iii. → b.; iv. → c.

The optical path difference between the two waves arriving at P is

$$\delta = (SS_2 + S_2P) - (SS_1 + S_1P)$$

$$= (SS_2 - SS_1) + (S_2P - S_1P)$$

$$= d\sin\theta_0 + d\sin\theta = \frac{dy_0}{D_1} + \frac{dy}{D_2}$$



$D = 20$ mm, $y_0 = 2$ mm, $D_2 = 2$ m, $D_1 = 1$ m, $y = 10$ mm

$$\therefore \quad \delta = \frac{20 \times 2}{1000} + \frac{20 \times 10}{2000} = 0.14 \text{ mm}$$

For a bright fringe, $\delta = n\lambda$

$$\Rightarrow \quad n = \frac{\delta}{\lambda} = \frac{0.14}{0.5 \times 10^{-3}} = 280$$

At the origin O, $\delta' = \dfrac{dy_0}{D_1} = 0.04$ mm

$$n' = \frac{\delta'}{\lambda} = \frac{0.04}{0.5\times10^{-3}} = 80$$

Due to transparent paper, the change in optical path is

$$(\mu-1)\,t = (1.45-1)\,(0.02)\text{ mm} = 0.009\text{ mm}$$

$$\delta'' = 0.14\text{ mm} - 0.009\text{ mm} = 0.131\text{ mm}$$

$$\Rightarrow \quad n = \frac{0.131}{0.5\times10^{-3}} = 262$$

Due to transparent paper, the path difference at O,

$$\delta = \delta' - (\mu-1)\,t = (0.04-0.009)\text{ mm} = 0.031\text{ mm}$$

$$\Rightarrow \quad n = \frac{0.031}{0.5\times10^{-3}} = 62$$

(1) Path difference due to slab-1 $= \Delta x = t(u-1) = 5\left[\frac{1}{2}\right] = \frac{5}{2}\mu\text{m}$

$\therefore$ Phase difference $\Delta\phi = \dfrac{2\pi\,\Delta x}{\lambda}$

$$= \frac{2\pi}{500\times10^{-9}}\times\frac{5}{2}\times10^{-6}\text{ m} = 10\pi$$

Path difference due to slab (2) $= \Delta x = t|\mu-1| = \frac{3}{2}\left[\frac{3}{3}\right] = \frac{9}{4}\mu\text{m}$

$$\Delta\phi = \frac{2\pi}{\lambda}\times\frac{9}{4}\times10^{-6} = 9\pi$$

Path difference due to slab (3) $= \Delta x = t|\mu-1| = \frac{1}{4}[1] = \frac{1}{4}\mu\text{m}$

$$\Delta\phi = \frac{2\pi}{500\times10^{-9}}\times\frac{1}{4}\times10^{-6} = \pi$$

(i) $10\pi \rightarrow$ fifth maxima

(ii) $(10\pi-\pi) = 9\pi \rightarrow$ fifth minima

(iii) $(10\pi-(9\pi+\pi)) = 0 \rightarrow$ central maxima

(iv) $((10\pi+\pi)-9\pi) = 2\pi \rightarrow$ first maxima

7. (2) $\phi = \dfrac{2\pi}{\lambda}\Delta x$ and then apply, $I = 4I_0\cos^2\left(\dfrac{\phi}{2}\right)$

8. (4) $I = I_{\text{max}} = 4I_0 \Rightarrow \quad I_0 = \dfrac{I}{4}$

(i) For $y = \dfrac{2D}{d}$, $\Delta x = \dfrac{yd}{D} = \dfrac{\lambda}{2}$

$\therefore \quad I_{S_3} = I_{S_4} = 0$ for $\Delta x = \dfrac{\lambda}{2}$

$\therefore \quad I_0 = 0$

(ii) For $y = \dfrac{\lambda D}{6d}$, $\quad \Delta x = \dfrac{yd}{D} = \dfrac{\lambda}{6}$

$$\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x) = \frac{2\pi}{6} = 60^\circ$$

Now, $I_{S_3} = I_{S_4} = I_{\text{max}}\cos^2\dfrac{\theta}{2} = I\cos^2 30^\circ = \dfrac{3}{4}I$

$\therefore \quad I_0 = 4I_{S_3}$ or $I_{S_4} = 3I$

Same explanations can be given for (iii) and (iv) options.

9. (3)

(i) Fringe pattern will shift in the direction of slab.

(ii) No interference pattern will be obtained due to single slit.

$\therefore \quad I = I_{S_3} = I_0 =$ uniform

(iii) Fringe pattern will shift in the direction of slab.

(iv) No interference pattern will be obtained due to two real incoherent sources.

$\therefore \quad I = I_{S_1} + I_{S_2} =$ uniform

Numerical Value Type

1. (5) Let intensity of individual slit be I_0, then intensity of central maxima is $4I_0$.

Intensity at distance y from the central maxima is

$$I = 4I_0\cos^2\left(\frac{\phi}{2}\right) \text{ where } \phi = \frac{2\pi}{\lambda}\frac{dy}{D}$$

Given $I = 2I_0 \Rightarrow \quad 2I_0 = 4I_0\cos^2\left(\dfrac{\phi}{2}\right)$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}} \Rightarrow \frac{\phi}{2} = \frac{\pi}{4} \Rightarrow \phi = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{2\pi}{\lambda}\frac{dy}{D} = \frac{\pi}{2}$$

$$\Rightarrow \quad y = \frac{D\lambda}{4d} = \frac{500\times10^{-9}\times4}{4\times10^{-3}} = 5\times10^{-4}\text{ m}$$

2. (0) Take a point at a distance y from the center of the slit. Path difference between waves reaching this point,

$$\Delta x = \frac{dy}{D} - (\mu-1)t$$

For center of slit: $y = 0$

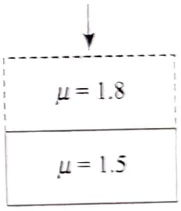
So $\quad \Delta x = -(\mu-1)t$

Phase difference, $\phi = \dfrac{2\pi}{\lambda}\Delta x = -\dfrac{2\pi}{\lambda}(\mu-1)t$

$$I = 4I_0\cos^2\left(\frac{\phi}{2}\right) = 4I_0\cos^2\left(\frac{\pi}{\lambda}(\mu-1)t\right)$$

$$= 4I_0\cos^2\left[\frac{\pi(1.5-1)1.5\times10^{-6}}{500\times10^{-10}}\right] = 4I_0\cos^2\left(\frac{3\pi}{2}\right) = 0$$

3. (9) Path difference between rays reflected from upper and lower faces of layer $= 2\mu t\cos r = 2\mu t$ (for normal incidence.) But there is abrupt change in path of $\lambda/2$ of light at upper surface. So actual path difference is $2\mu t - \lambda/2$



For constructive interference $2\mu t - \dfrac{\lambda}{2} = n\lambda$

$t = \dfrac{(2n+1)\lambda}{4\mu}$. For least thickness $n = 0$.

$$\therefore \quad t_{\text{min}} = \frac{\lambda}{4\mu} = \frac{648}{4\times1.8}\text{ nm} = 90\text{ nm}$$

4. (5) According to the question, shift = 5 (fringe width)

$$\therefore \quad \frac{(\mu-1)tD}{d} = \frac{5\lambda D}{d}$$

$$\therefore \quad t = \frac{5\lambda}{\mu-1} = \frac{25000}{1.5-1} = 50{,}000\text{ Å} = 5\times10^{-6}\text{ m}$$

5. (7) Number of fringes is $t\dfrac{(\mu-1)D/d}{D\lambda/d} = \dfrac{(\mu-1)t}{\lambda} = 7$

6. (2.00)

For coherent sources:

Intensity $\propto$ (Amplitude)^2

The amplitude at the mid-point $= A + A = 2A$

[constructive interference]

$$\Rightarrow I' \propto (2A)^2 \Rightarrow I' \propto 4A^2 \qquad \ldots\text{(i)}$$

For incoherent sources:

The intensities add up normally (no interference). Therefore,

The total intensity $I'' = 2I$.

$$I'' \propto 2A^2 \qquad (\because I \propto A^2) \qquad \ldots\text{(ii)}$$

$$\therefore \quad \frac{I'}{I''} = \frac{4A^2}{2A^2} = 2$$

7. (36.00)

Path difference due to the glass slab, $\Delta x = (\mu - 1)t$

Thirty fringes are displaced due to the slab. Hence, $\Delta x = 30\lambda$

$$\therefore \quad (\mu - 1)t = 30\lambda$$

$$t = \frac{30\lambda}{\mu - 1} = \frac{30 \times 6 \times 10^{-5}}{1.5 - 1} = 3.6 \times 10^{-3}\ \text{cm} = 36\ \mu\text{m}$$

8. (5.00)

According to the question, shift = 5 (fringe width)

$$\therefore \quad \frac{(\mu - 1)tD}{d} = \frac{5\lambda D}{d}$$

$$\therefore \quad t = \frac{5\lambda}{\mu - 1} = \frac{25000}{1.5 - 1} = 50{,}000\ \text{Å} = 5\ \mu\text{m}$$

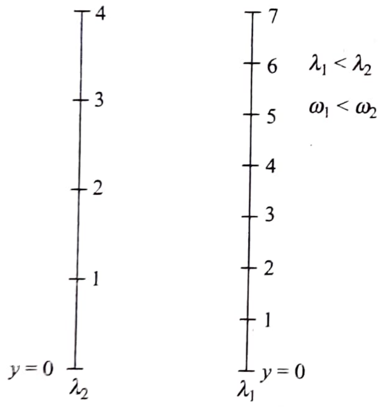
9. (0.700)

Let n_1 bright fringes of λ_1 overlaps with n_2 bright fringes of λ_2. Then,

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

$$\text{or} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{700}{400} = \frac{7}{4}$$

The ratio implies that 7th bright fringe of λ_1 will overlap with 4th bright fringe of λ_2. Similarly, 14th bright fringe of λ_1 will overlap with 8th bright fringe of λ_2 and so on. So, the minimum order of λ_1 which overlaps with λ_2 is 7.



10. (5.00)

For central bright fringe at C

$$\left(\frac{4}{3}\times 2\right) + \left(\frac{5}{3}\times 1\right) + \frac{\mu}{3}(D - 1) = 2 + \left(\frac{8}{3}\times\frac{3}{2}\right) + \frac{\mu}{3}\left(D - \frac{3}{2}\right)$$

$$\mu = 5$$

11. (0.125)

$$\frac{I_{max}}{2} = I = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos\phi$$

$$\frac{4I_0}{2} = 2I_0 + 2I_0\cos\phi$$

$$\therefore \quad \cos\phi = 0$$

$$\phi = \frac{\pi}{2} \Rightarrow \Delta x = \left(\frac{\lambda}{2\pi}\right)\left(\frac{\pi}{2}\right) = \frac{\lambda}{4}$$

For path difference $\frac{\lambda}{4}$, $d\cos\theta = \frac{\lambda}{4}$

$$\Rightarrow \cos\theta = \frac{1}{8}$$

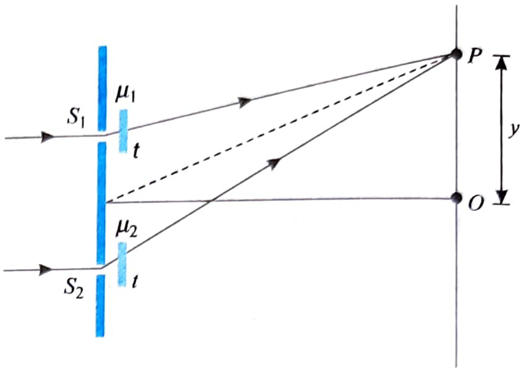
12. (8.00)

Path difference at a point P,

$$\Delta x = (S_2P - t + \mu_2 t) - (S_1P - t + \mu_1 t)$$

$$= (S_2P - S_1P) - (\mu_2 - \mu_1)t = \frac{dy}{D} + (\mu_2 - \mu_1)t$$

$$\left(\because \ S_2P - S_1P = d\sin\theta \simeq d\tan\theta = \frac{dy}{D}\right)$$



For 5th bright fringe, $\Delta x = 5\lambda$; $5\lambda = \frac{dy}{D} + (\mu_2 - \mu_1)t$

Given that 5th bright pattern is formed at the point where central bright fringe was formed before introducing the glass sheet.

$$\therefore \quad 5\lambda = (\mu_2 - \mu_1)t \qquad (\because y = 0)$$

$$\Rightarrow \quad t = \frac{5\lambda}{\mu_2 - \mu_1} = \frac{5 \times 4800 \times 10^{-10}}{0.3} = 8 \times 10^{-6}\ \text{m}$$

13. (30.00)

$$I_0 = 4I_0\cos^2\frac{\phi}{2} \Rightarrow \cos\frac{\phi}{2} = \frac{1}{2} \Rightarrow \phi = \frac{2\pi}{3}$$

Therefore, path difference of the two beams producing intensity I_0 on the screen is $\lambda/3$.

Optical path difference between two beams meeting on the screen is

$$\Delta x = \left\{\left(S_2P - t + \frac{4}{3}t\right) - \left(S_1P - t + \frac{3}{2}t\right)\right\}$$

$$= (S_2P - S_1P) + t\left(\frac{4}{3} - \frac{3}{2}\right)$$

$$\Rightarrow \quad \Delta x = d\sin\theta - \frac{t}{6}$$

$$\therefore \quad \frac{dy}{D} - \frac{t}{6} = \frac{\lambda}{3} \Rightarrow \frac{dy}{D} = \frac{\lambda}{3} + \frac{t}{6}$$

$$\Rightarrow \quad y = \frac{D}{d}\left[\frac{\lambda}{3} + \frac{t}{6}\right] = \frac{1}{10^{-3}}\left(\frac{6000 \times 10^{-10}}{3} + \frac{0.6 \times 10^{-6}}{6}\right)$$

$$= \frac{1}{10^{-3}} \times 3 \times 10^{-7} = 3 \times 10^{-4}\ \text{m}$$

$$\therefore \quad y = 30\ \text{mm}$$

14. (5769)

In a double-slit interference pattern, the distance x of a dark fringe from the central achromatic fringe is

$$x = \frac{D}{2d}(2n + 1)\frac{\lambda}{2}, n = 0, 1, 2, \ldots$$

$$\lambda = \frac{75000}{2n + 1}\text{Å}\,(\text{where } n = 0, 1, 2, \ldots)$$

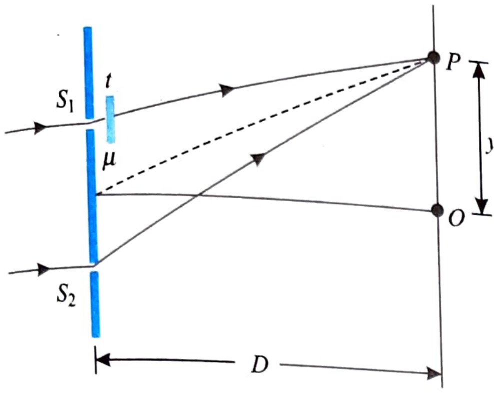
Thus, in the range 4000–8000 Å, the absent wavelengths are 6818, 5769, 5800, 4421

(1.50)

Path difference at a point F,

$$\Delta x = (S_2P) - (S_1P - t + \mu t)$$

$$= (S_2P - S_1P) + (t - \mu t) = \frac{dy}{D} - t(\mu - 1)$$



For finding intensity at the center of the screen, Put $y = 0$.

$\therefore \quad |\Delta x| = t(\mu - 1)$

$\therefore \quad \Delta\phi = \frac{2\pi}{\lambda}|\Delta x| = \frac{2\pi}{\lambda} \times t(\mu - 1)$

$$I = I_0 \cos^2 \frac{\Delta\phi}{2}$$

Putting the values of $\Delta\phi$ in the above equation we get, $I = 0$

For lateral shift, Put $n = 0$ (for the central maxima)

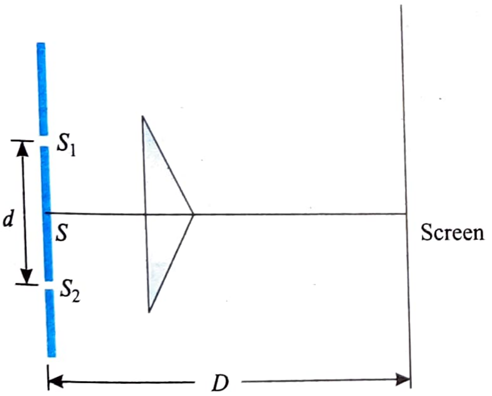
$\therefore \quad \Delta x = \pm n\lambda = 0 \Rightarrow \frac{dy}{D} = t(\mu - 1)$

$\Rightarrow \quad y = \frac{Dt(\mu - 1)}{d}$

Putting respective values, $y = 1.5$ mm

16. (5850)

Biprism experiment:



Here, S is the source of light and S_1 and S_2 are the images of S through the biprism. d = Distance between S_1 and S_2; D = Distance between slits and the screen.

Then $\beta(\text{fringe width}) = \frac{\lambda D}{d}$

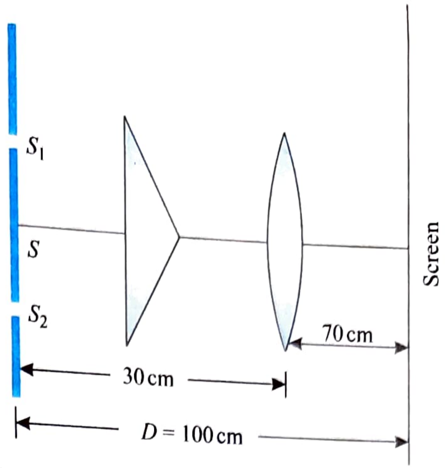
Introduce a convex lens 30 cm away from shift between biprism. From the given diagram, for convex lens, $u = -30$ m; $v = 70$ cm

Then, $m = \frac{v}{u} = \frac{d'}{d} = \frac{h_i}{h_0} \Rightarrow \frac{70}{-30} = \frac{0.7}{d}$

Given that, $d' = 0.7$ cm $\Rightarrow d = -0.3$ cm

–ve sign shows that S_1 and S_2 positions have been interchanged. Also β = fringe width = 0.0195 cm

$\therefore \quad \beta = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{d\beta}{D}$



Putting values in the above equation we get,

$$\lambda = 5.85 \times 10^{-7}\,\text{m} = 5850\,\text{Å}$$

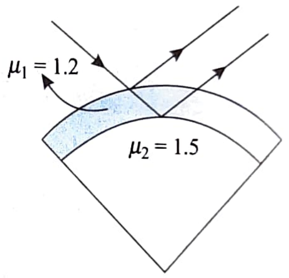
17. (0.100)

Path difference between two reflected ray in air,

$$\Delta x = 2\mu_1 t + \left(\frac{\lambda}{2} - \frac{\lambda}{2}\right) = 2\mu_1 t$$

For the minimum intensity, $\Delta x = (2n-1)\frac{\lambda}{2}$

$\Rightarrow \quad 2\mu_1 t = (2n-1)\frac{\lambda}{2}$



$$t = \frac{(2n-1)\lambda}{4\mu_1} = \frac{\lambda}{4\mu_1} \text{ (For minimum thickness, } n = 1)$$

$$t = \frac{\lambda}{4\mu_1} = \frac{4800 \times 10^{-10}}{4 \times 1.2} = 10^{-7}\,\text{m}$$

18. (16)

Ratio of intensities of the sources S_1 and S_2, $\frac{I_1}{I_2} = 9$

Let $I_2 = I_0$ and $I_1 = 9I_0$.

Intensity at any point on the screen,

$$I_1 = 9I_0 + I_0 + 2\sqrt{9I_0 I_0}\cos 0 = 16I_0$$

Path difference at P, $\Delta x_P = \frac{yd}{D} + t(\mu_2 - \mu_1) = 0$

$\Rightarrow \quad \frac{yd}{D} = t(\mu_1 - \mu_2)$ ...(i)

If transparent plates are interchanged, the central maxima will shift downward by some amount y.

When μ_2 and μ_1 are interchanged, the path difference at P,

$\Delta x_{P_1} = \frac{yd}{D} + t(\mu_1 - \mu_2) = 2t(\mu_1 - \mu_2)$ ...(ii)

From (i) and (ii), we get

Phase difference at P, $\phi = \frac{2\pi}{\lambda}2(\mu_1 - \mu_2)t = \frac{44}{3}\pi$

Now, intensity at P, $I_2 = I_0 + I_0 + 2I_0\cos\frac{44}{3}\pi = I_0$

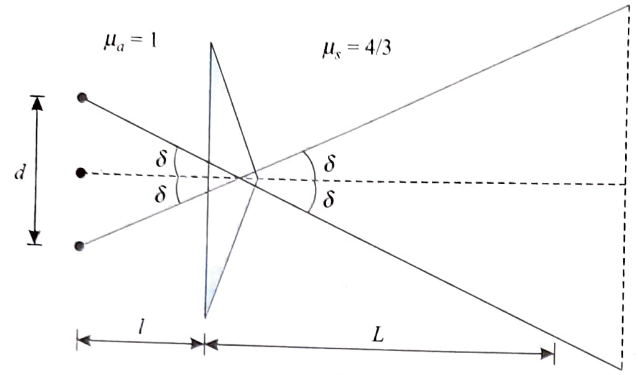
Hence, ratio of intensities at P, $\frac{I_1}{I_2} = 16$

19. (0.1125)

Angle of deviation of the prism with water on one side,

$$\delta = A\left[\frac{\mu_g}{\mu_m} - 1\right] = \frac{2}{180} \times \frac{22}{7}\left[\frac{3/2}{4/3} - 1\right]$$

$$\delta = \frac{2\times 11}{90\times 7}\left[\frac{3}{2}\times\frac{3}{4} - 1\right] = \frac{2\times 11}{90\times 7}\times\left[\frac{1}{8}\right] = \frac{11}{90\times 4\times 7}$$



$$d = 2\delta l = \frac{2\times 11\times 57.5}{90\times 4\times 7} = \frac{11\times 115}{90\times 4\times 7}$$

$$D = l + L = 150\,\text{cm}$$

Fringe width, $\beta = \dfrac{11\times 23}{18\times 4\times 7}\,\text{cm} = 0.1125\ \text{mm}$

Archives

JEE Main

Single Correct Answer Type

1. (3) In Young's double slits experiment,

Path difference for bright fringes $\Delta x = \dfrac{xd}{D} = n\lambda$

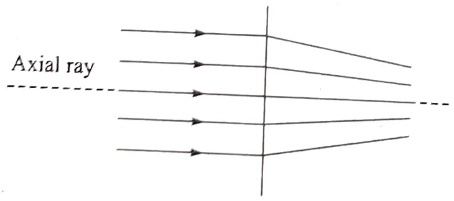
Hence position of any point on screen, $x = \dfrac{\Delta x D}{d} = \dfrac{n\lambda D}{d}$

The central fringes when $\Delta x = 0$ coincide for all wavelengths.

The third bright fringe of $\lambda_1 = 590$ nm coincides with the fourth bright fringe of unknown wavelength λ.

$$\therefore \quad x = \frac{3\times 590 D}{4d} = \frac{4\lambda D}{d} \Rightarrow \lambda = 442.5\,\text{nm}$$

2. (2) As the beam enters the medium, axial ray will travel slowest. So, it will lag behind. To compensate for the path, the rays will bend towards axis.



3. (4) As the beam is initially parallel, the shape of wave front is planar.

4. (1) The speed of light in a medium of refractive index μ is given by

$$v = \frac{\text{velocity of light in vaccum}}{\text{Refractive index of medium}} = \frac{c}{\mu(I)}$$

The refractive index is given as $\mu(I) = \mu_0 + \mu_2 I$

It is given that intensity decreases with increasing radius, hence the intensity is maximum at the axis of beam. It means the refractive index will be maximum on the axis of the beam. Hence the speed of the light will be minimum at the axis of the light beam.

5. (2) As light enters from air to glass it suffers a phase change on π and therefore at centre there will be destructive interference.

Statement 1 is true as light enters from air to glass it suffers a phase change on π ($\because$ rarer to denser propagation)

Statement 2 is true, the centre of interference pattern is dark, showing that the phase difference between two interfering waves is π.

6. (4) It is given, $A_2 = 2A_1$

We know, Intensity $\propto$ (Amplitude)^2

Hence $\dfrac{I_2}{I_1} = \left(\dfrac{A_2}{A_1}\right)^2 = \left(\dfrac{2A_1}{A_1}\right)^2 = 4$

$\Rightarrow \quad I_2 = 4I_1$

Maximum intensity, $I_m = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$$= \left(\sqrt{I_1} + \sqrt{4I_1}\right)^2 = \left(3\sqrt{I_1}\right)^2 = 9I_1$$

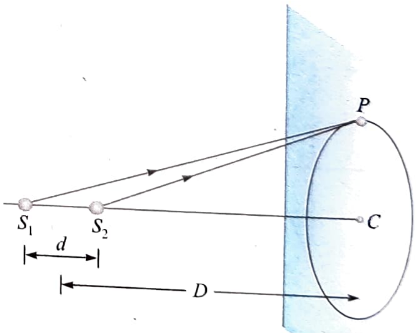
Hence $I_1 = \dfrac{I_m}{9}$

Resultant intensity,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$= I_1 + 4I_1 + 2\sqrt{I_1(4I_1)}\cos\phi$$
$$= I_1 + 4I_1 + 4I_1\cos\phi$$
$$= I_1 + 4I_1(1 + \cos\phi)$$
$$= I_1 + 8I_1\cos^2\left(\frac{\phi}{2}\right) \qquad \left(\because\ 1 + \cos\phi = 2\cos^2\frac{\phi}{2}\right)$$
$$I = \frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$$

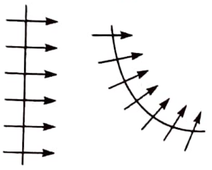
Putting the value of I_1 from Eq. (i), we get $I = \dfrac{I_m}{9}\left(1 + 8\cos^2\dfrac{\phi}{2}\right)$

7. (3) For bright fringe $S_1P - S_2P = n\lambda$



So fringes are concentric circles (centre of origin).

8. (4) Consider a plane wavefront travelling horizontally. As it moves, its different parts move with different speeds. So, its shape will change as shown.



$\Rightarrow$ Light bends upward

9. (4) For λ_1, $y = \dfrac{m\lambda_1 D}{d}$

For λ_2, $y_2 = \dfrac{n\lambda_2 D}{d}$

The point where the bright fringes due to both the wavelengths coincide, $y_1 = y_2$

$$\Rightarrow \quad \frac{m}{n} = \frac{\lambda_2}{\lambda_1} = \frac{4}{5}$$

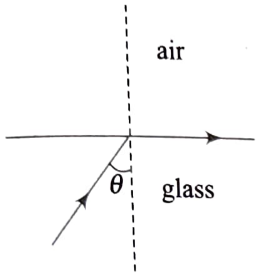
The least distance from the common central maximum for λ_1

$$y = \frac{4 \times 650 \times 10^{-9} \times 0.15}{0.5 \times 10^{-3}} = 7.8 \times 10^{-3}\ \text{m} = 7.8\ \text{mm}$$

Advanced

le Correct Answer Type

(3) Initially, most of part will be transmitted. When $\theta > i_C$, all the light rays will be total internal reflected. So transmitted intensity = 0.



(4) Fringe width, $\beta = \frac{\lambda D}{d}$

Hence, $\beta \propto \lambda$.

As we know $\lambda_R > \lambda_G > \lambda_B$

Hence, $\beta_R > \beta_G > \beta_B$

(2) $\frac{I_{\text{max}}}{2} = I_{\text{max}} \cos^2\left(\frac{\phi}{2}\right)$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{4}$$

$$\Rightarrow \quad \phi = \frac{\pi}{2}(2n+1)$$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{2\pi}\phi = \frac{\lambda}{2\pi} \times \frac{\pi}{2}(2n+1) = \frac{\lambda}{4}(2n+1)$$

Multiple Correct Answers Type

1. (1), (2), (3)

$$\beta = \frac{D\lambda}{d}$$

$\because \quad \lambda_2 > \lambda_1 \Rightarrow \beta_2 > \beta_1$

Also $m_1\beta_1 = m_2\beta_2 \Rightarrow m_1 > m_2$

Also $3\left(\frac{D}{d}\right)(600\ \text{nm}) = (2 \times 5 - 1)\left(\frac{D}{2d}\right)400\ \text{nm}$

Angular width $\theta = \frac{\lambda}{d}$

2. (1), (3)

From theory fringes will be semicircular

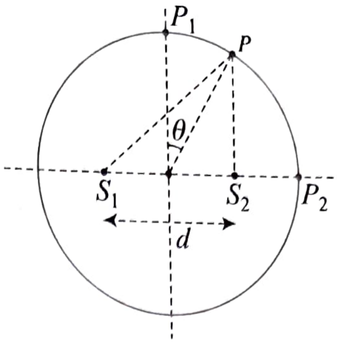
and $\frac{d}{\lambda} = 1000 + \frac{1}{2}$

at O $\Delta x = 1000\lambda + \frac{\lambda}{2}$

So at O it will be dark.

3. (3), (4)

$$d = 1.8 \times 10^{-3}\ \text{m} = 18 \times 10^{-4}\ \text{m and } \lambda = 6 \times 10^{-7}\ \text{m}$$



Path difference at point P (as shown)

$\Delta x = S_1P - S_2P = d\sin\theta$, where θ angle is measured from vertical line as shown.

For bright fringe, $d \sin\theta = m\lambda$ ...(i)

Point P_1 is the point of central maxima.

At point P_2, path difference $(\Delta x) = d$

If P_2 is the point of bright fringe, then

$$d = m\lambda \Rightarrow m = \frac{d}{\lambda} = 3000$$

$\therefore$ Total number of fringes = 3000

At P_2, the order of fringe will be maximum.

$\therefore$ Options (3) and (4) are correct.

On differentiating Eq. (i)

$d\cos\theta(\Delta\theta) = (\Delta m)\lambda$ = constant for consecutive bright fringe.

If cos θ decreases, Δθ increases as θ varies from 0 to $\frac{\pi}{2}$

$\therefore$ Option (2) is incorrect.

Linked Comprehension Type

1. (2) $n = \frac{c}{v}$

For metamaterials, $v = \frac{c}{|n|}$

2. (3) Metamaterial has a negative refractive index.

$\therefore \quad \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 \quad \Rightarrow \quad n_2$ is negative.

$\therefore \quad \theta_2$ is negative.

Matrix Match Type

1. i. → a., d.; ii. → b.; iii. → e.; iv. → c., d., e.

(i) This is Young's double slit experiment. The path difference is zero for P_0

$\therefore \quad I(P_0) > I(P_1)$

$\delta(P_0) = 0$ and $I(P_0) > I(P_1)$.

(ii) The path difference for the second beam is less for P_0. At P_1, path difference is zero. Therefore P_1 is the brightest central fringe and $\delta(P_1) = 0$.

(iii) Now the extra path difference is $\lambda/2$.

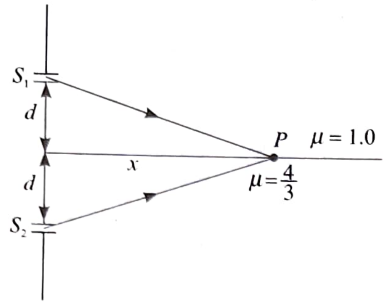
$I(P_2) > I(P_1)$

(iv) $(\mu - 1)t = \frac{3\lambda}{4} \quad \therefore \ \delta(P_0) = \frac{3\pi}{2}$

$I(P_1) = 0,\ I(P_2) > 0,\ I(P_0) > 0$

Numerical Value Type

1. (3) If light travels a distance x in medium, then the effective distance travelled in vacuum is μx. For the given case, consider two rays undergoing interference at a point P.



For S_1P: optical path: $l_1 = \sqrt{x^2 + d^2}$

For S_2P: optical path: $l_2 = \mu\sqrt{x^2 + d^2}$

Path difference: $\Delta l = l_2 - l_1 = (\mu - 1)\sqrt{x^2 + d^2}$

For maxima, $\Delta l = m\lambda$

$$\therefore \quad m\lambda = (\mu - 1)\sqrt{x^2 + d^2} = \frac{1}{3}\sqrt{a^2 + d^2}$$

Solving this, we get $x^2 = 9\,m^2\lambda^2 - d^2$

i.e., $p = 3$

Chapter 4

Concept Application Exercises

ercise 4.1

1. Here, $a = 0.15\text{ cm} = 0.15 \times 10^{-2}\text{ m}$,

$$\lambda = 5\times10^{-5}\text{ cm} = 5\times10^{-7}\text{ m}, D = 2.1\text{ m}$$

Now, $\beta = \dfrac{2D\lambda}{a} = \dfrac{2\times2.1\times5\times10^{-7}}{0.15\times10^{-2}} = 1.4\times10^{-3}\text{ m} = 1.4\text{ mm}$

2. Here, $\lambda = 5900\times10^{-10}\text{ m}$

$$a = 11.8\times10^{-5}\text{ cm} = 11.8\times10^{-7}\text{ m}$$

The angular position of first minimum is given by

$$\sin\theta_1 = \frac{\lambda}{a} = \frac{5900\times10^{-10}}{11.8\times10^{-7}} = 0.5$$

or $\theta_1 = 30°$

The angular width of central maximum, $2\theta_1 = 2\times30° = 60°$

3. Here, wavelength of microwaves, $\lambda = \dfrac{c}{v} = \dfrac{3\times10^{10}}{12\times10^{9}} = 2.5\text{ cm}$

Width of the slit, $a = 10$ cm

The direction of the first minimum on either side of the central maximum is given by $\theta = \dfrac{\lambda}{a}$.

In other words, the angular spread of the central maximum is given by

$$2\theta = \frac{2\lambda}{a} = \frac{2\times2.5}{10} = 0.5\text{ rad}$$

$$= (0.5\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right) = 28.7°$$

4. For third minimum, $\theta = \dfrac{3\lambda}{d}$

$$\theta = \frac{3(4.00\times10^{-7}\text{ m})}{5.0\times10^{-5}\text{ m}} = 0.024\text{ rad} = 0.024\left(\frac{180°}{\pi}\right) = 1.4°$$

Width of central maximum $= \dfrac{2\lambda D}{d}$

$$= \frac{2(4.0\times10^{-7}\text{ m})(1.0\text{ m})}{5.0\times10^{-5}\text{ m}} = 1.6\times10^{-2}\text{ m} = 1.6\text{ cm}$$

5. Angular spread of central maximum on either side of incident light is given by, $\sin\theta = \dfrac{\lambda}{a}$

$\therefore$ Slit width, $a = \dfrac{\lambda}{\sin\theta} = \dfrac{5000\times10^{-10}}{\sin 30°} = 10^{-6}\text{ m}$

For $\theta = 90°$, we have $a = \dfrac{\lambda}{\sin 90°} = \dfrac{5000\times10^{-10}}{1} = 5\times10^{-7}\text{ m}$.

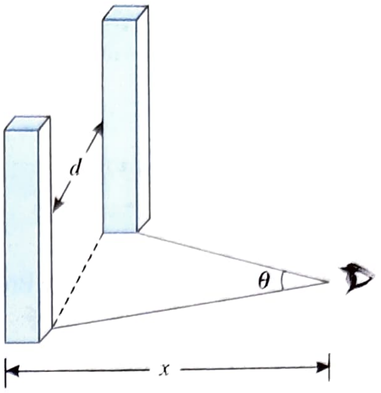
6. Here $d = 3\text{ mm} = 3\times10^{-3}\text{ m}$, $\lambda = 500\text{ nm} = 500\times10^{-9}\text{ m}$

The distance up to which a beam of light can travel without significant broadening is called Fresnel distance and its value is given by

$$D_F = \frac{d^2}{\lambda} = \frac{(3\times10^{-3})^2}{500\times10^{-9}} = 18\text{ m}$$

xercise 4.2

1. As limit of resolution of eye is $\left(\dfrac{1}{60}\right)^{\circ}$, the pillars will be seen distinctly if $\theta > \left(\dfrac{1}{60}\right)^{\circ}$ i.e., $\dfrac{d}{x} > \left(\dfrac{1}{60}\right)\times\dfrac{\pi}{180}$



$$\Rightarrow \quad d > \frac{\pi\times x}{60\times180}$$

$$\Rightarrow \quad d > \frac{3.14\times11\times10^{3}}{60\times180} \Rightarrow d > 3.2\text{ m}$$

2. Here $d\theta = 4.6\times10^{-6}\text{ rad}$,

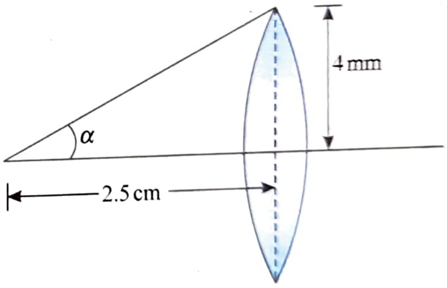
$$\lambda = 5460\text{Å} = 5460\times10^{-10}\text{ m}$$

As $\quad d\theta = \dfrac{1.22\lambda}{D}$

$\therefore$ Aperture of the telescope,

$$D = \frac{1.22\lambda}{d\theta} = \frac{1.22\times5460\times10^{-10}}{4.6\times10^{-6}} = 0.1488\text{ m}$$

3. We assume that the object is placed a little beyond its focal distance, say 2.5 cm.



Refer figure, $\tan\alpha = \dfrac{4\text{ mm}}{2.5\text{ cm}} = \dfrac{0.4}{2.5} = 0.16$

Since $\sin\alpha$ is small, $\sin\alpha \approx \tan\alpha = 0.16$

Thus, $\Delta x = \dfrac{1.22\lambda}{2\mu\sin\alpha} = \dfrac{1.22(5500\times10^{-10}\text{ m})}{2\times1\times0.16} \approx 2\times10^{-6}\text{ m}$

4. $\Delta\theta = 1.22\left(\dfrac{\lambda}{a}\right) = 1.22\left(\dfrac{5500\times10^{-8}\text{ cm}}{200\text{ cm}}\right) = 3.35\times10^{-7}\text{ rad}$

5. Here, diameter of the human eye, $D = 2\text{ mm} = 2\times10^{-3}\text{ m}$
distance of the two points from the eye, $y = 50$ m
and wavelength of light used = 5000 Å = 5×10^{-7} m

The angular diameter of the two points at the eye, $d\theta = \dfrac{d}{y}$

Now, resolving power the eye, $\dfrac{1}{d\theta} = \dfrac{D}{1.22\lambda}$

or $\quad \dfrac{y}{d} = \dfrac{D}{1.22\lambda}$

or $\quad d = \dfrac{1.22\lambda}{D}\times y = \dfrac{1.22\times5\times10^{-7}}{2\times10^{-3}}\times50$

$$= 1.525\times10^{-2}\text{ m} = 1.525\text{ cm}$$

Exercise 4.3

1. (c) At polarizing angle, the reflected and refracted rays are mutually perpendicular to each other.
2. (a) When unpolarized light is made incident at polarizing angle, the reflected light is plane polarized in a direction perpendicular to the plane of incidence.
   Therefore $\vec{E}$ in reflected light will vibrate in vertical plane with respect to plane of incidence.
3. (a) At $i_p + r_p = 90°$

   $\therefore \quad r_p = 90° - i_p = 90° - 53° = 37°$

   (b) $\mu = \tan i_p = \tan 53° = \frac{4}{3}$
4. Refer to figure in question. Incident light at the Brewster's angle (i_p) has the greatest polarization, where

   $$i_p = \tan^{-1}(\mu) = \tan^{-1}\frac{4}{3} = 53°$$

   If θ is the altitude angle for greatest polarization,

   $$\theta = 90° - i_p = 90° - 53° = 37°$$
5. Let I_0 be the intensity when the axis of analyzer is parallel to the axis of polarizer. By the law of Malus, $I = I_0 \cos^2\theta$.

   (i) When $\theta = 45°$, $I = I_0 \cos^2 45° = \frac{I_0}{2}$.

   (ii) When $\theta = 90°$, $I = I_0 \cos^2 90° = 0$.

   (iii) When $\theta = 180°$, $I = I_0 \cos^2 180° = I_0$.
6. Here $I = \frac{I_0}{2}$

   Using Malus law, $I = I_0 \cos^2\theta$

   $\therefore \quad \frac{I_0}{2} = I_0 \cos^2\theta$

   or $\quad \cos\theta = \pm\frac{1}{\sqrt{2}}$

   Hence $\theta = \pm 45°, \pm 135°$

   The same effect occurs no matter which sheet is rotated or in which direction it is rotated.

Exercise 4.4

1. (b) Shifting towards ultraviolet region shows that apparent wavelength decreased. Therefore, the source is moving towards the earth.
2. (c) Blue radiations have the wavelength around 4600 Å. It shows that apparent wavelength is smaller than the real wavelength. It means that the star is proceeding towards earth.
3. $\Delta\lambda = \frac{v}{c}\cdot\lambda = \frac{9\times10^6}{3\times10^8}\times 6000\text{Å} = 180\text{Å}$

   Apparent wavelength, $\lambda' = \lambda - \Delta\lambda = 6000 - 180 = 5820\,\text{Å}$.
4. $v' = v\left(\frac{c+v}{c}\right), \quad \frac{v'}{v} = 1 + \frac{v}{c}$

   or $\quad \frac{\lambda}{\lambda'} = 1 + \frac{v}{c}$

   or $\quad \lambda' = \frac{\lambda}{1 + v/c} = \frac{6\times10^{-7}}{(4/3)}$

   or $\quad \lambda' = 4.5\times10^{-7}\,\text{m}$
5. $\frac{\Delta\lambda}{\lambda} = \frac{v_s}{c}$

   or $\quad v_s = \left(\frac{\Delta\lambda}{\lambda}\right)c = \frac{(589.6 - 589.0)\,\text{nm}}{589.0\,\text{nm}}\times(3\times10^8\,\text{m/s})$

   $= 3.1\times10^5\,\text{m/s}$

Exercises

Single Correct Answer Type

1. (2) The magnitude of electric field vector varies periodically with time because it is in the form of electromagnetic wave.
2. (1) Band width $\propto \lambda$,

   $\because \lambda_{\text{blue}} < \lambda_{\text{red}}$, hence for blue light the diffraction bands become narrower and crowded together.
3. (1) Intensity of the polarized light coming out of polarizing sheet will be $I = \int_0^{2\pi} I_0 \cos^2\theta\, d\theta$

   On solving, we get $I = \frac{I_0}{2}$
4. (1) $\beta = \frac{\lambda.D}{d}$, where D = distance of screen from wire, d = diameter of wire
5. (3) Angular width $= \frac{2\lambda}{d} = \frac{2\times6000\times10^{-10}}{12\times10^{-5}\times10^{-2}} = 1\,\text{rad}$
6. (1) $2\theta = \frac{2\lambda}{d}$ (where d = slit width)

   As d decreases, θ increases.
7. (2) Thickness of the film must be of the order of wavelength of light falling on film (i.e. visible light)
8. (2) $\Delta\lambda = 5200 - 5000 = 200\text{Å}$

   Now $\quad \frac{\Delta\lambda}{\lambda'} = \frac{v}{c}$

   $\Rightarrow \quad v = \frac{c\Delta\lambda}{\lambda'} = \frac{3\times10^8\times200}{5000} = 1.2\times10^7\,\text{m/s} \approx 1.15\times10^7\,\text{m/s}$
9. (4) $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

   $\Rightarrow \quad v = \frac{c}{\lambda}\Delta\lambda = \frac{c}{\lambda}(\lambda' - \lambda) = c\times\frac{0.01}{100}$

   $= 3\times10^4\,\text{m/s} = 30\text{ km/s}$
10. (4) $\Delta\lambda = \frac{v}{c}\lambda = \frac{3600\times10^3}{3\times10^8}\times5896 = 70.75\,\text{Å}$

    So the increased wavelength of light is observed.
11. (2) Observed frequency $v' = v\left(1 - \frac{v}{c}\right)$

    $\Rightarrow \quad v' = 6\times10^{14}\left(1 - \frac{0.8\,c}{c}\right) = 1.2\times10^{14}\,\text{Hz}$
12. (3) According to Doppler's principle, $\lambda' = \lambda\sqrt{\frac{1 - v/c}{1 + v/c}}$

    For $v = c$,

    $$\lambda' = 5500\sqrt{\frac{1 - 0.8}{1 + 0.8}} = 1833.3$$

    $\therefore \quad$ Shift = 5500 – 1833.3 = 3167 Å
13. (4) $\Delta\lambda = \frac{v_s}{c}\lambda$

    $\Rightarrow \quad v_s = \frac{\Delta\lambda.c}{\lambda} = \frac{47\times3\times10^8}{4700} = 3\times10^6\,\text{m/s}$

    away from earth
14. (4) $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow v = \frac{\Delta\lambda}{\lambda}.c = \frac{5}{6563}\times(3\times10^8) = 2.29\times10^5\,\text{m/s}$

(3) For first minima, $\theta = \dfrac{\lambda}{a}$

or $a = \dfrac{\lambda}{\theta}$

$\therefore\ a = \dfrac{6500 \times 10^{-8} \times 6}{\pi}$ (As $30^\circ = \dfrac{\pi}{6}$ radian)

$= 1.24 \times 10^{-4}\,\text{cm} = 1.24$ micron

(1) The angular half width of the central maxima is given by

$\sin\theta = \dfrac{\lambda}{a}$

$\Rightarrow\ \theta = \dfrac{6328 \times 10^{-10}}{0.2 \times 10^{-3}}$ rad

$= \dfrac{6328 \times 10^{-10} \times 180}{0.2 \times 10^{-3} \times \pi}$ degree $= 0.18^\circ$

Total width of central maxima $= 2\theta = 0.36^\circ$

(3) Width of central maxima $= \dfrac{2\lambda D}{d}$

$= \dfrac{2 \times 2.1 \times 5 \times 10^{-7}}{0.15 \times 10^{-2}} = 1.4 \times 10^{-3}\,\text{m} = 1.4$ mm

(1) Using $d\sin\theta = n\lambda$, for $n = 1$

$\sin\theta = \dfrac{\lambda}{d} = \dfrac{550 \times 10^{-9}}{0.55 \times 10^{-3}} = 10^{-3} = 0.001$ rad

(1) For single slit diffraction pattern, $d\sin\theta = \lambda$ ($d =$ slit width)

Angular width $= 2\theta = 2\sin^{-1}\left(\dfrac{\lambda}{d}\right)$

It is independent of D i.e. distance between screen and slit

(3) Width of central bright fringe

$= \dfrac{2\lambda D}{d} = \dfrac{2 \times 500 \times 10^{-9} \times 80 \times 10^{-2}}{0.20 \times 10^{-3}} = 4 \times 10^{-3}\,\text{m} = 4\,\text{mm}$

(4) $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$, Now $\Delta\lambda = \dfrac{0.5}{100}\lambda \Rightarrow \dfrac{\Delta\lambda}{\lambda} = \dfrac{0.5}{100}$

$\therefore\ v = \dfrac{0.5}{100} \times c = \dfrac{0.5}{100} \times 3 \times 10^{8} = 1.5 \times 10^{6}\,\text{m/s}$

Increase in λ indicates that the star is receding.

(2) $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c} = \dfrac{6 \times 10^{7}}{3 \times 10^{8}} = 0.2$

$\Delta\lambda = \lambda' - \lambda = 0.2\lambda \Rightarrow \lambda' = 1.2\lambda = 1.2 \times 4600 = 5520$ Å

(2) Diffraction is obtained when the slit width is of the order of wavelength of e.m. waves (or light). Here wavelength of X-rays (1–100 Å) is very-very lesser than slit width (0.6 mm). Therefore, no diffraction pattern will be observed.

(1) For secondary maxima, $d\sin\theta = \dfrac{5\lambda}{2} \Rightarrow d\theta = d.\dfrac{x}{D(\approx f)} = \dfrac{5\lambda}{2}$

$\Rightarrow\ 2x = \dfrac{5\lambda f}{d} = \dfrac{5 \times 0.8 \times 10^{-7}}{4 \times 10^{-4}} = 6 \times 10^{-3}\,\text{m} = 6$ mm

(2) $\lambda_{\text{Blue}} < \lambda_{\text{Red}}$

Therefore fringe pattern will contract because fringe width $\propto \lambda$

(1) Angular width of central maxima $= \dfrac{2\lambda}{d}$

$= \dfrac{2 \times 589.3 \times 10^{-9}}{0.1 \times 10^{-3}}$ rad $= 0.0117 \times \dfrac{180^\circ}{\pi} = 0.68^\circ$

27. **(4)** Distance between the first dark fringes on either side of central maxima = width of central maxima

$= \dfrac{2\lambda D}{d} = \dfrac{2 \times 600 \times 10^{-9} \times 2}{1 \times 10^{-3}} = 2.4$ mm

28. **(4)** For n^{th} secondary maxima path difference

$d\sin\theta = (2n+1)\dfrac{\lambda}{2} \Rightarrow a\sin\theta = \dfrac{3\lambda}{2}$

29. **(4)** The phase difference (φ) between the wavelets from the top edge and the bottom edge of the slit is $\varphi = \dfrac{2\pi}{\lambda}(d\sin\theta)$ where d is the slit width. The first minima of the diffraction pattern occurs at $\sin\theta = \dfrac{\lambda}{d}$ so $\varphi = \dfrac{2\pi}{\lambda}\left(d \times \dfrac{\lambda}{d}\right) = 2\pi$

30. **(1)** For second dark fringe $d\sin\theta = 2\lambda$

$\Rightarrow\ 24 \times 10^{-5} \times 10^{-2} \times \sin 30^\circ = 2\lambda$

$\Rightarrow\ \lambda = 6 \times 10^{-7}\,\text{m} = 6000$ Å

31. **(3)** For the first minima $d\sin\theta = \lambda$

$\Rightarrow\ \sin\theta = \dfrac{\lambda}{d} \Rightarrow \theta = \sin^{-1}\left(\dfrac{5000 \times 10^{-10}}{0.001 \times 10^{-3}}\right) = 30^\circ$

32. **(3)** Position of first minima = position of third maxima

i.e., $\dfrac{1 \times \lambda_1 D}{d} = \dfrac{(2 \times 3 + 1)\,\lambda_2 D}{2\,d} \Rightarrow \lambda_1 = 3.5\lambda_2$

33. **(1)** Position of n^{th} minima, $x_n = \dfrac{n\lambda D}{d}$

$\Rightarrow\ 5 \times 10^{-3} = \dfrac{1 \times 5000 \times 10^{-10} \times 1}{d}$

$\Rightarrow\ d = 10^{-4}\,\text{m} = 0.1$ mm

34. **(3)** In single slit experiment,

Width of central maxima $(y) = 2\lambda D/d$

$\Rightarrow\ \dfrac{y'}{y} = \dfrac{\lambda'}{d'} \times \dfrac{d}{\lambda} = \dfrac{600}{d/2} \times \dfrac{d}{400} \Rightarrow y' = 3y$

35. **(4)** By using $\mu = \tan\theta_p \Rightarrow \mu = \tan 60^\circ = \sqrt{3}$,

also $C = \sin^{-1}\left(\dfrac{1}{\mu}\right) \Rightarrow C = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

36. **(2)** $I = I_0\cos^2\theta = I_0\cos^2 45^\circ = \dfrac{I_0}{2}$

37. **(3)** Intensity of polarized light from first polarizer $= \dfrac{100}{2} = 50$

$I = 50\cos^2 60^\circ = \dfrac{50}{4} = 12.5$

38. **(2)** $I' = \dfrac{I}{2}\cos^2\theta = \dfrac{I}{6}$

or $\cos\theta = \dfrac{1}{\sqrt{3}}$

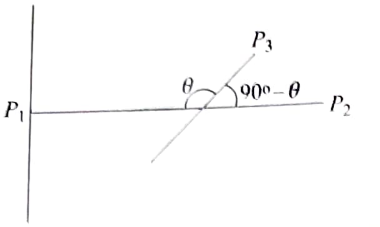
$\therefore\ \theta = 55^\circ$

39. **(1)** According to Brewster's law, when a beam of ordinary light (i.e., unpolarized) is reflected from a transparent medium (like glass), the reflected light is completely plane polarized at certain angle of incidence called the angle of polarization.

40. **(3)** From Brewster's law, $\mu = \tan i_p \Rightarrow \dfrac{c}{v} = \tan 60^\circ = \sqrt{3}$

$\Rightarrow\ v = \dfrac{c}{\sqrt{3}} = \dfrac{3 \times 10^{8}}{\sqrt{3}} = \sqrt{3} \times 10^{8}\,\text{m s}^{-1}$

41. (1) No light is emitted from the second polaroid, so P_1 and P_2 are perpendicular to each other.



Let the initial intensity of light is I_0. So, intensity of light after transmission from first polaroid $=\frac{I_0}{2}$.

Intensity of light emitted from P_3, $I_1=\frac{I_0}{2}\cos^2\theta$

Intensity of light transmitted from last polaroid i.e. from $P_2 = I_1\cos^2(90°-\theta) = \frac{I_0}{2}\cos^2\theta.\sin^2\theta = \frac{I_0}{8}(2\sin\theta\cos\theta)^2$

$=\frac{I_0}{8}\sin^2 2\theta$

42. (2) Angular width $\beta=\frac{2\lambda}{d}\Rightarrow\beta\propto\lambda$

$$\Rightarrow\frac{\beta_1}{\beta_2}=\frac{\lambda_1}{\lambda_2}\Rightarrow\frac{\beta}{\frac{70}{100}\beta}=\frac{6000}{\lambda_2}\Rightarrow\lambda_2=4200\,\text{Å}$$

43. (4) If I is the final intensity and I_0 is the initial intensity then

$$I=\frac{I_0}{2}(\cos^2 30°)^5$$

or $\frac{I}{I_0}=\frac{1}{2}\times\left(\frac{\sqrt{3}}{2}\right)^{10}=0.12$

44. (4) When the slit width is doubled, the half angular width of the central maximum, which is λ/a, reduces to half. The area of the central diffraction band becomes 1/4. So the intensity of the central maximum becomes four times.

45. (2) Momentum of the electron will increase. So the wavelength ($\lambda = h/p$) of electrons will decrease and fringe width decreases as $\beta\propto l$.

46. (1) $\Delta\lambda=\lambda\frac{v}{c}$ and $v=r\omega$

$$v=7\times10^8\times\frac{2\pi}{25\times24\times3600},\ c=3\times10^8\text{ m/s}$$

$\therefore\quad \Delta\lambda=0.04\,\text{Å}$

47. (2) $v=\frac{c\Delta\lambda}{\lambda}=\frac{3\times10^8\times(706-656)}{656}=\frac{1500}{656}\times10^7=2\times10^7\text{m/s}$

48. (2) In this case, we can assume as if both the source and the observer are moving towards each other with speed v.

Hence $v'=\frac{c-u_o}{c-u_s}v=\frac{c-(-v)}{c-v}v=\frac{c+v}{c-v}v$

$$=\frac{(c+v)(c-v)}{(c-v)^2}v=\frac{c^2-v^2}{c^2+v^2-2vc}v$$

Since $v<<c$, therefore $v'=\frac{c^2v}{c^2-2vc}=\frac{c}{c-2v}v$

9. **(1)** $\Delta\lambda=\lambda.\frac{v}{c}$ where $v=r\omega=r\times\left(\frac{2\pi}{T}\right)$

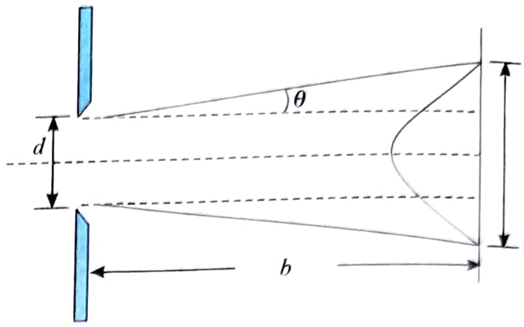
$\therefore\quad \Delta\lambda=\frac{4320\times7\times10^8\times2\times3.14}{3\times10^8\times22\times86400}=0.033\,\text{Å}$

50. (3) The direction in which the first minima occurs is θ (say).

Then $d\sin\theta=\lambda$

or $d\theta=\lambda$

or, $\theta=\frac{\lambda}{d}$ $\quad(\because\ \theta=\sin\theta$ when θ small)



Width of central maximum $=2b\theta+d=\frac{2b\lambda}{d}+d$

51. (1) In a single slit diffraction experiment, position of minima is given by $d\sin\theta=n\lambda$

So for first minima of red, $\sin\theta=1\times\left(\frac{\lambda_R}{d}\right)$ and as first maxima is midway between first and second minima, for wavelength λ', its position will be

$$d\sin\theta'=\frac{\lambda'+2\lambda'}{2}\Rightarrow\sin\theta'=\frac{3\lambda'}{2d}$$

According to given condition $\sin\theta=\sin\theta'$

$\Rightarrow\quad \lambda'=\frac{2}{3}\lambda_R$

So $\quad\lambda'=\frac{2}{3}\times6600=440\text{nm}=4400\,\text{Å}$

52. (3) $I=I_0\left[\frac{\sin\alpha}{\alpha}\right]^2$, where $\alpha=\frac{\phi}{2}$

For n^{th} secondary maxima, $d\sin\theta=\left(\frac{2n+1}{2}\right)\lambda$

$$\Rightarrow\quad\alpha=\frac{\phi}{2}=\frac{\pi}{\lambda}[d\sin\theta]=\left(\frac{2n+1}{2}\right)\pi$$

$$\Rightarrow\quad I=I_0\left[\frac{\sin\left(\frac{2n+1}{2}\right)\pi}{\left(\frac{2n+1}{n}\right)\pi}\right]^2=\frac{I_0}{\left\{\frac{(2n+1)}{2}\pi\right\}^2}$$

So $\quad I_0:I_1:I_2=I_0:\frac{4}{9\pi^2}I_0:\frac{4}{25\pi^2}I_0=1:\frac{4}{9\pi^2}:\frac{4}{25\pi^2}$

53. (1) $I=4I_0\cos^2\frac{\phi}{2}$

At central position $I_1=4I_0$ $\quad$...(i)

Since the phase difference between two successive fringes is 2x, the phase difference between two points separated by a distance equal to one quarter of the distance between the two, successive fringes is equal to $\delta=(2\pi)\left(\frac{1}{4}\right)=\frac{\pi}{2}$ radian

$$\Rightarrow \quad I_2 = 4I_0\cos^2\left(\frac{\frac{\pi}{2}}{2}\right) = 2I_0 \qquad \ldots\text{(ii)}$$

Using Eqs. (i) and (ii), $\frac{I_1}{I_2} = \frac{4I_0}{2I_0} = 2$

(1) Using Malus law, $I = I_0\cos^2\theta$

As here polarizer is rotating i.e. all the values of θ are possible.

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} I\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} I_0\cos^2\theta\,d\theta$$

On integration we get $I_{av} = \frac{I_0}{2}$

where $I_0 = \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{p}{A} = \frac{10^{-3}}{3\times10^{-4}} = \frac{10}{3}\frac{\text{watt}}{\text{m}^2}$

$$\therefore \quad I_{av} = \frac{1}{2}\times\frac{10}{3} = \frac{5}{3}\text{watt}$$

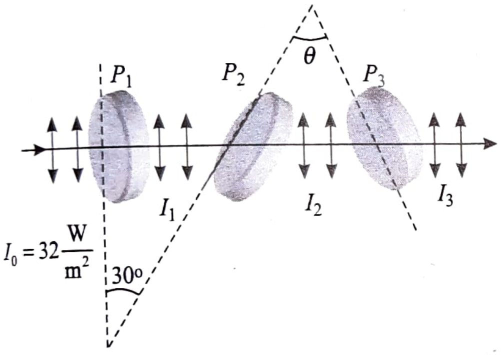
and time period $T = \frac{2\pi}{\omega} = \frac{2\times3.14}{31.4} = \frac{1}{5}$ s

$\therefore$ Energy of light passing through the polarizer per revolution

$$= I_{av}\times\text{Area}\times T = \frac{5}{3}\times3\times10^{-4}\times\frac{1}{5} = 10^{-4}\text{ J}$$

(2) Angle between P_1 and $P_2 = 30°$ (given)

Angle between P_2 and $P_3 = \theta = 90° - 30° = 60°$



The intensity of light transmitted by P_1 is $I_1 = \frac{I_0}{2} = \frac{32}{2} = 16\,\text{W m}^{-2}$

According to Malus law the intensity of light transmitted by P_2 is

$$I_2 = I_1\cos^2 30° = 16\left(\frac{\sqrt{3}}{2}\right)^2 = 12\,\text{W m}^{-2}$$

Similarly intensity of light transmitted by P_3 is

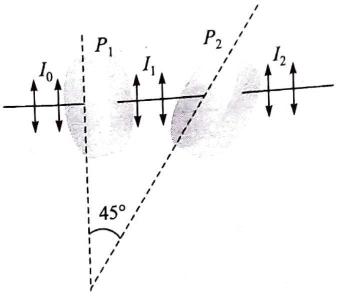
$$I_3 = I_2\cos^2\theta = 12\cos^2 60° = 12\left(\frac{1}{2}\right)^2 = 3\,\text{W m}^{-2}$$

Archives

EE Main

Single Correct Answer Type

1. (2) Intensity of light after passing polaroid P_1 is $I_1 = \frac{I_0}{2}$



Now this light pass through the second polaroid P_2 whose axis is inclined at an angle of 45° to the axis of polaroid P_1. So in accordance with Malus law, the intensity of light emerging from polaroid B is

$$I_2 = I_1\cos^2 45° = \left(\frac{I_0}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{4}$$

2. (2) By law of Malus, $I = I_0\cos^2\theta$

Now, $I'_A = I_A\cos^2 30°$

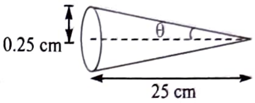
and $I'_B = I_B\cos^2 60°$

As $I'_A = I'_B$

$$\Rightarrow \quad I_A\times\frac{3}{4} = I_B\times\frac{1}{4}$$

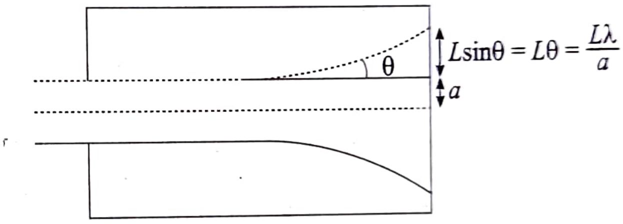
$$\frac{I_A}{I_B} = \frac{1}{3}$$

3. (2) Resolving power, RP $= \frac{1.22\lambda}{2\mu\sin\theta} = \frac{1.22\times(500\times10^{-9}\text{ m})}{2\times1\times\left(\frac{1}{100}\right)}$



or RP $= 3.05\times10^{-5}$ m $= 30\ \mu$m

4. (3) Let b is the radius of spot = geometrical spread + spread due to diffraction



$$b = a - \frac{\lambda L}{a}$$

Since, $\frac{db}{da} = 0$

$$\Rightarrow \quad 1 - \frac{\lambda}{a^2 L} = 0$$

$$\Rightarrow \quad a^2 = L\lambda$$

or $a = \sqrt{L\lambda}$

Hence $b_{\min} = \sqrt{L\lambda} + \frac{\lambda L}{\sqrt{L\lambda}}$

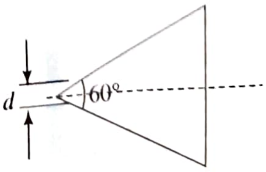
$$b_{\min} = 2\sqrt{L\lambda}$$

or $b_{\min} = \sqrt{4L\lambda}$

5. (2) In case of single slit diffraction $d\sin 30° = \lambda$

$$d\times\frac{1}{2} = \lambda$$

or $\lambda = \frac{d}{2} \Rightarrow \lambda = \frac{10^{-6}}{2}$ m



We know Young's fringe width $\beta = \frac{D\lambda}{d'}$

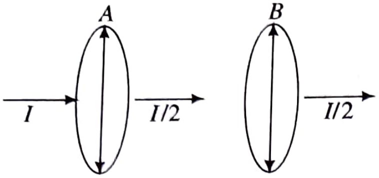
where d' = separation between two slits.

Substituting the corresponding values

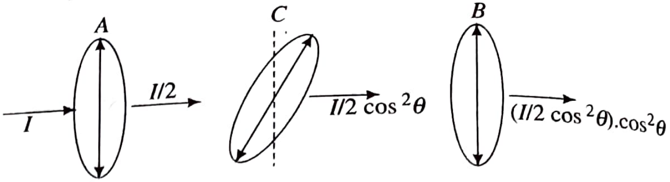
$$10^{-2} = \frac{\left(50\times10^{-2}\times10^{-6}\right)/2}{d'} = \frac{25\times10^{-8}}{d'}$$

We get $d' = 25\,\mu\text{m}$

6. (4) Axis of transmission of A and B are parallel.



Now


A
C
B
I
I/2
I/2 cos ²θ
(I/2 cos ²θ).cos²θ

$$\frac{I}{2}\cos^4\theta = \frac{I}{8} \Rightarrow \cos^4\theta = \frac{1}{4}$$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

Chapter 5

oncept Application Exercises

rcise 5.1

For completely absorbing surface,

$$P_{\text{rod}} = \frac{I}{c} = \frac{1.4\times10^{3}}{3.0\times10^{8}} = 4.66 \times 10^{-6}\ \text{Nm}^{-2}$$

Here, $\lambda = 632.8\ \text{nm} = 632.8 \times 10^{-9}$ m;

Power, $P = 9.42\ \text{mW} = 9.42 \times 10^{-3}$ W

(a) Now, energy of each photon in the light beam,

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{632.8\times10^{-9}}$$

$$= 3.14 \times 10^{-19}\ \text{J} = \frac{3.14\times10^{-19}}{1.6\times10^{-9}} = 1.963\ \text{eV}$$

Also, momentum of each photon in the light beam,

$$p = \frac{h}{\lambda} = \frac{6.62\times10^{-34}}{632.8\times10^{-9}} = 1.046 \times 10^{-27}\ \text{kg m s}^{-1}$$

(b) The number of photons falling on the target per second,

$$N = \frac{P}{E} = \frac{9.42\times10^{-3}}{3.14\times10^{-19}} = 3 \times 10^{16}\ \text{s}^{-1}$$

(c) The required speed of the hydrogen atom,

$$v = \frac{p}{m_{\text{H}}} = \frac{1.046\times10^{-27}}{1.66\times10^{-27}} = 0.63\ \text{m s}^{-1}$$

3. Here, $h = 6.63 \times 10^{-34}$ Js; $\lambda = 1.0\ \text{nm} = 10^{-9}$ m

(a) The electron and the proton will possess the same momentum, which is given by

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{10^{-9}} = 6.63 \times 10^{-25}\ \text{kg m s}^{-1}$$

(b) The energy of photon,

$$E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{10^{-9}} = 1.99 \times 10^{-16}\ \text{J}$$

$$= \frac{1.99\times10^{-16}}{1.6\times10^{-19}} = 1243.1\ \text{eV}$$

(c) The kinetic energy of electron,

$$E = \frac{p^{2}}{2m} = \frac{(6.63\times10^{-25})^{2}}{2\times9.1\times10^{-31}} = 2.415 \times 10^{-19}\ \text{J}$$

$$= \frac{2.415\times10^{-19}}{1.6\times10^{-19}} = 1.51\ \text{eV}$$

4. Energy incident on the plate $= \dfrac{1200\times100}{60} = 2000\ \text{J s}^{-1}$

$\therefore \quad nh\nu = 2000 \qquad$ ...(i)

Initial momentum of photons $= \dfrac{nh\nu}{c}$

Final momentum of photons $= \dfrac{40}{100}\times\dfrac{nh\nu}{c} = \dfrac{0.4nh\nu}{c}$

Force acting on the plate = Rate of change of momentum

$$= \frac{0.4nh\nu}{c} - \left(-\frac{nh\nu}{c}\right) = \frac{1.4nh\nu}{3\times10^{8}}, \qquad \text{...(ii)}$$

Substituting the value of $nh\nu$ from Eq. (i) in Eq. (ii), we get the force acting on the plate $= 1.86 \times 10^{-5}$ N

5. Energy of photon,

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{600\times10^{-9}} \approx 3.3 \times 10^{-19}\ \text{J}$$

The number of photons emitted per second,

$$n = \frac{200}{3.3\times10^{-19}} = 6.1 \times 10^{20}\ \text{s}^{-1}$$

The radiation is spherically symmetrical, the number of photons entering the sensor per second is the product of n and ratio of aperture area to the area of sphere of radius 10 m

$$(6.1 \times 10^{20})\ \frac{\lambda\times(0.01)^{2}}{4\times\pi\times10^{2}} = 1.53 \times 10^{14}\ \text{photon s}^{-1}$$

Therefore, number of photons entering the sensor in 1 s $= (0.1) \times (1.53 \times 10^{14}) = 1.53 \times 10^{13}$.

6. We know that force is rate of change of momentum: $F = \dfrac{\Delta p}{\Delta t}$

Since the photons are incident normally on a perfectly reflecting screen, Δp is twice of incident momentum. If n is the number of photons per second.

$$F = \frac{\Delta p}{\Delta t} = n\times2\times\frac{h}{\lambda} = 1\ \text{N} \quad \left(\text{since } p = \frac{h}{\lambda}\right)$$

$$\therefore \quad n = \frac{\lambda}{2h} = \frac{662\times10^{-9}}{2\times(6.62\times10^{-34})} = 5 \times 10^{26}\ \text{photons}$$

7. (a) As energy flux, $I = [E/(S \times t)] = [P/S]$

$$P = I\times S = (1.4\times10^{3})\times(8\times20) = 2.24\times10^{5}\ \text{W} = 224\ \text{kW}$$

(b) As power $P = (E/t)$,

$$E = P\times t = (2.24\times10^{5})\times(60\times60)$$

$$= 8.064\times10^{8} = 806.4\ \text{MJ}$$

(c) For perfectly absorbing surface radiation pressure,

$$P = \frac{I}{c} = \frac{1.4\times10^{3}}{3\times10^{8}} = 4.7\times10^{-6}\ \text{Nm}^{-2} \text{ and as } P = (F/S)$$

$$F = P\times S = 4.7\times10^{-6}\times(8\times20) = 7.52\times10^{-4}\ \text{N}$$

8. For equilibrium, the force exerted by the light beam should balance the weight of plate.

$$F_{\text{photon}} = mg \quad \left(F_{\text{photon}} = \frac{IA}{c} = \frac{P}{c}, \text{ where power } P = IA\right)$$

$$\Rightarrow \quad \frac{P}{c} = 10\times10^{-3}\times10 \Rightarrow P = 3\times10^{7}\ \text{W}$$

9. $I = 1400\ \text{Wm}^{-2}$; $\lambda = 6000$ Å

(a) Energy of the photon, $E = hf = \dfrac{hc}{\lambda} \quad \left(c = 3\times10^{8}\ \text{m s}^{-1}\right)$

Let n be the number of photons received per second per unit area.

$$\therefore \quad n = \frac{IA}{E} = \frac{(1400\times1)\times\left(6000\times10^{-10}\right)}{6.63\times10^{-34}\times3\times10^{8}}$$

$$= 4.22\times10^{21}$$

(b) Total energy emitted per second = power (W)

Number of photons emitted from the sun per second,

$$n = \frac{\text{Power of sun (W)}}{\text{Energy of a photon}}$$

$$= \frac{I\times\left(4\pi(1.49\times10^{11})^2\right)\times\left(6000\times10^{-10}\right)}{6.63\times10^{-34}\times3\times10^{8}} = 1.178\times10^{45}$$

(R: average radius of earth's orbit)

10. Energy of one photon, $E = hc/\lambda$

If power of source is P, the number of photons incident on the metallic surface is

$$N = \frac{P}{E} = \frac{\lambda P}{hc}$$

Momentum of incident photons = h/λ

Change in momentum due to reflection = $2h/\lambda$

The total momentum imparted to the surface per second is

$$\frac{2h}{\lambda}\frac{\lambda P}{hc} = \frac{2P}{c}$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \left(\frac{2P}{c}\right)\Big/A = \frac{2P}{cA}$$

$$= \frac{2\times5\times10^{-3}}{3\times10^{8}\times10^{-6}} = 3.33\times10^{-5}\ \text{N m}^{-2}$$

11. The momentum of a photon, $p = \frac{hf}{c} = \frac{E}{c}$

Momentum of reflected photon $= \rho\left(\frac{E}{c}\right)$

$\therefore$ Change of momentum $= \left(\frac{E}{c}\right) + \left(\frac{\rho E}{c}\right) = \frac{(1+\rho)E}{c}$

Force exerted, $F = \frac{\left[(1+\rho)\frac{E}{c}\right]}{T}$

Pressure, $P = \frac{F}{A} = \frac{4\left[(1+\rho)\frac{E}{c}\right]}{T\pi d^2}$

$$= \frac{4(1+\rho)E}{\pi cTd^2} = \frac{4(1+0.5)\times10}{\pi\times3\times10^8\times10^{-4}\times10^{-10}}$$

$$= 6.37\times10^{6}\,\text{N m}^{-2}$$

12. If S is the area of the mirror on which light falls, the transverse section of the incident beam is $S\cos\theta$. Momentum of the incident photons per second, $p = \left(\frac{I}{c}\right)S\cos\theta$

Normal component of momentum incident per second

$$= (p_n)_{\text{incident}} = \left(\frac{IS\cos\theta}{c}\right)\cos\theta = \frac{IS\cos^2\theta}{c}.$$

Momentum of reflected photons per second

$(p_n)_{\text{reflected}} = \rho\left(\frac{IS\cos^2\theta}{c}\right)$ in the opposite direction.

$\therefore$ Rate of change of momentum = force $= \frac{\Delta p}{\Delta t} = \frac{(1+\rho)IS\cos^2\theta}{c}$

$\therefore$ Normal pressure $P = \frac{F}{A} = \frac{(1+\rho)I\cos^2\theta}{c}$

$\Rightarrow$ Required pressure,

$$P = \frac{(1+0.5)(0.5\times10^4\cos^2 45°)}{3\times10^8} = 1.25\times10^{-5}\text{Nm}^{-2}$$

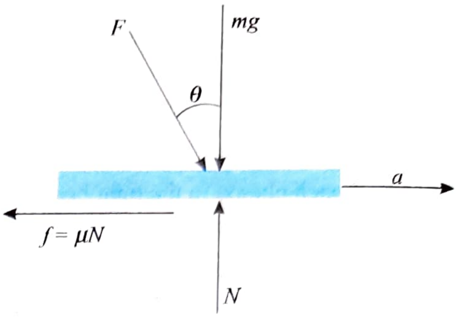
13. E = Energy received by surface per second is $IA\cos\theta = Iab\cos\theta$

Number of photons received by the surface per second,

$$n = \frac{E}{hc/\lambda} = \frac{Iab\cos\theta\lambda}{hc}$$

$$\Delta p_{\text{one photon}} = \frac{h}{\lambda} \Rightarrow F_{\text{radian}} = \Delta p_{\text{one photon}}\cdot\lambda = \frac{Iab\cos\theta}{c}$$

Let us draw the FBD of the plank (see figure)



$$F\sin\theta - \mu N = m\alpha \qquad \text{...(i)}$$

$$N = mg + F\cos\theta \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $\alpha = \frac{F\sin\theta - \mu(mg + F\cos\theta)}{m}$

Exercise 5.2

1. For an electron, de Broglie wavelength is given by

$$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{25}} = \sqrt{6}\ \text{Å} \approx 2.5\ \text{Å}$$

2. The value of de Broglie wavelength associated with a moving particle is given by

$$\lambda = \frac{h}{mv}$$

Consider a ball of 0.5 kg moving with a velocity of 10 ms^{-1}. It is an object from our daily observations. The de Broglie wavelength of the ball will be

$$\lambda = \frac{6.625\times10^{-34}}{0.5\times10} = 13.25\times10^{-34}\ \text{m}$$

The value of λ is very small as compared to the size a ball of 0.5 kg would possess. Hence, wave nature of matter is not apparent in our daily observations.

3. If λ is the de Broglie wavelength of the particle of kinetic energy K, then

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Suppose that the de Broglie wavelength of the particle becomes λ', when its kinetic energy is $K/4$, then

$$\lambda' = \frac{h}{\sqrt{2mK/4}} = 2\left(\frac{h}{\sqrt{2mK}}\right) = 2\lambda$$

4. In terms of accelerating potential V, the de Broglie wavelength of a charged particle is given by

$$\lambda = \frac{h}{\sqrt{2meV}} \qquad \text{...(i)}$$

where e is charge and m is mass of the particle.

Equation (i) represents a straight line, whose slope is $h/\sqrt{2me}$. The slope of the line is inversely proportional to $\sqrt{m}$. Since the slope of line A is lesser, it represents the particle of heavier mass.

5. $W_0 = 4.5$ eV, $\lambda = 200$ nm

Stopping potential or energy, $E - W_0 = \dfrac{hc}{\lambda} - W$

Minimum 1.7 V is necessary to stop the electron

The minimum KE = 2 eV

[Since the electric potential of 2 V is required to accelerate the electron to reach the plates] the maximum KE = (2 + 1.7) eV = 3.7 eV.

6. Let W_0 be the work function of the metal. Then maximum kinetic energies of the photoelectrons ejected in the two cases will be,

$$eV_1 = hf_1 - W_0$$

And, $eV_2 = hf_2 - W_0 \therefore e(V_1 - V_1) = h(f_2 - f_1)$

or, $$h = \frac{e(V_2 - V_1)}{f_2 - f_1}$$

If f_0 is the threshold frequency, then $eV_1 = hf_1 - W_0 = hf_1 - hf_0$

or $$f_0 = f_1 - \frac{eV_1}{h} = f_1 - \frac{V_1(f_2 - f_1)}{V_2 - V_1}$$

$$= \frac{f_1V_2 - f_1V_1 - f_2V_1 + f_1V_1}{V_2 - V_1} = \frac{f_1V_2 - f_2V_1}{V_2 - V_1}$$

7. Let W_0 be the work function of the metal. Let E_1 and E_2 be the kinetic energies of photoelectrons corresponding to frequencies f and $2f$ of the incident radiation.

Using Einstein's photoelectric equation, we find $hf = E_1 + W_0$

and $2hf = E_2 + W_0$

On dividing, $2 = \dfrac{E_2 + W_0}{E_1 + W_0}$

or $2E_1 + 2W_0 = E_2 + W_0$

or $E_2 = 2E_1 + W_0$

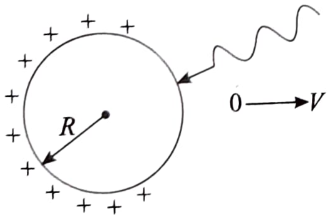
Thus kinetic energy of photoelectrons is increased more than double on doubling the frequency of incident radiation.

8. As the copper sphere loses photoelectrons, it becomes positively charged. Let it lose n electrons. Then its charge will be ne. Hence, the potential of the copper ball is

$$V_{\text{stop}} = \frac{1}{4\pi\varepsilon_0}\frac{ne}{R} \quad \text{...(i)}$$

This is the stopping potential which is equal to the maximum KE of the photoelectrons.

According to Einstein's photoelectric Eq. (i), $\dfrac{hc}{\lambda} - W_0 = \dfrac{1}{2}mv^2_{\text{max}}$



or $$\frac{hc}{\lambda} - W_0 = eV_{\text{stop}} \quad \text{...(ii)}$$

By using Eqs. (i) and (ii), $\dfrac{hc}{\lambda} - W_0 = e\left(\dfrac{1}{4\pi\varepsilon_0}\right)\dfrac{ne}{R}$

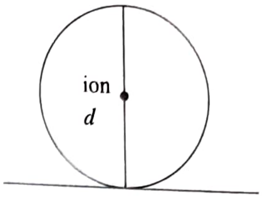
or $$n = \left(\frac{hc}{\lambda} - W_0\right)\frac{4\pi\varepsilon_0 R}{e^2}$$

On substituting corresponding values we get $n = 4.86 \times 10^9$

9. Work function = W_0, distance = d

The particle will move in a circle.

When the stopping potential is equal to the potential due to the singly charged ion at that point.



$$eV_0 = \frac{hc}{\lambda} - W_0$$

$$\Rightarrow V_0 = \left(\frac{hc}{\lambda} - W_0\right)\frac{1}{e}$$

$$\Rightarrow \frac{Ke}{2d} = \left(\frac{hc}{\lambda} - W_0\right)\frac{1}{e}$$

$$\Rightarrow \frac{Ke^2}{2d} = \frac{hc}{\lambda} - W_0$$

$$\Rightarrow \frac{hc}{\lambda} = \frac{Ke^2}{2d} + W_0 = \frac{Ke^2 + 2dW_0}{2d}$$

$$\Rightarrow \lambda = \frac{hc\,2d}{Ke^2 + 2dW_0} = \frac{2hcd}{(1/4\pi\varepsilon_0)e^2 + 2dW_0} = \frac{8\pi\varepsilon_0 hcd}{e^2 + 8\pi\varepsilon_0 dW_0}$$

Exercise 5.3

1. (i) de-Broglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mqV}}$

For same V, $\lambda \propto \dfrac{1}{\sqrt{mq}} \therefore \dfrac{\lambda_p}{\lambda_\alpha} = \sqrt{\dfrac{m_\alpha q_\alpha}{m_p q_p}}$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m_p}{m_p}\cdot\frac{2e}{e}} = \sqrt{8} = 2\sqrt{2}$$

Hence $\lambda_p > \lambda_\alpha$ i.e., proton has a greater value of de-Broglie wavelength.

(ii) Kinetic energy, $K = qV$

For same V, $K \propto q$

$$\therefore \quad \frac{K_p}{K_\alpha} = \frac{q_p}{q_\alpha} = \frac{e}{2e} = \frac{1}{2}$$

Hence, $K_p < K_\alpha$, i.e., proton has less kinetic energy.

2. (i) de-Broglie wavelength, $\lambda = \dfrac{h}{\sqrt{2mqV}}$

For same V, $\lambda \propto \dfrac{1}{\sqrt{mq}}$

$$\therefore \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p \times e}{m_e \times e}} = \sqrt{\frac{m_p}{m_e}}$$

As $m_p > m_e$, so $\lambda_e > \lambda_p$ i.e., electron has a greater de-Broglie wavelength.

(ii) Momentum, $p = \sqrt{2meV}$

or $p \propto \sqrt{m}$

As $m_e < m_p$, $p_e < p_p$ i.e., electron has less momentum.

3. The kinetic energy of a particle, $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\dfrac{(mv)^2}{m} = \dfrac{p^2}{2m}$

$\therefore$ Linear momentum, $p = \sqrt{2mK}$

de-Broglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}}$

For the particles possessing same kinetic energy, $\lambda \propto \dfrac{1}{\sqrt{m}}$

As $m_e << m_p < m_\alpha \therefore \lambda_e >> \lambda_p > \lambda_\alpha$

Hence the α-particle has the shortest de-Broglie wavelength.

4. The kinetic energy gained by a charged particle when accelerated through a potential difference of V volts is given by

$$\frac{1}{2}mv^2 = qV$$

or $m^2v^2 = 2mqV$

or $p = mv = \sqrt{2mqV}$

$\therefore$ de Broglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mqV}}$

For the same potential difference V,

$$\frac{\lambda(\alpha-\text{particle})}{\lambda(\text{proton})} = \sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}} = \sqrt{\frac{m_p \times e}{4m_p \times 2e}} = \frac{1}{2\sqrt{2}}$$

5. If α particle is accelerated through a potential difference V, then

$$qV = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

or $p = \sqrt{2mqV}$

$\therefore$ $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mqV}}$

As $\lambda_p = \lambda_\alpha$

or $\dfrac{h}{\sqrt{2m_p q_p V_p}} = \dfrac{h}{\sqrt{2m_\alpha q_\alpha V_\alpha}}$

or $\dfrac{V_p}{V_\alpha} = \dfrac{m_\alpha q_\alpha}{m_p q_p} = \dfrac{4m_p \cdot 2e}{m_p \cdot e} = 8$

6. $\lambda_{\text{de Broglie}} = \dfrac{h}{\sqrt{2mK}} = \dfrac{h}{mv}$

$$v = u + at = 0 + \left(\frac{eE}{m}\right)t \Rightarrow \lambda = \frac{h}{eEt}$$

$$\Rightarrow \frac{d\lambda}{dt} = -\frac{h}{eEt^2}$$

7. $\lambda_{\text{de Broglie}} = \dfrac{h}{mV_{\text{mp}}}$ ...(i)

$V_{\text{mp}} = \sqrt{\dfrac{2kT}{m}}$ ...(ii)

Then, $\lambda_{\text{de Broglic}} = \dfrac{h}{m\sqrt{\dfrac{2kT}{M}}}$

$$= \frac{6.3\times10^{-34}}{\left(\sqrt{\dfrac{2\times1.38\times10^{-23}\times300}{2\times1.67\times10^{-27}}}\right)\times2\times1.67\times10^{-27}}$$

$$= 1.2\times10^{-7}\,\text{m}$$

Exercises

Single Correct Answer Type

1. (1) Energy $= \dfrac{1}{2}mv^2 = 5000\text{ eV} = 5000 \times 1.6 \times 10^{-19}$ J

$$mv = \sqrt{2\times5000\times(1.6\times10^{\;19})} = 4\times10^{-8}\times\sqrt{m}$$

Number of electrons striking per second is

$$n = \frac{q}{e} = \frac{It}{e} = \frac{50\times10^{-6}\times1}{1.6\times10^{-19}} = 31.25\times10^{13}$$

Force = change of momentum per second

$= n\,(mv) = 31.25\times10^{13}\times4\times10^{-8}\sqrt{m}$

$= 125\times10^{5}\sqrt{9.1\times10^{-31}}$

$= 1.1924\times10^{-8}$ N

2. (2) Number of photons falling per second.

$$N_p = \frac{10^{-3}}{\dfrac{6.6\times10^{-34}\times3\times10^{8}}{5000\times10^{-10}}} = 2.5\times10^{15}$$

Let N_e is the number of photoelectrons emitted per second.

$$\therefore \quad I = \frac{q}{t} = \frac{N_e e}{1} \Rightarrow N_e = \frac{I}{e} = \frac{0.16\times10^{-6}}{1.6\times10^{-19}} = 10^{12}$$

Percentage of photons producing photoelectrons,

$$= \frac{N_e}{N_p}\times100 = \frac{10^{12}}{2.5\times10^{15}}\times100 = 0.04\%$$

3. (3) Using photoelectric equation, $hf - hf_0 = \dfrac{1}{2}mv^2 = eV_s$

or $\left(\dfrac{hc}{\lambda} - \dfrac{hc}{\lambda_0}\right) = eV_s$

For the first case, $\dfrac{hc}{\lambda} - \dfrac{hc}{\lambda_0} = e(3V_0)$ ...(i)

For the second case, $\dfrac{hc}{2\lambda} - \dfrac{hc}{\lambda_0} = e(V_0)$ ...(ii)

Solving $\lambda_0 = 4\lambda$

4. (3) Einstein's equation for photoelectric effect is

$$hf - hf_0 = \frac{1}{2}mv^2_{\max}$$

When $f = 2f_0$, $f_{\max} = 4\times10^8$ cm s^{-1}

$2hf_0 - hf_0 = (1/2)\,m\,(4\times10^8)^2$

$hf_0 = \dfrac{1}{2}m\,(4\times10^8)^2$ ...(i)

When $f = 5f_0$, $v_{\max} = v'$

$h(5f_0) - hf_0 = \dfrac{1}{2}mv'^2$ ...(ii)

Dividing Eq. (ii) by Eq. (i), we get $v' = 8\times10^8$ cm s^{-1}

5. (2) $E - W_0 = \dfrac{1}{2}mv^2 = eV_s$

or $\dfrac{hc}{\lambda} - W_0 = eV_s$

Hence, $\dfrac{hc}{0.6\times10^{-6}} - W_0 = e(0.5)$ ...(i)

and $\dfrac{hc}{0.4\times10^{-6}} - W_0 = e(1.5)$ ...(ii)

Solving, we get $W_0 = 1.5$ eV

6. (3) $n = \dfrac{\text{power}}{hc/\lambda} = \dfrac{1\times300\times10^{-9}}{6.6\times10^{-34}\times3\times10^{8}}$

$= 1.51\times10^{18}\,\text{m}^{-2}\,\text{s}^{-1} = 1.51\times10^{14}\,\text{cm}^{-2}\,\text{s}^{-1}$

As only 1 percent of photons cause emission of photoelectrons, number of photoelectrons is

$n_e = 1.51\times10^{12}$ s^{-1}

7. (4) $\dfrac{1}{2}mv^2 = \dfrac{hc}{\lambda} - W_0$ ...(i)

Let the speed of the fastest electron be v_1 when excitation wavelength is changed to $3\lambda/4$.

$\therefore$ $\dfrac{1}{2}mv_1^2 = \dfrac{4hc}{3\lambda} - W_0$

$\Rightarrow$ $\dfrac{1}{2}mv_1^2 = \dfrac{4}{3}\left(\dfrac{hc}{\lambda} - W_0\right) + \dfrac{W_0}{3}$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{W_0}{3}$$ [using Eq. (i)]

$$\Rightarrow \quad v_1^2 = \frac{4v^2}{3} + \frac{2W_0}{3m}$$

$$\therefore \quad v_1 > \sqrt{\frac{4}{3}}\,v$$

(2) $hf_1 - hf_0 = \frac{1}{2}mv_1^{\,2}$

$$hf_2 - hf_0 = \frac{1}{2}mv_2^2$$

$$\therefore \quad h(f_1 - f_2) = \frac{1}{2}m(v_1^2 - v_2^2)$$

$$\therefore \quad v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$$

(4) Since the number of photoelectrons emitted is directly proportional to the intensity of incident radiation, the number of photoelectrons emitted becomes four times.

The energy of photoelectrons does not change with the intensity of light.

(3) $\frac{hc}{\lambda_{\max}} = 3 \times 1.6 \times 10^{-19}$ J

$$\Rightarrow \quad \lambda_{\max} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3 \times 1.6 \times 10^{-19}} = 4.125 \times 10^{-7}\text{ m}$$

(1) $K_{\max} = h\nu - W$

ω is the intercept on y-axis and h is the slope.

$$\therefore \quad h = \frac{2.4 \times 10^{-15}}{4 \times 10^{18}} = 6 \times 10^{-34}\text{ Js}$$

$$W = 2 \times 10^{-15}\text{ J}$$

$$\Rightarrow \quad h\nu_0 = 2 \times 10^{-15}$$

or $\quad \nu_0 = 3.33 \times 10^{18}$ Hz

(3) According to Einstein's equation, $E = W_0 + K$

$$W_{0\max} = 4.2\text{ eV}$$

$$K = 2.6\text{ eV}$$

$$\therefore \quad E_{\min} = W_{0\max} + K = (4.2 + 2.6)\text{ eV} = 6.8\text{ eV}$$

3. (1) As work function $W = h\nu_0$, where ν_0 is the threshold frequency. Greater the work function, greater is the threshold frequency. Therefore, the threshold frequency of sodium will be lesser than that for aluminium.

4. (2) The maximum KE of the photoelectron is given by

$$\left(\frac{1}{2}mv^2\right)_{\max} = hf - W$$

Now, $f = \frac{c}{\lambda}$ and $\left(\frac{1}{2}mv^2\right) = eV$

$$\therefore \quad eV = \frac{hc}{\lambda} - W$$

or $\quad V = \left(\frac{hc}{e}\right)\frac{1}{\lambda} - \frac{W}{e}$ ...(i)

Slope of straight line = hc/e

15. (2) Since V is represented along y-axis and $(1/\lambda)$ along x-axis, Eq. (i) represents a straight line.

Intercept of straight line = $-(W/e)$

16. (3) The ratio of electric force to magnetic force is

$$\frac{F_e}{F_B} = \frac{qE}{qvB} = \frac{3 \times 10^6}{2.4 \times 10^6 \times 1.5} = \frac{5}{6}$$

Now, magnetic force on beta particles acts downward, whereas electric force acts upward and both are in the plane of diagram. But since magnetic force is larger, so beta particles are deflected downward.

17. (4) By photoelectric equation: $\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$, where λ is the wavelength of incident radiation and λ_0 is the threshold wavelength.

$$\frac{1}{2}m(2v)^2 = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda_0}\right)$$

Solving for λ', we get $\lambda' = 300$ nm

18. (2) By Einstein's particle (photon) theory, the maximum kinetic energy $E_{\max}$ of the emitted electrons from the cathode is proportional to the frequency f of the light. This is expressed in the Einstein's photoelectric equation below

$$E_{\max} = hf - \phi = hf - hf_0 = h(f - f_0) \quad \text{...(i)}$$

where h is the Planck's constant, ϕ_0 is the work function of the metal and is related to the threshold frequency f_0 by $\phi_0 = hf_0$. It is the minimum amount of work or energy necessary to take a free electron out of the metal against the attractive forces of surrounding positive ions.

At a particular negative potential difference V applied to the anode A, the current becomes zero. This is value of the negative potential difference which just stops the electrons with maximum energy from reaching A. V is called the stopping potential. Therefore,

$$eV = E_{\max} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have $eV = E_{\max} = h(f - f_0)$

$$V = \frac{h}{e}(f - f_0) \text{ or } y = \frac{h}{e}(x - x_0)$$

The variation of V (or y) is thus a straight line of gradient h/e, when it is plotted against f (or x) at f_0. It is best represented in graph (2).

19. (2) On increasing intensity, only saturation current increases, whereas retarding potential remains the same because wavelength of light is unchanged.

20. (3) $eV_s = \frac{hc}{\lambda} - \phi_0$ or $eV_s + \phi_0 = \frac{hc}{\lambda}$

or $\quad \lambda = \frac{hc}{eV_s + \phi_0}$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_1} = \frac{eV_{s_1} + \phi_0}{eV_{s_2} + \phi_0}$$

or $\quad \frac{4500}{4000} = \frac{1.3 + \phi_0}{0.9 + \phi_0}$

or $\quad \phi_0 = 851.3 - 9 \times 0.9$

Solving, $\phi_0 = (10.4 - 8.1) = 2.3$ eV

21. (1) The photoelectrons will be emitted because wavelength of incident radiation is less than threshold wavelength $(\lambda < \lambda_0)$

22. (2) $\left(\frac{1}{2}mv^2\right)_{\max} = hf - W$

When f is doubled (W remains same), $\left(\frac{1}{2}mv^2\right)_{\max}$ i.e., (KE) is increased. The photoelectric current is directly proportional to the intensity of incident light.

23. (3) Let $hf_0 - W_0 = K$ ...(i)

If frequency is doubled, let kinetic energy of photoelectrons be K_1.

$$2hf_0 - W_0 = K_1 \quad \text{... (ii)}$$

$$\Rightarrow \quad 2(hf - W_0) + W_0 = K_1$$

$$\Rightarrow \quad 2K + W_0 = K_1$$

i.e., kinetic energy is more than doubled.

24. (3) If E is the energy of incident photon and W_0 the work function, then $E - W_0$ = available energy.

$$E - W_0 = \frac{1}{2}mv^2$$

or $$v = \sqrt{\frac{2(E - W_0)}{m}}$$

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{1 \quad 0.5}{2.5 \quad 0.5}} = \sqrt{\frac{0.5}{2}} = \frac{1}{2}$$

25. (4) $\lambda_p = \lambda_\alpha$

or $$\frac{h}{\sqrt{2m_pQ_pV}} = \frac{h}{\sqrt{2m_\alpha Q_\alpha V_\alpha}}$$

$$\therefore \quad m_pQ_pV_p = m_\alpha Q_\alpha V_\alpha$$

$$\therefore \quad V_\alpha = \left(\frac{m_p}{m_\alpha}\right)\left(\frac{Q_p}{Q_\alpha}\right)V = \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)V = \frac{V}{8}$$

26. (3) $I \propto 1/d^2$

When source is placed 2 m away, then $I' = (I/4)$. The number of electrons emitted $\propto$ intensity. Hence, the number of emitted electrons is reduced to one-fourth.

27. (1) Energy radiated as visible light is $\frac{5}{100} \times 100 = 5$ J s^{-1}

Let n be the number of photons emitted per second.

Then, $$\frac{nhc}{\lambda} = 5$$

$$\therefore \quad n = \frac{5\lambda}{hc} = \frac{5 \times 5.6 \times 10^{-7}}{(6.62 \times 10^{-34})(3 \times 10^{8})} = 1.4 \times 10^{19}$$

28. (1) $hf = 5 \text{ eV} + 2.2 \text{ eV} = 7.2 \text{ eV}$

$$7.2 = \frac{12375}{\lambda(\text{in Å})}$$

or $$\lambda\,(\text{in Å}) = \frac{12375}{7.2} \approx 1719$$

29. (3) $\phi_0 = hf_0 = \frac{hc}{\lambda_0}$

$\phi_0\lambda_0$ = constant

$4.5 \times \lambda = 2.3 \times 5460$

or $$\lambda = \frac{2.3 \times 5460}{4.5} \text{ Å} = 2790.7 \text{ Å} \approx 2791 \text{ Å}$$

30. (4) Work function, $W_0 = \frac{hc}{\lambda_0}$

$$\Rightarrow \quad W_0 = \frac{2\times10^{-25}}{2\times10^{-7}} = 10^{-18} \text{ J} = \frac{10^{-18}}{1.6\times10^{-19}} \text{ eV} = 6.25 \text{ eV}$$

Energy of incident ratiation is $E = \frac{hc}{\lambda} = 31.25$ eV

$\therefore$ KE of photoelectrons = $E - W_0$ = 25 eV

31. (4) $\frac{1}{2}mv^2_{\max} = \frac{hc}{\lambda} - \phi_0$

$$\frac{1}{2}mv^2_{\max} = \frac{12375 \text{ eV}}{3000} - 1 \text{ eV} = 3.125 \times 1.6 \times 10^{-19}$$

$$v_{\max} = \sqrt{\frac{2 \times 3.125 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \approx 10^6 \text{ m s}^{-1}$$

32. (3) Let us calculate energy corresponding to 0.2×10^{-6} m

or $0.2 \times 10^{-6} \times 10^{10}$ Å

or $$2000 \text{ Å} = \frac{12375}{2000} = 6.1875 \text{ eV}$$

$$\frac{1}{2}mv^2_{\max} = (6.1875 - 4) \text{ eV}$$

or $$mv^2_{\max} = \frac{2 \times 2.1875 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$$

or $$mv^2_{\max} = 0.769 \times 10^{12} \Rightarrow v = 0.876 \times 10^6 = 8.76 \times 10^5 \text{ m s}^{-1}$$

33. (3) Effective power $= \frac{25}{100} \times 200 \text{ W} = 50 \text{ W}$

Now, $$50 = nh\nu = \frac{nhc}{\lambda}$$

$$n = \frac{50\lambda}{hc} = \frac{50 \times 0.6 \times 10^{-6}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 1.5 \times 10^{20}$$

34. (4) If q is the charge on the particle and V the potential difference through which it is accelerated, then

$$qV = \frac{1}{2}mv^2$$

or $$mv = \sqrt{2mqV}$$

de Broglie's wavelength, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

$$\therefore \quad \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$$

35. (2) We know that mass m in motion and the rest mass m_0 is related through the equation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As $$v = c,\ m = \frac{m_0}{\sqrt{1-1}} = \frac{m_0}{0} = \infty$$

Therefore, de Broglie wavelength is $\lambda = \frac{h}{mv} = \frac{h}{(\infty)(c)} = 0$

36. (2) $W = hf - eV_s$

hf = energy of incident photon

Here $$hf = \frac{12400}{1240} \text{ eV} = 10 \text{ eV}$$

$$\therefore \quad W = 10 - 8 = 2 \text{ eV}$$

So, λ_0 = Threshold wavelength $= \frac{12400}{2}$ Å = 6200 Å

37. (1) $\lambda_1 = \frac{12375}{E_1(\text{eV})}$ Å = 1000 Å

$$\therefore \quad E_1 = 12.375 \text{ eV}$$

Similarly, $E_2 = \frac{12375}{\lambda_2(\text{Å})}$ eV $= \frac{12375}{2000} = 6.1875$ eV

Now, $E_1 - W_0 = eV_s$

and $E_2 - W_0 = eV'_s$

Hence, $12.375 - W_0 = 7.7$eV

and $6.1875 - W_0 = eV'_s$

Solving, we get $V'_s = 1.5$ V

38. (3) Energy is given by $E = \frac{m_0c^2}{\sqrt{1 - (v^2/c^2)}}$

or $E^2 = \dfrac{m^2{}_0c^2}{c^2 - v^2}$

Momentum p is given by $p = \dfrac{m_0 v}{\sqrt{1-(v^2/c^2)}}$

or $p^2c^2 = \dfrac{m_0^2 c^4 v^2}{c^2 - v^2}$

$\therefore$ $E^2 - p^2c^2 = m_0^2 c^4$ or $E^2 = p^2c^2 + m_0^2 c^4$

For photon, rest mass $m_0 = 0$, so $E = pc$

For electron, $m_0 \neq 0$, so $E \neq pc$

(2) $eV = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda_0}\right)$ ...(i)

$$\frac{eV}{3} = hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get $\lambda_0 = 4\lambda$

(3) $\dfrac{hc}{\lambda} = E + \phi_0$

$$\frac{hc}{\lambda'} = 2E + \phi_0$$

Dividing, we get $\dfrac{\lambda'}{\lambda} = \left(\dfrac{E+\phi_0}{2E+\phi_0}\right)$ or $\dfrac{\lambda'}{\lambda} < 1$

$\therefore$ $\lambda' < \lambda$ or $\lambda > \lambda'$ ...(i)

Also, $\dfrac{\lambda'}{\lambda} = \dfrac{1}{2}\left[\dfrac{E+\phi_0}{E+\dfrac{\phi_0}{2}}\right]$

or $\dfrac{\lambda'}{\lambda} > \dfrac{1}{2}$

or $\lambda' > \dfrac{\lambda}{2}$ ...(ii)

It follows from Eqs. (i) and (ii) that $\lambda > \lambda' > \dfrac{\lambda}{2}$

(4) $\lambda = \dfrac{h}{\sqrt{2meV}}$

$$p = \sqrt{2meV}$$

$$\frac{p_e}{p_\alpha} = \sqrt{\frac{2m_e eV}{2m_\alpha(2e)V}}$$

$$\frac{p_e}{p_\alpha} = \sqrt{\frac{m_e}{2m_\alpha}}$$

(3) $E = \dfrac{hc}{\lambda} - \phi_0$

$$2E = \frac{hc}{\lambda'} - \phi_0$$

Solving $\lambda' = \dfrac{hc\lambda}{E\lambda + hc}$

(1) $E = \dfrac{hc}{\lambda/2} - \phi_0$

$$2E = \frac{hc}{\lambda/3} - \phi_0$$

or $2\left(\dfrac{2hc}{\lambda} - \phi_0\right) = \dfrac{3hc}{\lambda} - \phi_0$

or $\dfrac{4hc}{\lambda} - 2\phi_0 = \dfrac{3hc}{\lambda} - \phi_0$

or $\phi_0 = \dfrac{hc}{\lambda}$

44. (4) Order of magnitude calculation is enough.

$(2m_e eV)^{1/2} = (2\times 9\times 10^{-31}\times 100\times 1.6\times 10^{-19})^{1/2} \approx 5\times 10^{-24}$ kg m s^{-1}

and $h \approx 6\times 10^{-34}$ J s

So, $\lambda \approx 10^{-10}$ m

Mid-wavelength in visible region is $\lambda_0 \approx 5000\times 10^{-10}$ m

Thus, $\lambda = \lambda_0/5000$

45. (3) $8\times 10^{14}\, h = \phi_0 + 0.5$

$12\times 10^{14}\, h = \phi_0 + 2$

Dividing, we get $\dfrac{12}{8} = \dfrac{\phi_0 + 2}{\phi_0 + 0.5}$

$$\frac{3}{2} = \frac{\phi_0 + 2}{\phi_0 + 0.5}$$

$$3\phi_0 + 1.5 = 2\phi_0 + 4$$

or $\phi_0 = 2.5$ eV

46. (1) $\lambda = \dfrac{h}{\sqrt{2mkT}} = \dfrac{6.62\times 10^{-34}}{\sqrt{2\times 1.67\times 10^{-27}\times 1.38\times 10^{-23}\,T}}$ m

$$= \frac{6.62\times 10^{-34}}{2.15\times 10^{-25}\sqrt{T}} \text{ m}$$

$$= \frac{3.079}{\sqrt{T}}\times 10^{-9}\text{ m} = \frac{30.79}{\sqrt{T}}\text{ Å} \approx \frac{30.8}{\sqrt{T}}\text{ Å}$$

47. (4) $\lambda = \dfrac{h}{mv}$

Here, $0\times M = m_1v_1 + m_2v_2$

Clearly, $m_1v_1 = -m_2v_2$

In magnitude, mv = constant

$\therefore$ $\dfrac{\lambda_1}{\lambda_2} = \dfrac{1}{1}$

48. (2) Energy of electron in nth orbit, $E_n = -\dfrac{13.6}{n^2}$ eV

For first Bohr orbit, $n = 1$; $E_1 = -13.6$ eV

For second Bohr orbit, $n = 2$; $E_2 = -\dfrac{13.6}{4}$ eV

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

or $\dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{1}{4}} \Rightarrow \dfrac{\lambda_1}{\lambda_2} = \dfrac{1}{2}$

49. (4) Each photon has associated with it an energy wave given by

$$E = hf = \frac{hc}{\lambda}$$

and graph of E vs λ is a hyperbola.

Thus, $E \propto \dfrac{1}{\lambda}$

50. (2) Energy corresponding to 2000 Å = (12375/2000) eV = 6.2 eV

Maximum kinetic energy is (6.2 – 5.01) eV = 1.19 eV

Now, $\dfrac{1}{2}\times 9.1\times 10^{-31}\times v_{\max}^2 = 1.19\times 1.6\times 10^{-19}$

or $v_{\max}^2 = \dfrac{1.19\times 1.6\times 10^{-19}\times 2}{9.1\times 10^{-31}}$

$$= 0.418\times 10^{12} = 41.8\times 10^{10}$$

or $v_{\max} = 6.46\times 10^5$ m s^{-1}

51. (3) $\lambda = \dfrac{h}{\sqrt{2mqV}}$

$$mV = \text{constant}$$

$$1837\,V' = 1 \times V$$

or $$V' = \frac{V}{1837}\text{ volt}$$

52. (3) $E_k = \dfrac{12375}{4000} - \phi = 3.1 - \phi_0$

$$2E_k = \frac{12375}{3100} - \phi = 3.99 - \phi_0$$

$$6.2 - 2\phi_0 = 3.99 - \phi_0$$

or $$\phi_0 = 6.2 - 3.99 = 2.21\text{ eV}$$

53. (3) $K_A = \dfrac{hc}{\lambda_A} - \phi_0;\quad K_B = \dfrac{hc}{\lambda_B} - \phi_0$

But $\lambda_A = 2\lambda_B$, therefore

$$\therefore\quad K_A = \frac{hc}{2\lambda_B} - \phi_0$$

$$K_A = \frac{1}{2}[K_B + \phi_0] - \phi_0$$

or $$K_A = \frac{K_B}{2} - \frac{\phi_0}{2}$$

$$\therefore\quad K_A < \frac{K_B}{2}$$

54. (2) $\lambda = \dfrac{h}{\sqrt{2meV}}$

$$\frac{hc}{\lambda_{\min}} = eV$$

$$\lambda \times \frac{hc}{\lambda_{\min}} = \frac{h}{\sqrt{2meV}}eV \quad\text{or}\quad \frac{\lambda}{\lambda_{\min}} \propto \sqrt{V}$$

55. (2) $\dfrac{1}{2}mv^2 = hf - \phi_0$

$$\frac{1}{2}mv'^2 = 4hf - \phi_0$$

$$\frac{v'^2}{v^2} = \frac{4hf - \phi_0}{hf - \phi_0}$$

or $$\frac{v'^2}{v^2} = \frac{4[hf - \phi] + 3\phi_0}{hf - \phi_0}$$

Clearly, $v' > 2v$

56. (1) Energy received by the eye, $E = \dfrac{nhc}{\lambda}$

$$= \frac{5 \times 10^4 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} = 0.2 \times 10^{-13}\text{ W m}^{-2}$$

So, eye is more sensitive by a factor of $\dfrac{1}{0.200} = 5.00$

57. (3) $E = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 4 \times 1.67 \times 10^{-27} \times (0.1 \times 10^{-10})^2} \times \frac{1}{1.6 \times 10^{-19}}\text{ eV}$$

$$= \frac{43.82 \times 10^{-68}}{21.376 \times 10^{-68}} = 2.05\text{ eV}$$

58. (2) The momentum of photon = h/λ

If n is the number of photons falling per second on the plate, then total momentum per second of the incident photons is

$$p = n \times \frac{h}{\lambda}$$

Since the plate is blackened, all photons are absorbed by it.

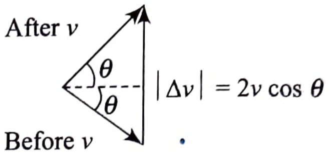
$$\frac{\Delta p}{\Delta t} = n\frac{h}{\lambda}$$

Since $F = \dfrac{\Delta p}{\Delta t} = n\dfrac{h}{\lambda}$

$$\therefore\quad n = \frac{F\lambda}{h} = \frac{6.62 \times 10^{-5} \times 5 \times 10^{-7}}{6.62 \times 10^{-34}} = 5 \times 10^{22}$$

59. (1) Since the molecules rebound from the wall, the component of velocity perpendicular to the wall is reversed, while its velocity parallel to the wall does not change. The change in velocity of molecules is parallel to normal N. The magnitude of change is

$$|\Delta\vec{v}| = 2v\cos\theta$$



The change in momentum of a molecule is

$$|\Delta\vec{p}| = m|\Delta\vec{v}| = 2mv\cos\theta$$

in the direction of normal N. Let n be the number of molecules per unit volume. The number of molecules arriving at an area A of the wall per unit time is the number in a slanted cylinder whose length is equal to the velocity v and whose cross section is $A\cos\theta$.

Number of molecules = $n\,(Av\cos\theta)$

Each molecule suffers a change of momentum $2mv\cos\theta$.

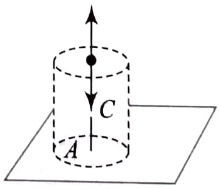
Change of momentum of a stream of gas in a direction perpendicular to the wall is equal to $(nAv\cos\theta) \times (2mv\cos\theta) = 2A\,nmv^2\cos^2\theta$.

Hence, force exerted on stream of gas by the wall,

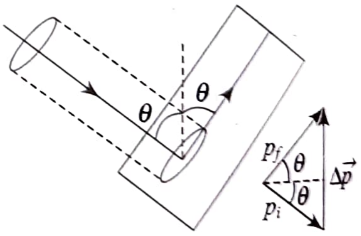
$$F = 2Anmv^2\cos^2\theta$$

This is also the force exerted by gas molecules on the wall.

$$\text{Pressure} = \frac{\text{Normal force}}{\text{Area}} = \frac{F}{A} = 2nmv^2\cos^2\theta$$



(a) Radiation pressure at normal incidence



(b) Radiation pressure at oblique incidence

Remark: For oblique incidence, the change in momentum of the radiation per unit volume at the perfectly reflecting surface is $2p\cos\theta$ and the corresponding radiation pressure is

$$P_{\text{rad}} = 2pc\cos^2\theta = 2E\cos^2\theta$$

60. (4) Momentum corresponding to incident protons normal to the surface,

$$\left(\frac{dp}{dt}\right)_{\text{incident}} = \frac{I}{c}dA\cos^2\theta$$

Since reflection coefficient is ρ, so the momentum of the reflected photons per second normal to surface,

$$\left(\frac{dp}{dt}\right)_{\text{reflected}} = -\frac{I}{c}dA\rho\cos^2\theta$$

Hence, rate of change of momentum of the photons,

$$\left(\frac{dp}{dt}\right)_{\text{photons}} = -\frac{I}{c}\, dA(\rho + 1)\cos^2\theta$$

From Newton's third law, $\left(\frac{dp}{dt}\right)_{\text{surface}} = -\frac{I}{c} dA(\rho + 1)\cos^2\theta$

Hence, pressure exerted on surface,

$$p = \frac{dF}{dA} = \frac{I}{c}\, dA(\rho + 1)\cos^2\theta$$

On substituting values, we get $P = 0.5$ N cm^{-2}.

(1) Imagine the sphere to be made of thin circular rings of radius r, thickness $ds = R\, d\theta$ and subtending an angle of θ at the centre.

Momentum per second of incident photons,

$$\left(\frac{dp}{dt}\right)_{\text{incident}} = \frac{I}{c} dA \cos^2\theta$$

Since surface of mirror is considered to be ideal, i.e., reflection coefficient is unity, photons suffer momentum change in normal direction only.

$$\left(\frac{dp}{dt}\right)_{\text{photon}} = \frac{2I}{c} dA\cos^2\theta$$

$$dF_n = \left(\frac{dp}{dt}\right)_{\text{ball}} = +\frac{2I}{c} dA\cos^2\theta$$

This force may be resolved into horizontal and vertical components. The vertical component $dF_n \sin\theta$ is cancelled because every element on the upper half has a symmetrically placed element in the lower half. So, resultant force on the ball,

$$F = \int dF_n \cos\theta = \int \frac{2I}{c}\, dA\cos^3\theta$$

$$dA = (2\pi R\sin\theta)R\, d\theta$$

$$F = \int_0^{\pi/2} 4\pi \frac{I}{c} R^2 \cos^3\theta \sin\theta\, d\theta$$

$$= \frac{4\pi R^2 I}{c}\int_0^{\pi/2} \cos^3\theta\sin\theta\, d\theta = \frac{\pi R^2 I}{c}$$

On substituting values, we get $F = 0.8$ μN.

(3) Photons have momentum ($p = h/\lambda$) which they carry away; the spacecraft will acquire momentum in the opposite direction according to law of conservation of momentum.

Number of photons per second from laser $= n$

Then, from energy considerations, $0.5 \times 10^{-3} = nh\left(\frac{c}{\lambda}\right)$

$n = (0.5 \times 10^{-3})\lambda/(ch)$

Rate of change of momentum of spacecraft

$$= np = n\frac{h}{\lambda} = (0.5\times 10^{-3})\frac{\lambda}{ch}\left(\frac{h}{\lambda}\right) = \frac{0.5\times 10^{-3}}{c}$$

From Newton's second law, $\frac{nh}{\lambda} = ma$

$$1000a = \frac{0.5\times10^{-3}}{3.00\times10^{8}} = \frac{1}{6}\times10^{-11}$$

$v = at$

$$t = \frac{v}{a} = \frac{1000}{\left(\frac{1}{1000}\right)\times\frac{1}{6}\times10^{-11}}\ \text{s} = 6\times10^{17}\ \text{s}$$

63. (2) The gain of kinetic energy by an electron is eV.

$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19})(50)}{(9.11\times10^{-31})}} = 4.19\times10^6\ \text{m s}^{-1}$$

Thus, the electron's de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(4.19\times19^{6})} = 1.74\times10^{-10}\ \text{m}$$

64. **(4)** Magnetic force experienced by a charged particle in a magnetic field is given by

$$F_B = q\vec{v}\times\vec{B} = qvB\sin\theta$$

In our case, $F_B = qvB$ [as $\theta = 90°$]

Hence, $Bqv = \frac{mv^2}{r} \Rightarrow mv = qBr$

The de Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{qBr}$

$$\frac{\lambda_{\alpha\text{-particle}}}{\lambda_{\text{proton}}} = \frac{q_p r_p}{q_\alpha r_\alpha}$$

Since $\frac{r_\alpha}{r_p} = 1$ and $\frac{q_\alpha}{q_p} = 2$

$$\Rightarrow \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2}$$

65. **(1)** Kinetic energy gained by a charge q after being accelerated through a potential difference V volt.

$$qV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$mv = \sqrt{2mqV}$$

de Broglie wavelength $= \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}$$

Putting $V_\alpha = V_p$, $\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4\times2}{1\times1}} = 2\sqrt{2}$

66. **(4)** The electrons ejected with maximum speed $v_{\max}$ are stopped by electric field $E = 4$ N/C after traveling a distance $d = 1$ m.

$$\frac{1}{2}mv^2_{\max} = eE\,d = 4\ \text{eV}$$

The energy of incident photon $= \frac{1240}{200} = 6.2$ eV

From equation of photoelectric effect $\frac{1}{2}mv^2_{\max} = hv - \phi_0$

$\therefore\quad \phi_0 = 6.2 - 4 = 2.2$ eV

67. **(1)** We have KE $= \frac{p^2}{2m_e} = \frac{hc}{\lambda_{\min}}$

$$p = \sqrt{\frac{2hcm_e}{\lambda_{\min}}}$$

Also, $\lambda_{\text{de Broglie}} = \frac{h}{p} = \sqrt{\frac{h\lambda_{\min}}{2m_e c}}$

For $\lambda_{\min} = 10$ Å,

$\lambda_{\text{de Broglie}} \cong 0.3$ Å

68. **(3)** $K_{\max} = hf - \phi$

$$\Rightarrow \frac{1}{2}mv^2_{\max} = hf - \phi$$

$$\Rightarrow \quad v_{\max} = \sqrt{\frac{2(hf-\phi)}{m}}$$

Hence, (1) is incorrect.

Since $n = (IA/hf)$, therefore rate of emission of electrons is proportional to the intensity (I).

$$K_{\max} = hf - \phi$$

Hence, (3) is true.

69. (2) The intensity of light at the location of your eye is

$$I = \frac{P}{4\pi r^2} = \frac{60}{4\pi \times 4^2}\ \text{W m}^{-2}$$

The energy entering into your eye per second is

$$P_1 = I \times \frac{\pi d^2}{4}$$

where d is the diameter of pupil.

$$P_1 = \frac{60}{4\pi \times 4^2} \times \frac{\pi \times (2\times 10^{-3})^2}{4} = 9.375 \times 10^{-7}\ \text{J s}^{-1}$$

Let n be the number of photons entering into the eye per second, then

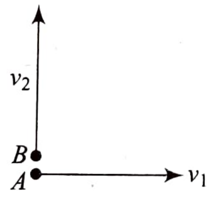
$$P_1 = n \times \frac{hc}{\lambda}$$

$$9.375 \times 10^{-7} = n \times \frac{1240 \times 1.6 \times 10^{-19}}{600}$$

$$n = 2.84 \times 10^{12}\ \text{photon s}^{-1}$$

So, the number of photons entering the eye in 0.1 s = 0.1 n = 2.84×10^{11}

70. (2) Let m be the mass of each particle, then $\lambda_1 = (h/mv_1)$ and $\lambda_2 = (h/mv_2)$, where v_1 and v_2 are the velocities of two particles as shown in the figure.



$$\vec{v}_{CM} = \frac{m\vec{v}_1 + m\vec{v}_2}{2m} = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

Velocity of A w.r.t. C frame is

$$\vec{v}_{1c} = \vec{v}_1 - \vec{v}_{CM} = \frac{\vec{v}_1 - \vec{v}_2}{2}$$

$$|\vec{v}_{1c}| = \frac{\sqrt{\vec{v}_1 - \vec{v}_2}}{2} = |\vec{v}_{2c}|$$

So, required wavelength is

$$\lambda = \frac{h}{m|\vec{v}_{1c}|} = \frac{h}{m} \times \frac{2}{\frac{h}{m}\sqrt{\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

71. (4) $K_{\max} = hf - \phi$

$$= \left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.3 \times 10^{-6} \times 1.6 \times 10^{-19}} - 2.46\right)\text{eV} = 1.68\ \text{eV}$$

Cut-off wavelength, $\lambda_0 = \dfrac{hc}{\phi} = 505$ nm

The minimum energy required to eject the photo-electrons is equal to work function.

72. (1) Let m be the mass of particle.

$$\frac{mv^2}{2} = \frac{hc}{\lambda_{\text{photon}}},$$ where symbols have their usual meanings.

$$\frac{p^2}{2m} = \frac{hc}{\lambda_{\text{photon}}}$$

and $p = \dfrac{h}{\lambda_{\text{particle}}} \Rightarrow \dfrac{h^2}{2m\lambda_{\text{particle}}^2} = \dfrac{hc}{\lambda_{\text{photon}}}$

$$\Rightarrow \quad \frac{\lambda_{\text{photon}}}{\lambda_{\text{particle}}} = \frac{2mc}{h} \times \lambda_{\text{particle}} = \frac{2mc}{h} \times \frac{h}{mv} = \frac{2c}{0.05c} = 40$$

73. (2) Resolving power is proportional to inverse of wavelength,

i.e., $R \propto \dfrac{1}{\lambda}$

and $\lambda \propto \dfrac{1}{p}$

So, $R \propto p = \sqrt{2mE}$

So, $R \propto \sqrt{E}$

$$\frac{R'}{R} = \sqrt{\frac{4\,\text{kV}}{16\,\text{kV}}} = \frac{1}{2}$$

$$R' = \frac{R}{2}$$

74. (4) Speed of electron which enters into electric field may increase or decrease while for second electron, it remains constant.

So, from $\lambda = \dfrac{h}{mv'}$

$\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$

75. (2) $\lambda_1 = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$

$$\lambda_2 = \frac{hc}{E}$$

So, $\dfrac{\lambda_1}{\lambda_2} \propto \dfrac{E}{\sqrt{E}} = E^{1/2}$

76. (1) When the positive potential of the ball is enough to hold back the most energetic photoelectron, the ball will not emit photoelectrons.

$$\frac{hc}{\lambda} - \phi = Ve$$

$$V = \frac{\frac{hc}{\lambda} - \phi}{e}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{200 \times 10^{-9} \times 1.6 \times 10^{-19}} - 3.74$$

$$= 6.216 - 3.74 = 2.5\ \text{V}$$

77. (1) $E = \dfrac{hc}{\lambda}$

Number of photons emitted is $\dfrac{Pt}{\left(\dfrac{hc}{\lambda}\right)} = n_0$

$$n_0 = \frac{P\lambda t}{hc}$$

Since the radiation is spherically symmetric, so total number of photons entering the sensor is n_0 times the ratio of aperture area to the area of a sphere of radius l.

$$N = n_0 \frac{\pi(2d)^2}{4\pi l^2} = \frac{P\lambda t}{hc}\frac{d^2}{l^2}$$

78. (2) $\lambda_{\text{ph}} = \lambda_{\text{e}}$

$$\frac{h}{p_{\text{ph}}} = \frac{h}{p_{\text{e}}}$$

$$\frac{E_{\text{ph}}}{c} = \frac{2E_{\text{e}}}{v}$$

$$\frac{E_{\text{e}}}{E_{\text{ph}}} = \frac{v}{2c}$$

(4) From conservation of linear momentum, both the particles will have equal and opposite momentum. The de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} \Rightarrow \lambda_1/\lambda_2 = 1$$

(1) $mvr = n\frac{h}{2\pi} = 2\left(\frac{h}{2\pi}\right)$ $\quad (n = 2)$

or $\quad mvr = \frac{h}{\pi}$

$\therefore \quad \lambda = \frac{h}{mv}$ = de Broglie wavelength

$$= \pi r = (3.14)\,(2.116\text{ Å}) = 6.64\text{ Å}$$

(2) $h(f_1 - f_2) = e(V_1 - V_2)$

$$\frac{h}{e} = \frac{1}{c}\frac{(V_1 - V_2)}{\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)} = \frac{10^{-9}}{3\times10^{8}}\left(\frac{4.6 - 0.08}{\frac{1}{185} - \frac{1}{546}}\right)$$

$$= \frac{4.42\times185\times546\times10^{-17}}{361\times31}$$

$$= 4.12\times10^{-15}\text{ Js C}^{-1}$$

(4) Momentum of photon of green light,

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}}{505\times10^{-9}} = 1.3\times10^{-27}\text{ kg ms}^{-1}$$

Velocity of bacterium, $v = \frac{p}{m} = \frac{1.3\times10^{-27}}{9.5\times10^{-15}} = 1.368\times10^{-13}\text{ ms}^{-1}$

(1) Energy of dissociation, $E_s = 10^5\text{ J mol}^{-1}$

Photon energy, $E_p = \frac{E_s}{N_a} = \frac{10^5}{6.02\times10^{23}} = 1.66\times10^{-19}\text{ J} = 1.04\text{ eV}$

Multiple Correct Answers Type

1. (1), (2), (3)

Existence of cut-off frequency and photoemission takes place even when intensity is low.

2. (2), (4)

$$eV_0 = K_{\text{max}} = \frac{hc}{\lambda} - W$$

3. (1), (3)

Use Einstein's photoelectric equation.

4. (1), (2)

For photoemission, $\lambda < \lambda_0$.

5. (1), (3), (4)

$$E = \frac{1240}{248}\text{eV}$$

$E_1 = 5 - 2.2 = 2.8$ eV

$E_2 = 5\times0.8 - 2.2 = 1.8$ eV

$E_3 = 5\times0.8^2 - 2.2 = 1$ eV

$E_4 = 5\times0.8^3 - 2.5 = 0.36$ eV

6. (1), (2), (3), (4)

Stopping potential $\propto$ frequency $\equiv \frac{1}{\text{wave length}}$

Saturation current $\propto$ rate of photoelectron emission.

Also, $K_{\text{max}} = h\nu - \phi,\ p = \sqrt{2mK}$

7. (3), (4)

The threshold wavelength is 5200 Å. For ejection of electrons, the wavelength of the light should be less than 5200 Å so that frequency increases and hence the energy of incident photon increases. UV light has less wavelength than 5200 Å.

8. (2), (4)

Since the stopping potential depends on the frequency and not on the intensity and the source is same, the stopping potential remains unaffected. The saturation current depends on the intensity of incident light on the cathode of the photocell which in turn depends on the distance of the source from cathode. The intensity of light is inversely proportional to the square of the distance between the light source and photocell.

Intensity, $I \propto 1/r^2$ and saturation current $\propto I$ (Intensity)

$\Rightarrow$ Saturation current $\propto \frac{1}{r^2}$

$$\Rightarrow \frac{(\text{Saturation current})_{\text{final}}}{(\text{Saturation current})_{\text{initial}}} = \frac{r^2_{\text{initial}}}{r^2_{\text{final}}}$$

$$\Rightarrow (\text{Saturation current})_{\text{final}} = \frac{0.2\times0.2}{0.6\times0.6}\times18 = 2\text{ mA}$$

9. (1), (2), (3)

For metal A: $4.25 = W_A + T_A$ ...(i)

Also, $T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}\frac{m^2v_A^2}{m} = \frac{p_A^2}{2m} = \frac{h^2}{2m\lambda_A^2}$ ...(ii)

$$\left[\because \lambda = \frac{h}{p}\right]$$

For metal B: $4.7 = (T_A - 1.5) + W_B$ ...(iii)

Also, $T_B = \frac{h^2}{2m\lambda_B^2}\times\frac{2m\lambda_A^2}{h^2} = \frac{\lambda_A^2}{\lambda_B^2}$

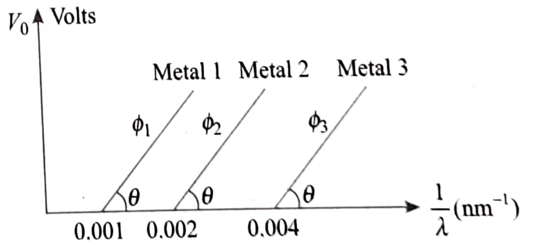
$\Rightarrow \frac{T_A - 1.5}{T_A} = \frac{\lambda_A^2}{(2\lambda_A)^2} = \frac{\lambda_A^2}{4\lambda_A^2} = \frac{1}{4}$ $\quad [\because \lambda_B = 2\lambda_A \text{ given}]$

$\Rightarrow 4T_A - 6 = T_A \Rightarrow T_A = 2$ eV

$\therefore \quad W_A = 2.25$ eV, $W_B = 4.20$ eV, $T_B = 3$ eV

10. (1), (3)

$$\phi_1 : \phi_2 : \phi_3 = eV_{0_1} : eV_{0_2} : eV_{0_3}$$



$$V_{0_1} : V_{0_2} : V_{0_3} = 0.001 : 0.002 : 0.004 = 1:2:4$$

Therefore, option (1) is correct.

By Einstein's photoelectric equation, $\frac{hc}{\lambda} - \phi = \text{eV}$

$\Rightarrow \quad V = \frac{hc}{e\lambda} - \frac{\phi}{e}$ ...(i)

Comparing Eq. (i) by $y = mx + c$, we get the slope of the line

$$m = \frac{hc}{e} = \tan\theta$$

$\Rightarrow$ Option (3) is correct.

From the graph it is clear that, $\frac{1}{\lambda_{0_1}} = 0.001\ \text{nm}^{-1}$

$$\Rightarrow \lambda_{0_1} = \frac{1}{0.001} = 1000\ \text{nm}$$

Also, $\frac{1}{\lambda_{0_2}} = 0.002\ \text{nm}^{-1} \Rightarrow \lambda_{0_2} = 500\ \text{nm}$

and $\lambda_3 = 250$ nm

Violet colour light will have wavelength less than 400 nm. Therefore, this light will be unable to show photoelectric effect on plate.

11. (1), (3), (4)

(a) For the first wavelength: $eV_{s_1} = hf - W$ ...(i)

W = work function

For the second surface: $eV_{s_2} = hf_2 - W$ ...(ii)

Subtracting, we get $V_{s_2} - V_{s_1} = \frac{h}{e}(f_2 - f_1)$

or $$V_{s_2} = V_{s_1} + \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = V_{s_1} + \frac{hc}{e}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right)$$

$$= 0.19 + 1240\left(\frac{550-190}{190 \times 550}\right) = 4.47\ \text{V}$$

(b) From Eq. (i): $W = \frac{hc}{\lambda_1} - eV_{s_1}$

Work function (in eV),

$$\frac{W}{e} = \frac{hc}{e\lambda_1} - V_{s_1} = \frac{1240}{550} - 0.19 = 2.07\ \text{eV}$$

(c) Threshold frequency,

$$f_0 = \frac{W}{h} = \frac{2.07 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 5 \times 10^{14}\ \text{Hz}$$

12. (1), (3), (4)

Let N be the number of photoelectrons per second.

$\therefore$ Intensity = N × energy of one photon $\Rightarrow I = N\frac{hc}{\lambda}$

$\therefore$ Number of incident photons,

$$N_i = \text{Incident intensity} \times \frac{\lambda}{hc}$$

$$= \frac{10^{-8} \times 365 \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^{8}} = 18.35 \times 10^{9}$$

Number of photon N_{ab} absorbed by the surface per unit area per unit time (i.e., absorbed flux),

$$N_{ab} = 0.8 \times 18.35 \times 10^{9} = 1.47 \times 10^{10}\ \text{m}^{-2}\text{s}^{-1}$$

Emitted flux = No. of electrons emitted $\text{m}^{-2}\text{s}^{-1} = 1.47 \times 10^{10}\ \text{m}^{-2}\text{s}^{-1}$

Power absorbed per m^2 = 0.8 × Incident intensity

$$= 0.8 \times 10^{-8} = 8 \times 10^{-9}\ \text{W m}^{-2}$$

From Einstein's photoelectric equation,

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{365 \times 10^{-9}} - 1.6 \times 1.6 \times 10^{-19}$$

$$= 2.89 \times 10^{-19}\ \text{J} = 1.80\ \text{eV}$$

13. (1), (3)

(a) The energy of each photon is

$$E = \frac{hc}{\lambda} = \frac{(4.14 \times 10^{-15}\ \text{eV-s}) \times (3 \times 10^{8}\ \text{m/s})}{500\ \text{nm}}$$

$$= \frac{1242\ \text{eV-nm}}{500\ \text{nm}} = 2.48\ \text{eV}$$

In 1 s, 10 J of energy passes through any cross section of the beam. Thus, the number of photons crossing a cross section per unit time is

$$n = \frac{10\ \text{J}}{2.48\ \text{eV}} = 2.52 \times 10^{19}$$

This is also the number of photons falling on the surface per second and being absorbed.

(b) The linear momentum of each photon is $p = h/\lambda = hf/c$

The total momentum of all the photons falling per second on the surface is

$$\frac{nhf}{c} = \frac{10\ \text{J}}{3 \times 10^{8}\ \text{m/s}} = 3.33 \times 10^{-8}\ \text{N-s}$$

As the photons are completely absorbed by the surface, this much momentum is transferred to the surface per second. The rate of change of the momentum of the surface, i.e., the force on it is

$$F = \frac{dp}{dt} = \frac{3.33 \times 10^{-8}\ \text{Ns}}{1\ \text{s}} = 3.33 \times 10^{-8}\ \text{N}$$

Linked Comprehension Type

For Problems 1–3

1. (2) $\lambda_0 = 3800$ Å

$$W = hf_0 = h\frac{c}{\lambda_0} = \frac{6.633 \times 10^{-34} \times 3 \times 10^{8}}{3800 \times 10^{-10}}$$

$$= 5.23 \times 10^{-19}\ \text{J} = 3.27\ \text{eV}$$

2. (1) Incident wavelength $\lambda = 2600$ Å

$$f = \text{incident frequency} = \frac{3 \times 10^{8}}{2600 \times 10^{-10}}\ \text{Hz}$$

Then, $K_{max} = hf - W_0$

$$hf = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2600 \times 10^{-10}} = 7.65 \times 10^{-19}\ \text{J} = 4.78\ \text{eV}$$

$$K_{max} = hf - W_0 = 4.78\ \text{eV} - 3.27\ \text{eV} = 1.51\ \text{eV}$$

3. (1) $$K_{max} = \frac{1}{2}mv_{max}^2$$

or $$v_{max} = \sqrt{\frac{2K_{max}}{m}} = 72.89 \times 10^{8}\ \text{m s}^{-1}$$

For Problems 4–6

4. (4) Energy of a photon of wavelength λ,

$$E = hf = (hc/\lambda)$$

$$E = \frac{(6.6 \times 10^{-34}) \times (3 \times 10^{8})}{6000 \times 10^{-10}} = 3.3 \times 10^{-19}\ \text{J}$$

So, if n is the number of photons emitted per second,

$$nE = \frac{\text{energy}}{\text{second}} = \text{power}\ (P)$$

Hence, $$n = \frac{P}{E} = \frac{100}{3.3 \times 10^{-19}} \approx 3 \times 10^{20},\ \text{photons s}^{-1}$$

5. (2) Intensity (i.e., energy flux) at a distance of 5 m from the source of power P,

$$I=\frac{P}{4\pi r^2}=\frac{100}{4\pi\times 5^2}=\frac{1}{\pi}\,\mathrm{Wm^{-2}}$$

The number of photons passing normally per unit area per second, i.e.,

$$\text{Photon flux}=\frac{I}{E}=\frac{1/\pi}{3.3\times10^{-19}}\approx 10^{18}\ \text{photons m}^{-2}\,\text{s}^{-1}$$

(3) Energy of photon

$$E=hf=\frac{hc}{\lambda}=\frac{6.6\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}}\approx 5\times10^{-19}\ \text{J}$$

So, number of photons emitted per second by light source,

$$n_p=\frac{P}{E}=\frac{1.5\times10^{-3}}{5\times10^{-19}}=3\times10^{15}$$

Now, as only 0.1% of the photons emit electrons, number of photoelectrons emitted per second,

$$n_e=\frac{0.1}{100}\times3\times10^{15}=3\times10^{12}$$

But current is the rate of flow of charge, i.e.,

$$I=\frac{q}{t}=\frac{Ne}{t}=n_e\times e\qquad\left[\text{as }\frac{N}{t}=n_e\right]$$

So, $I=3\times10^{12}\times1.6\times10^{-19}=0.48\ \mu\text{A}$

Problems 7–8

(3) From Einstein's photoelectric effect equation, $\dfrac{hc}{\lambda}=\phi_0+\dfrac{1}{2}mv^2$

For $\lambda_1=3000$ Å, $\dfrac{hc}{3000\times10^{-10}}=\phi_0+\dfrac{1}{2}m(3v)^2$ ...(i)

For $\lambda_1=6000$ Å, $\dfrac{hc}{6000\times10^{-10}}=\phi_0+\dfrac{1}{2}mv^2$ ...(ii)

Now, on multiplying Eq. (ii) by 9 and subtracting Eq. (i) from it, we get

$$8\phi_0=9\frac{hc}{6000\times10^{-10}}-\frac{hc}{3000\times10^{-10}}$$

Substituting the values, we get $\phi_0=1.23$ eV

(1) Maximum speed of the photoelectrons will be for the incident light of wavelength $\lambda=3000$ Å. From Eq. (i)

$$\frac{1}{2}m(3v)^2=\frac{6.62\times10^{-34}\times3\times10^{8}}{3000\times10^{-10}}-2.896\times10^{-19}$$

$$=3.724\times10^{-19}$$

$$\therefore\quad 3v=v_{\max}=\left(\frac{2\times3.724\times10^{-19}}{9.1\times10^{-31}}\right)^{1/2}=9\times10^{5}\ \text{m s}^{-1}$$

r Problems 9–10

(2) The energy of each photon is $E=hv$

Since the wavelength and frequency are related to the speed of light by

$$c=v\lambda$$

$$E=\frac{hc}{\lambda}=\frac{(6.626\times10^{-34})(3.00\times10^{8})}{632.8\times10^{-9}}=3.14\times10^{-19}\ \text{J}$$

$$1\ \text{eV}=1.602\times10^{-19}\ \text{J}$$

$$E=\frac{3.14\times10^{-19}}{1.602\times10^{-19}}=1.96\ \text{eV}$$

10. (1) The number of photons emitted per second is equal to the energy emitted by the laser each second divided by the energy of one photon.

$$N=\frac{1.00\times10^{-3}}{3.14\times10^{-19}}=3.18\times10^{15}\ \text{photon s}^{-1}$$

For Problems 11–13

11. (1) From Einstein's relation, $eV_s=hf-W$

As work function is a constant for a surface,

$$e(V_{s_2}-V_{s_1})=h(f_2-f_1)$$

$$V_{s_2}=V_{s_1}+\frac{h}{e}(f_2-f_1)=V_{s_1}+\frac{hc}{e}\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)$$

$$=0.19+1240\left(\frac{1}{190}-\frac{1}{550}\right)=4.47\ \text{V}$$

12. (2) $W=\dfrac{hc}{\lambda_1}-eV_{s_1}=\dfrac{1240}{550}-0.19=2.07$ eV

13. (1) $hf_c=W$

$$f_c=\frac{W}{h}=\frac{(2.07)(1.602\times10^{-19})}{6.626\times10^{-34}}\approx 500\times10^{12}\ \text{Hz}$$

For Problems 14–15

14. (4) Energy of photon, $E=\dfrac{hc}{\lambda}=\dfrac{1.24\times10^{3}}{400}=3.1$ eV

Remaining energy = 3.1 – 0.31 = 2.79 eV

Energy lost in first collision is $(3.1)\times\left(\dfrac{10}{100}\right)=0.31$ eV

Remaining energy is 3.1 – 0.31 = 2.79 eV

Energy lost in second collision is $(2.79)\times\left(\dfrac{10}{100}\right)=0.279$ eV

Total energy lost in two collisions is (0.31) + (0.279) eV = 0.589 eV

So, from conservation of energy, we have

$$\frac{hc}{\lambda}=\phi+K_{\max}+\text{energy lost in two collision}$$

$$3.1=2.2+K_{\max}+0.589$$

$$K_{\max}=0.31\ \text{eV}$$

Total energy after second collision is (2.79 – 0.279) = 2.511 eV

15. (3) Energy lost in third collision is $2.511\times\dfrac{10}{100}=0.2511$ eV

Remaining energy = (2.511 – 0.2155) = 2.2599 eV

Energy lost in fourth collision = $\left(2.2599\times\dfrac{10}{100}\right)=0.2259$ eV

Remaining energy = (2.2599 – 0.2259) = 2.034 eV

After the fourth collision, the electron does not have enough energy to overcome the work function, so it cannot come out.

For Problems 16–18

16. (4) If P is the power of point source of light, the intensity at a distance r is

$$I=\frac{P}{4\pi r^2}$$

The energy intercepted by the metallic sphere is

$$E=\text{intensity}\times\text{projected area of sphere}=\frac{P}{4\pi r^2}\times\pi R^2$$

If ε is the energy of the single photon and η the efficiency of the photon to liberate an electron, the number of ejected electrons is

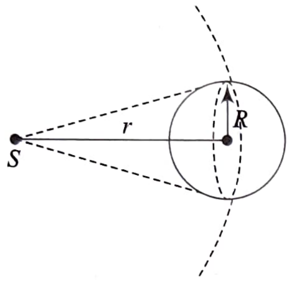
$$\eta\frac{PR^2}{4r^2\varepsilon}=\frac{(10^{-6})(3.2\times10^{-3})(8\times10^{-3})^2}{4\times(0.8)^2\times(5\times1.6\times10^{-19})}=10^5\text{ electron s}^{-1}$$

17. (2) The emission of electrons from a metallic sphere leaves it positively charged. As the potential of the charged sphere begins to rise, it attracts emitted electrons. The emission of electrons will stop when the kinetic energy of the electrons is neutralised by the retarding potential of the sphere. So, we have

$$eV=K_{\text{max}}$$

$$V=\left(\frac{K_{\text{max}}}{e}\right)$$

18. (1)



From Einstein's photoelectric equation,

$$K_{\text{max}}=h\nu-\phi=(5-3)=2\text{eV}$$

The potential of a charged sphere is $V=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}=\frac{1}{4\pi\varepsilon_0}\left(\frac{ne}{R}\right)$

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{ne}{R}\right)=2$$

$$n=\frac{4\pi\varepsilon_0 2R}{e}=\frac{2\times8\times10^{-3}}{9\times10^9\times1.6\times10^{-19}}=1.11\times10^7$$

The photoelectric effect will stop when 1.11 × 10⁷ electrons have been emitted.

The time taken by it to emit 1.11 × 10⁷ electrons,

$$t=\frac{1.11\times10^7}{10^5}=111\text{ s}=1.85\text{ min}$$

For Problems 19–20

19. (2) Pressure exerted by absorbed light $=\frac{1}{2}\left(\frac{S}{c}\right)$

Pressure exerted by reflected light $=\frac{1}{2}\left(\frac{2S}{c}\right)$

Total radiation pressure on the surface is

$$P_{\text{rad}}=\frac{\frac{3}{2}S}{c}=\frac{1.5\times10^3}{3\times10^8}=5\times10^{-6}\text{ Pa}$$

20. (1) $\frac{P_{\text{rad}}}{P_0}=\frac{5\times10^{-6}}{1\times10^5}=5\times10^{-11}$

For Problems 21–25

21. (3) $Q=CV\ \Rightarrow ne=\frac{\varepsilon_0 A}{d}V$

$$n=\frac{2.85\times10^{-12}\times10}{0.5\times10^{-3}\times1.6\times10^{-19}}\times16$$

$$n=8.85\times10^9$$

22. (3) Equivalent resistance $R=\frac{V}{I}=\frac{16\text{ V}}{2\times10^{-6}\text{ A}}=8\times10^6\ \Omega$

23. (4) $P=\frac{nhc}{e\lambda}$

where n = number of photons incident per unit time.

Also, $I=ne$

$$\Rightarrow\quad P=\frac{Ihc}{e\lambda}$$

$$\lambda=\frac{(2\times10^{-6})(6.6\times10^{-34})(3\times10^8)}{(4\times10^{-6})(1.6\times10^{-19})}$$

$$=\frac{9.9}{1.6}\times10^{-7}\text{ m}=\frac{9900}{1.6}\text{Å}=6187\text{ Å}$$

which is in the range of orange light.

24. (3) The range of wavelength for red light is beyond the wavelength of incident light.

25. (2) Stopping potential, $V_s=8$ V

and $\text{KE}=eV_s$

$\therefore\quad \text{KE}=8$ eV

For Problems 26–28

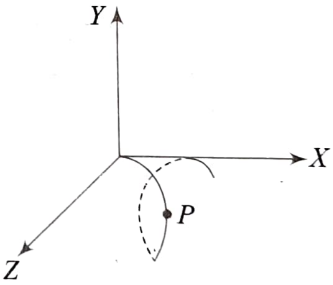
26. (1) At any time t the location of electron is shown as P. In two dimensional view of electron in YZ-plane, the situation is more clear.

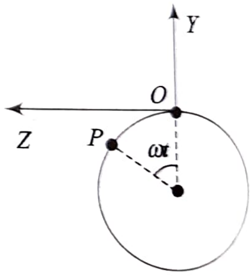
$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(h\nu-\phi)}{m}}$$

$$p=v\cos\theta\frac{2\pi m}{qB_0}$$

$$pqB_0=2\pi\cos\theta\, m\sqrt{\frac{2(hf-\phi)}{m}}=2\pi\cos\theta\sqrt{2m(hf-\phi)}$$

27. (2)





X-coordinate, $x=v\cos\theta\times t$

Y-coordinate, $y=-[R-R\cos\omega t]$

Z-coordinate, $z=R\sin\omega t$

So, $$z=\frac{mv\sin\theta}{qB_0}\times\sin\left[\frac{qB_0}{m}t\right]$$

$$=\frac{\sqrt{2m(hf-\phi)}\times\sin\theta}{qB_0}\times\sin\left[\frac{qB_0t}{m}\right]$$

28. (3) From $x=v\cos\theta\times t=\sqrt{\frac{2(hf-\phi)}{m}}\times\cos\theta\times t$

As f increases, slope of x versus t graph (a straight line) increases.

For Problems 29–31

29. (2) The energy of the incident photon is

$$E=\frac{hc}{\lambda}=\frac{6.6\times10^{-34}\times3\times10^8}{3300\times10^{-10}}=3.75\text{ eV}$$

A will not emit photoelectrons because energy of incident photon is less than work function of A.

(3) $eV_0 = h(f - f_0)$

When $V_0 = 0, f = f_0$, the threshold frequency.

From the graph it follows that $f_0 = 4 \times 10^{15}$ Hz

Therefore, work function is $\phi = hf_0 = 6.6 \times 10^{-34} \times 4 \times 10^{15}$

$$= 16.5 \text{ eV}$$

(1) $eV_0 = hf - hf_0$

$$V_0 = \frac{h}{e}f - \frac{h}{e}f_0$$

$$y = mx + c$$

$$\text{Slope} = \frac{h}{e}$$

Problems 32–34

(3) $\frac{1}{2}mv^2 = eV$

$$v = \sqrt{\frac{2eV}{m}}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}}$$

$$\text{Fringe width } \beta = \frac{\lambda D}{d} = \frac{hD}{d\sqrt{2meV}}$$

(2) As v increases, λ decreases.

(3) Central maxima will remain at the same position.

Problems 35–37

(2) Energy required, $E = ms\,\Delta\theta + mL$

$$= 2 \times 10^{-9} \times 4.2 \times 10^3 \times 70 + 2 \times 10^{-9} \times 2.25 \times 10^6$$

$$= 5.088 \times 10^{-3} \text{ J}$$

(2) Power output of the laser,

$$P = \frac{E}{t} = \frac{5.088 \times 10^{-3}}{450 \times 10^{-6}} = 11.3 \simeq 11 \text{ W}$$

(1) $N = \frac{E}{hv} = \frac{E\lambda}{hc} = \frac{(3.18 \times 10^{16} \text{ eV})(585 \text{ nm})}{(1240 \text{ eV nm})} = 1.5 \times 10^{16}$

Problems 38–39

(4) $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$ where E = kinetic energy

$$\frac{1}{\lambda_p} : \frac{1}{\lambda_d} : \frac{1}{\lambda_\alpha} = 1 : \frac{1}{\sqrt{2}} : \frac{1}{2} = 2 : \sqrt{2} : 1$$

39. (1) $\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{150}}\,\text{Å} = 1\,\text{Å}$

Matrix Match Type

1. i. → a., c.; ii. → b., d.; iii. → d.; iv. → a., c.

(i) From $eV_0 = hf - \phi$ and $K_{\max} = hf - \phi$

If f increases keeping ϕ constant, then V_0 and $K_{\max}$ increase.

(ii) If I increases, more photons/time are incident on the metal surface and more photoelectrons would be liberated. Hence, saturation photocurrent increases. Stopping potential and $K_{\max}$ will remain the same.

(iii) If separation between cathode and anode is increased, then there is no effect on V_0, $K_{\max}$ or current.

(iv) If ϕ decreases keeping f and I constant, then V_0 and $K_{\max}$ increase.

2. i. → d.; ii. → a., b., c., d.; iii. → b., c., d.; iv. → a., c., d.

(i) Maximum kinetic energy of ejected electron is given by Einstein's photoelectric equation $K_{\max} = hf - (hf/3) = 2hf/3$. As no potential difference is applied across target and collector and vaccum is there in the tube, so this maximum KE remains same at all locations.

(ii), (iii) and (iv) Kinetic energy of ejected photoelectrons can be anything from 0 to $K_{\max}$ (as found for (i)). It remains the same at all locations (reasoning is same as for above).

3. i. → c.; ii. → a., b.; iii. → a., b.; iv. → d.

(i) If intensity changes, then number of photons/time incident on the metal surface change and hence number of photoelectrons liberated change, so saturation photocurrent changes. Stopping potential and $K_{\max}$ will remain the same.

(ii) From $eV_0 = hf - \phi$ and $K_{\max} = hf - \phi$

If f changes, then V_0 and $K_{\max}$ change.

(iii) From $eV_0 = hf - \phi$ and $K_{\max} = hf - \phi$

If target material changes, then ϕ changes, then V_0 and $K_{\max}$ change.

(iv) If we change the potential difference between emitter and collector, then time taken for electrons to eject changes.

4. i. → a., c.; ii. → a., c.; iii. → b.; iv. → d.

Consider two equations.

$$eV_s = \frac{1}{2}mv^2_{\max} = hf - \phi_0 \quad \text{...(i)}$$

Number of photoelectrons ejected per second ∝ intensity ...(ii)

(i) As frequency is increased keeping intensity constant, V_s will increase and hence, $1/2m(v^2_{\max})$ will increase.

(ii) As frequency is increased and intensity is decreased, V_s will increase and hence $1/2m(v^2_{\max})$ will increase and saturation current will decrease.

(iii) It work function is increased, photoemission may stop.

(iv) If intensity is increased, then saturation current will increase.

5. i. → a., b., c., d.; ii. → a., b., c., d.; iii. → a, c.; iv. → a.

(i) Bichromatic light source is having two wavelengths and hence from energy point of view, two types of photons are possible. As light propagates in different directions, the photon can have different or same momenta depending upon the magnitude and direction of photon motion.

(ii) Same reasoning as for (i).

(iii) For a monochromatic light source, all photons have same energy but momenta can be different due to different directions.

(iv) Laser is a very narrow beam of monochromatic light, so all photons have nearly same energy and momenta.

Numerical Value Type

1. (1) $r \propto \frac{p}{q}$ $\left(\text{since, } r = \frac{p}{Bq}\right)$

where p = momentum, Given $r_\alpha = \frac{1}{2}r_e$

$$\frac{p_\alpha}{B2e} = \frac{1}{2}\left(\frac{p_e}{Be}\right)$$

or $\quad p_\alpha = p_e$

$$\lambda \propto \frac{1}{p} \quad \left(\text{since, } \lambda = \frac{h}{p}\right)$$

So, $\lambda_\alpha = \lambda_e$

or $n = 1$

2. (4) Speed of photon (c) = 3×10^8 ms^{-1}. Let λ be the wavelength of the photon. The de Broglie wavelength of the electron is h/mv.

Given $\lambda = \frac{h}{mv}$. Now,

$$\frac{\text{KE of photon}}{\text{KE of electron}} = \frac{hf}{(1/2)mv^2} = \frac{2hc}{mv^2\lambda} = \frac{2c}{v} = \frac{2 \times 3 \times 10^8}{1.5 \times 10^8} = 4$$

$$\left(\because \lambda = \frac{h}{mv}\right)$$

3. (5) For K_α X-ray, $(Z-1)^2\lambda$ = constant.

Hence, $(9-1)^2\lambda = (Z-1)^2(4\lambda)$

$$(Z-1)^2 = \frac{64}{4} = 16$$

$$Z - 1 = 4$$

or $Z = 5$

4. (4) The energy of each photon $= \frac{200\ \text{J/s}}{4 \times 10^{20}/\text{s}} = 5 \times 10^{-19}$ J

$$\text{Wavelength} = \lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\ \text{J-s}) \times (3 \times 10^8\ \text{m/s})}{(5 \times 10^{-19}\ \text{J})}$$

$$= 4.0 \times 10^{-7}\ \text{m}$$

5. (1) $\lambda = 663 \times 10^{-9}$m, $\theta = 60°$, $n = 1.0 \times 10^{19}$

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{6.63 \times 10^{-9}} = 10^{-27}$$

Force exerted on the wall is

$$n \times 2 \times p \cos\theta = 2 \times 1 \times 10^{19} \times 10^{-27} \times \frac{1}{2} = 1 \times 10^{-8}\ \text{N}$$

6. (1) Let n photons (each of frequency f) per second are emitted from source. Then power of source is $P = nhf$

But only 30% of the photons go towards mirrors.

Then force exerted on mirror is

$$F = 2\left[\frac{30}{100}n\right]\frac{h}{\lambda} = \frac{3}{5}\frac{nhf}{c} = \frac{3}{5}\frac{P}{c}$$

and this force should be equal to weight of mirror,

so $\frac{3}{5}\frac{P}{c} = 20 \times 10^{-3} g$

$$\Rightarrow \quad P = \frac{5 \times 3 \times 10^8 \times 20 \times 10^{-3} \times 10}{3} = 10^8\ \text{W}$$

7. (3) Given $\lambda = 200$ nm $= 2 \times 10^{-7}$ m

Energy of one photon is

$$\frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7}} = 9.945 \times 10^{-19}$$

$$\text{Number of photons is } \frac{1 \times 10^{-7}}{9.945 \times 10^{-19}} = 1 \times 10^{11}$$

Hence, number of photoelectrons emitted is

$$\frac{1 \times 10^{11}}{10^4} = 1 \times 10^7$$

Net amount of +ve charge 'q' developed due to the outgoing electrons $= 1 \times 10^7 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-12}$ C.

Now potential developed at the centre as well as at the surface due to these charges is

$$\frac{Kq}{r} = \frac{9 \times 10^9 \times 1.6 \times 10^{-12}}{4.8 \times 10^{-2}} = 3 \times 10^{-1}\ \text{V} = 0.3\ \text{V}$$

8. (9) Given: Fringe width, $y = 1.0$ mm $\times 2 = 2.0$ mm

$d = 0.24$ mm, $W_0 = 2.2$ eV, $D = 1.2$ m

$$y = \frac{\lambda D}{d}$$

or $$\lambda = \frac{yd}{D} = \frac{2 \times 10^{-3} \times 0.24 \times 10^{-3}}{1.2} = 4 \times 10^{-7}\ \text{m}$$

$$E = \frac{hc}{\lambda} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{4 \times 10^{-7}} = 3.105\ \text{eV}$$

Stopping potential $eV_0 = 3.105 - 2.2 = 0.905$ eV

$$V_0 = \frac{0.905}{1.6 \times 10^{-19}} \times 1.6 \times 10^{-19}\ \text{V} = 0.905\ \text{V}$$

9. (2) Momentum of electron, $p_e = \sqrt{2meV}$

Momentum of particle, $p_p = \sqrt{2\frac{m}{4} \cdot eV}$

$$\therefore \quad \frac{\lambda_p}{\lambda_e} = \frac{h/p_p}{h/p_e} = \sqrt{\frac{m}{m/4}} = 2$$

10. (38) Incident energy of photons $= \frac{hc}{200} = 6.2$ eV

$$6.2 = K_1 + W_0, K_1 = 6.2 - W_0$$

$$K_2 = K_1 + 10e = 16.2 - W_0$$

$$K_2 = \frac{hc}{\lambda_{\min}} = 12.4 \Rightarrow W_0 = 3.8\text{eV, given } W_0 = \frac{x}{10}$$

$$\therefore x = 38$$

11. (2.00)

$K_{\max} = (5 - \phi)$ eV

When electrons are accelerated through 5 V, they will reach the anode with energy $= (5 - \phi + 5)$ eV

$\therefore \quad 10 - \phi = 8$

or $\quad \phi = 2$ eV

12. (0.50)

Momentum corresponding to incident photons normal to the surface,

$$\left(\frac{dp}{dt}\right)_{\text{incident}} = \frac{I}{c}dA\cos^2\theta$$

Since refection coefficient is ρ, so the momentum of the reflected photons per second normal to surface,

$$\left(\frac{dp}{dt}\right)_{\text{reflected}} = \frac{I}{c}dA\rho\cos^2\theta$$

Hence, rate of change of momentum of the photons,

$$\left(\frac{dp}{dt}\right)_{\text{photons}} = -\frac{I}{c}dA(\rho - 1)\cos^2\theta$$

$$\text{From Newton's third law, } F = \left(\frac{dp}{dt}\right)_{\text{surface}} = \frac{I}{c}dA(\rho + 1)\cos^2\theta$$

$$\text{Hence, pressure exerted on surface, } P = \frac{dF}{dA} = \frac{I}{c}(\rho + 1)\cos^2\theta$$

On substituting values, we get $P = 0.5$ N cm^{-2}

(0.148)

Maximum kinetic energy of ejected electrons

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{hc}{\lambda} - W$$

$$\therefore\ K_{\max} = \frac{(6.62\times10^{-34})(3\times10^8)}{(180\times10^{-9})} - (2\times1.6\times10^{-19})$$

$$= 7.8\times10^{-19}\ \text{J}$$

$$\therefore\ v_{\max} = \sqrt{\left(\frac{2K_{\max}}{m}\right)} = \sqrt{\left(\frac{2\times(7.8\times10^{-19})}{1.3\times10^6}\right)} = 1.3\times10^6\ \text{m s}^{-1}$$

$$\text{Now, } Bev_{\max} = \frac{mv_{\max}^2}{r}$$

$$\Rightarrow\ r = \frac{mv_{\max}}{Be} = \frac{(9.1\times10^{-31})(1.3\times10^6)}{(5\times10^{-5})(1.6\times10^{-19})} = 0.148\ \text{m}$$

(0.63)

de-Broglie wavelength, $\lambda = \dfrac{h}{p}$

Mean KE of helium atom $= \dfrac{3kT}{2}$

$$\text{or}\quad E = \frac{3\times(1.38\times10^{-23})\times400}{2}$$

In terms of momentum, the energy is given by $E = \dfrac{p^2}{2m}$

or $\quad p^2 = 2mE$

or $\quad p = \sqrt{2mE}$

$$\therefore\quad p = \sqrt{2\times(4.002\times1.66\times10^{-27})\times3\times(1.38\times10^{-23})\times200}$$

(where mass of helium 4.002 amu $= 4.002 \times 1.66 \times 10^{-27}$ kg)

Now, $\lambda = \dfrac{6.625\times10^{-34}}{p}$

Substituting the value of p and solving it, we get

$$\lambda = 0.63\times10^{-10}\,\text{m}$$

5. **(450)**

We know that, de-Broglie wavelength

$$\lambda = \frac{h}{mv} \quad \text{and} \quad E = \frac{1}{2}mv^2$$

$$\therefore\ \lambda = \frac{h}{\sqrt{2mE}}$$

In first case, $100\times10^{-12} = \dfrac{h}{\sqrt{2mE_1}}$ ... (i)

In second case, $50\times10^{-12} = \dfrac{h}{\sqrt{2mE_2}}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get $2 = \sqrt{\left(\dfrac{E_2}{E_1}\right)}$

or $\quad E_2 = 4E_1$

So, energy to be added $= 4E_1 - E_1 = 3E_1$

$$\text{Now,}\ \frac{h}{\sqrt{2mE_1}} = 100\times10^{-12}$$

$$\text{or}\quad \sqrt{2mE_1} = \frac{6.625\times10^{-34}}{10^{-10}}$$

$$\text{or}\quad \sqrt{2mE_1} = 6.625\times10^{-24}$$

$$\text{or}\quad E_1 = \frac{\left(6.625\times10^{-24}\right)^2}{2\times\left(9.1\times10^{-31}\right)} = 150\ \text{eV}$$

Therefore, energy added $= 3E_1 = 450$ eV

16. (23.55)

Total number of ions in the rod

$=$ number of ions per unit volume $\times$ volume of rod

$= (2 \times 10^{25}) \times \{3.14 \times (0.005)^2 \times (5 \times 10^{-2})\}$

$= 7.85 \times 10^{19}$

The number of photons excited in one direction is equal to the total number of ions because all ions are excited. Now, excited energy = number of excited photons × energy of photon

$$= \left(7.85\times10^{19}\right)\times\frac{hc}{\lambda}$$

$$= \left(7.85\times10^{19}\right)\times\frac{\left(6.6\times10^{-34}\right)\times\left(3\times10^8\right)}{6.6\times10^{-7}}$$

$$= 23.55\ \text{W}$$

Archives

JEE Main

Single Correct Answer Type

1. (2) $\dfrac{1}{2}mv^2 = 1.68$ eV

$$hv = \frac{hc}{\lambda} = \frac{1240\ \text{eV nm}}{400\ \text{nm}} = 3.1\ \text{eV}$$

$$\Rightarrow\quad 3.1\ \text{eV} = W_0 + 1.68\ \text{eV}$$

$$\therefore\quad W_0 = 1.42\ \text{eV}$$

2. (4) Since the frequency of ultraviolet light is less than the frequency of X-rays, the energy of each incident photon will be more for X-rays.

KE of photoelectron $= hv - \varphi$

Stopping potential stops the fastest photoelectron.

$$V_0 = \frac{hv}{e} - \frac{\varphi}{e}$$

So $K_{\max}$ and V_0 both increase.

But KE ranges from zero to $K_{\max}$ because of loss of energy due to subsequent collisions before getting ejected and not due to the range of frequencies in the incident light.

3. (1) Here, power of the source, $P = 4$ kW $= 4 \times 10^3$ W

Number of photons emitted per second, $N = 10^{20}$

Energy of photon, $E = hf = \dfrac{hc}{\lambda}$ ...(i)

$\therefore$ Also energy of photon, $E = \dfrac{P}{N}$ ...(ii)

Equating Eqs. (i) and (ii), $\dfrac{hc}{\lambda} = \dfrac{P}{N}$

$$\text{or}\quad \lambda = \frac{Nhc}{P} = \frac{10^{20}\times6.63\times10^{-34}\times3\times10^8}{4\times10^3}$$

$$= 4.972 \times 10^{-9}\text{m} = 49.72\ \text{Å}$$

It lies in the X-ray region.

4. (3) $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

$$U(x = \infty) = 0$$

$$F = -\frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$$

At equilibrium, $F = 0$.

$$x^6 = \frac{2a}{b}$$

$$U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$$

$$D = [U(x=\infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

5. **(4)** The maximum kinetic energy of the electron $K_{\max} = hf - hf_0$

Here, f_0 is threshold frequency.

The stopping potential is $eV_0 = K_{\max} = hf - hf_0$

Therefore, if v is doubled $K_{\max}$ and V_0 is not doubled.

6. **(3)** Davisson–Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystals. This shows the wave nature of electrons as waves can exhibit interference and diffraction.

7. **(3)** As λ is increased, there will be a value of λ above which photoelectrons will be ceased to come out. So photocurrent will become zero. Hence, (3) is the correct answer.

8. **(3)** Franck– Hertz experiment—Discrete energy level.

Photoelectric effect—Particle nature of light

Davison–Germer experiment—Diffraction of electron beam.

9. **(1)** $K_{\max}$ for fastest emitted electron

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi \qquad \text{...(i)}$$

$$\frac{1}{2}m(v')^2 = \frac{hc}{\frac{3\lambda}{4}} - \phi$$

$$\frac{1}{2}m(v')^2 = \frac{4}{3}\frac{hc}{\lambda} - \phi \qquad \text{...(ii)}$$

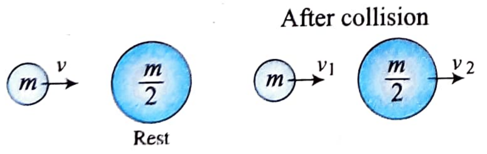
From Eqs. (i) and (ii)

$$\frac{1}{2}m(v')^2 = \frac{4}{3}\left(\frac{1}{2}mv^2 + \phi\right) - \phi$$

$$\frac{1}{2}m(v')^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}$$

or $$\frac{1}{2}m(v')^2 > \frac{4}{3}\left(\frac{1}{2}mv^2\right) \Rightarrow v' > \sqrt{\frac{4}{3}}v$$

10. **(4)**



By conservation of linear momentum $mv = mv_1 + \frac{m}{2}v_2$

$$2v = 2v_1 + v_2 \qquad \text{...(i)}$$

By law of collision $e = \frac{v_2 - v_1}{u_1 - u_2}$

$$\Rightarrow \quad u = v_2 - v_1 \qquad \text{...(ii)}$$

By Eqs. (i) and (ii) $v_1 = \frac{v}{3}$; $v_2 = \frac{4v}{3}$

$$\lambda_A = \frac{h}{p_1};\ \lambda_B = \frac{h}{p_2} \Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{2}{1}$$

Option (4) is correct.

JEE Advanced

Single Correct Answer Type

1. **(2)** $t = 100 \times 10^{-9}$ s, $P = 30 \times 10^{-3}$ W, $c = 3 \times 10^8$ m s^{-1}

$$\text{Momentum} = \frac{Pt}{c} = \frac{30 \times 10^{-3} \times 1000 \times 10^{-9}}{3 \times 10^8}$$

$$= 1.0 \times 10^{-17} \text{ kg m s}^{-1}$$

2. **(1)** $\frac{K_1}{K_2} = \left(\frac{u_1}{u_2}\right)^2 = 4$

$$K_1 = \frac{hc}{\lambda} - \phi = \frac{1240}{248} - \phi$$

$$K_1 = (5 - \phi)$$

$$K_2 = \frac{1240}{310} - \phi$$

$$K_2 = (4 - \phi)$$

$$\frac{K_1}{K_2} = \left(\frac{5-\phi}{4-\phi}\right) = 4 \Rightarrow 5 - \phi = 4(4 - \phi)$$

$$\Rightarrow \quad 5 - \phi = 16 - 4\phi$$

$$\Rightarrow \quad 3\phi = 11 \Rightarrow \phi = \frac{11}{3} = 3.7 \text{ eV}$$

3. **(2)** $K_{\max} = \frac{hc}{\lambda} - \phi$

$$eV_s = \frac{hc}{\lambda} - \phi$$

$$1.6 \times 10^{-19} \times 2 = \frac{h \times 3 \times 10^8}{3000 \times 10^{-10}} - \phi \qquad \text{...(i)}$$

$$1.6 \times 10^{-19} \times 1 = \frac{h \times 3 \times 10^8}{4000 \times 10^{-10}} - \phi \qquad \text{...(ii)}$$

From Eq. (ii), $\phi = \frac{h \times 3 \times 10^8}{4000 \times 10^{-10}} - 1.6 \times 10^{-19}$

$$1.6 \times 10^{-19} \times 2 = \frac{h \times 3 \times 10^8}{3000 \times 10^{-10}} - \frac{h \times 3 \times 10^8}{4000 \times 10^{-10}} + 1.6 \times 10^{-19}$$

$$1.6 \times 10^{-19} = \frac{h \times 3 \times 10^8}{10^{-7}}\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{h \times 3 \times 10^8}{10^{-7}}\left[\frac{4-3}{12}\right]$$

$$1.6 \times 10^{-19} = \frac{h \times 3 \times 10^8}{10^{-7}} \times \frac{1}{12}$$

$$1.6 \times 4 \times \frac{10^{-19} \times 10^{-7}}{10^8} = h$$

$$6.4 \times 10^{-34} \text{ Js} = h$$

4. **(2)** λ_e = Minimum de Broglie wave length

$$\frac{hc}{\lambda_{ph}} = \phi + \frac{hc}{\lambda_e} + eV$$

$$hc\left(\frac{-1}{\lambda_{ph}^2}d\lambda_{ph}\right) = hc\left(-\frac{1}{\lambda_e^2}d\lambda_e\right)$$

$$\frac{d\lambda_{ph}}{d\lambda_e} = \frac{\lambda_{ph}^2}{\lambda_e^2}$$

$$V >> \frac{\phi}{e}$$

Energy of electron $= eV$

$$\frac{p^2}{2m} = E \qquad \lambda = \frac{h}{p} = \frac{1}{\sqrt{2mE}}$$

$$p = \sqrt{2mE}$$

(1) According to photoelectric effect equation:

$$K_{\max}=\frac{hc}{\lambda}-\phi_0$$

$$\Rightarrow \quad \frac{p^2}{2m}=\frac{hc}{\lambda}-\phi_0$$

$$\Rightarrow \quad \frac{(h/\lambda_d)^2}{2m}=\frac{hc}{\lambda}-\phi_0$$

Assuming small changes, differentiating both sides,

$$\frac{h^2}{2m}\left(-\frac{2}{\lambda_d^3}\right)d\lambda_d=-\frac{hc}{\lambda^2}d\lambda \Rightarrow \frac{d\lambda_d}{d\lambda}\propto\frac{\lambda_d^3}{\lambda^2}$$

ed Comprehension Type

(1) $a=\frac{n\lambda}{2} \quad \Rightarrow \quad \lambda=\frac{2a}{n}$

$$\lambda_{\text{de Broglie}}=\frac{h}{p}$$

$$\frac{2a}{n}=\frac{h}{p} \quad \Rightarrow \quad p=\frac{nh}{2a}$$

$$E=\frac{p^2}{2m}=\frac{n^2h^2}{8a^2m}$$

$$\Rightarrow \quad E\propto 1/a^2$$

(2) $E=\frac{h^2}{8a^2m}=\frac{(6.6\times10^{-34})^2}{8\times(6.6\times10^{-9})^2\times10^{-30}\times1.6\times10^{-19}}=8\text{ meV}$

(4) $mv=\frac{nh}{2a}$

or $\quad v=\frac{nh}{2am}$

$$\Rightarrow \quad v\propto n$$

merical Value Type

. (3) de-Broglie wavelength, $\lambda=\frac{h}{\sqrt{2mK}}$

$$\lambda=\frac{h}{\sqrt{2mqV}} \qquad [\therefore \text{ Kinetic energy, } K=qV]$$

$$\lambda_p=\frac{h}{\sqrt{2m_pq_pV}},\ \lambda_\alpha=\frac{h}{\sqrt{2m_\alpha q_\alpha V}}$$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha}=\frac{h}{\sqrt{2m_pq_pV}}\times\frac{\sqrt{2m_\alpha q_\alpha V}}{h}$$

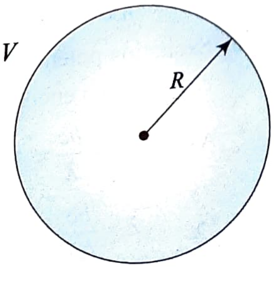
$$=\frac{\sqrt{m_\alpha q_\alpha}}{\sqrt{m_pq_p}}=\frac{\sqrt{(4m_p)(2q_p)}}{\sqrt{(m_pq_p)}}=\sqrt{8}=2\sqrt{2}\approx3$$

2. (7) $R=1$ cm

$f=4.7$ cm

$$\frac{hc}{\lambda}=\phi+eV$$

$$\frac{1240(eV)(\text{nm})}{200(\text{nm})}=4.7\ (\text{eV})+eV$$



$$\frac{1240}{200}e=4.7e+eV$$

$$6.2-4.7=V$$

$$\therefore \quad V=1.5\text{ volt}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}=1.5$$

$$(9\times10^9)\frac{Ne}{\frac{1}{100}}=1.5$$

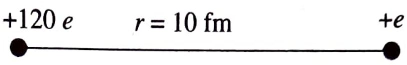
$$9\times10^{11}Ne=1.5$$

$$N=\frac{1.5}{9\times10^{11}\times1.6\times10^{-19}}=\frac{15}{16}\times\frac{1}{9}\times10^8$$

$$=\frac{5}{3\times16}\times10^8=\frac{50}{48}\times10^7$$

$$\therefore \quad Z=7$$

3. (7)



$$\frac{(9\times10^9)(120e)(e)}{10\times10^{-15}}=\frac{p^2}{2m}$$

$$\lambda=\frac{h}{p}$$

$$\lambda^2=\frac{h^2}{p^2}=\frac{h^2\times10\times10^{-15}}{2m\times9\times10^{-9}\times120e^2}$$

$$=\frac{4.2\times4.2\times10^{-30}\times10\times10^{-15}}{2(5/3)\times10^{-27}\times9\times10^{-9}\times120}$$

$$=\frac{42\times42}{36}\times10^{-30}$$

$$\Rightarrow \quad \lambda=7\times10^{-15}\text{ m}=7\text{ fm}$$

4. (1) According to Einstein's photoelectric equation, $K_{\max}=h\nu-\phi_0$ where the symbols have their usual meaning.

But $K_{\max}=eV_s$ where V_s is the stopping potential $V_s=\frac{h}{e}\nu-\frac{\phi_0}{e}$

Thus, the graph between $V_s-\nu$ is a straight line.

Compare the above relation with $y=mx+C$

$\therefore \quad$ Slope of $V_s-\nu$ graph $=\frac{h}{e}$

It is same for both the metals.

$\therefore \quad$ Ratio of the slopes = 1

5. (24) Power $=nhf$, $n=$ number of photons per second $=\frac{200\text{ W}}{6.25\text{ eV}}$

$$=\frac{200}{1.6\times10^{-19}\times6.25}=\frac{200}{6.25\times1.6\times10^{-19}}$$

As photon is just above threshold frequency, $\text{KE}_{\max}$ of the electron released is zero and they are accelerated by potential difference of 500 V.

$$\frac{p^2}{2m}=q\Delta V \Rightarrow \qquad p=\sqrt{2mq\Delta V}$$

Since efficiency is 100%, number of electrons = number of photons per second. As photon is completely absorbed force exerted $F=nmv=np$.

$$\text{Hence, } F=\frac{200}{6.25\times1.6\times10^{-19}}\times\sqrt{2(9\times10^{-31})\times1.6\times10^{-19}\times500}$$

$$=\frac{3\times200\times10^{-25}\sqrt{1600}}{6.25\times1.6\times10^{-19}}$$

$$=\frac{2\times40}{6.25\times1.6}\times10^{-4}\times3=24.00$$

Chapter 6

Concept Application Exercises

Exercise 6.1

1. **(b)** Because atom is hollow and whole mass of atom is concentrated in a small centre called nucleus.
2. **(d)** α-particles cannot be attracted by the nucleus.
3. **(a)** According to scattering formula

$$N \propto \frac{1}{\sin^4(\theta/2)} \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin(\theta_1/2)}{\sin(\theta_2/2)}\right]^4$$

$$\Rightarrow \quad \frac{N_2}{N_1} = \left[\frac{\sin\frac{90^\circ}{2}}{\sin\frac{60^\circ}{2}}\right]^4 = \left[\frac{\sin 45^\circ}{\sin 30^\circ}\right]^4$$

$$\Rightarrow \quad N_2 = (\sqrt{2})^4 \times N_1 = 4 \times 56 = 224$$

4. **(c)** At closest distance of approach,

Kinetic energy = Potential energy

$$\Rightarrow \quad 5\times10^6 \times 1.6\times10^{-19} = \frac{1}{4\pi\varepsilon_0} \times \frac{(Ze)(2e)}{r}$$

For uranium $Z = 92$, so $r = 5.3\times10^{-12}\,\text{cm}$

5. At the distance r_0 of closest approach,

KE of a proton = PE of proton and the gold nucleus

$$K = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze\cdot e}{r_0} \qquad [q_1 = Ze,\ q_2 = e]$$

Or $\quad r_0 = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Ze^2}{K}$

But $\quad K = 5.0\,\text{MeV} = 5.0\times1.6\times10^{-13}\,\text{J}$

For gold, $Z = 79$

$$\therefore \quad r_0 = \frac{9\times10^9\times79\times(1.6\times10^{-19})^2}{5.0\times1.6\times10^{-13}}$$

$$= 2.28\times10^{-14}\,\text{m} \simeq 2.3\times10^{-14}\,\text{m}$$

6. Here $r_0 = 41.3\,\text{fm} = 41.3\times10^{-15}\,\text{m}, Z = 79$

$$K = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{r_0}$$

$$= \frac{9\times10^9\times2\times79\times(1.6\times10^{-19})^2}{41.3\times10^{-15}}\,\text{J}$$

$$= 8.814\times10^{-13}\,\text{J} = \frac{8.814\times10^{-13}}{1.6\times10^{-13}}\,\text{MeV} = 5.51\,\text{MeV}$$

Exercise 6.2

1. The general expression for the radius of the electron orbit is given by

$$r = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}$$

Since $\dfrac{\varepsilon_0 h^2}{\pi m e^2}$ = a constant, for the same radius, $\dfrac{n^2}{Z}$ = constant.

$$\therefore \quad \frac{n_1^2}{Z_1} = \frac{n_2^2}{Z_2} \Rightarrow n_2^2 = n_1^2 \times \frac{Z_2}{Z_1}$$

For hydrogen, $n_1 = 1$ (ground state), $Z_1 = 1$ and for beryllium, $Z_2 = 4$.

Thus, $n_2^2 = (1)^2 \times 4/1 \Rightarrow n_2 = 2$

Further, the general expression for energy is $E_n = -13.6\dfrac{Z^2}{n^2}(\text{eV})$

Thus, $\quad \dfrac{E_2}{E_1} = \dfrac{Z_2^2/n_2^2}{Z_1^2/n_1^2}$

or $\quad \dfrac{E_2}{E_1} = \left(\dfrac{Z_2}{Z_1}\right)^2\left(\dfrac{n_1}{n_2}\right)^2 = \left(\dfrac{4}{1}\right)^2\left(\dfrac{1}{2}\right)^2 = 4$

2. As $E_n = -13.6\dfrac{Z^2}{n^2}$, for the same energy $\dfrac{Z^2}{n^2}$ = constant,

i.e., $\quad \dfrac{Z_1^2}{n_1^2} = \dfrac{Z_2^2}{n_2^2}$

For hydrogen (in the ground state), $Z_1 = 1, n_1 = 1$.

For Li^{++}, $Z_2 = 3$

Thus, $\quad \dfrac{1^2}{1^2} = \dfrac{(3)^2}{n_2^2}$

or $\quad n_2 = 3$

Further, as $r \propto \dfrac{n^2}{Z}, \dfrac{r_1}{r_2} = \dfrac{n_1^2/Z_1}{n_2^2/Z_2}$

Or $\quad \dfrac{r_1}{r_2} = \dfrac{(1)^2/1}{(3)^2/3} = \dfrac{1}{3}$ whence, $\dfrac{r_2}{r_1} = 3$

where the subscripts 1 and 2 stand for hydrogen and Li^{++} respectively.

3. Energy of photon, $hf = E_2 - E_1$

Frequency, $\quad f = \dfrac{E_2 - E_1}{h} = \dfrac{[-3.4-(-13.6)]\times1.6\times10^{-19}}{6.6\times10^{-34}}$

$$= \frac{10.2\times1.6\times10^{15}}{6.6} = 2.47\times10^{15}\,\text{Hz}$$

4. Given $E_1 = -13.6\,\text{eV}$

For third excited state, $n = 4$

$$\therefore \quad E_4 = \frac{E_1}{4^2} = -\frac{13.6}{16}\,\text{eV} = -0.85\,\text{eV}$$

$$\therefore \quad \text{KE} = -E_4 = 0.85\text{ eV}$$

and $\quad \text{PE} = 2E_4 = -1.7\,\text{eV}$

$$\Delta E = E_4 - E_1 = [-0.85-(-13.6)]\,\text{eV} = 12.75\,\text{eV}$$

$$\therefore \quad f = \frac{\Delta E}{h} = \frac{12.75\times1.6\times10^{-19}}{6.63\times10^{-34}} = 3\times10^{15}\,\text{Hz}$$

5. $E_n - E_1 = hf$

or $\quad E_n = hf + E_1 = 12.09 - 13.6 = -1.51\,\text{eV}$

As $\quad E_n = \dfrac{E_1}{n^2} \quad \therefore n^2 = \dfrac{E_1}{E_n} = \dfrac{-13.6}{-1.51} \simeq 9$

or $\quad n = 3$

6. Excitation potential = 2.1 V

$\therefore$ Excitation energy, $\Delta E = 2.1\,\text{eV} = 2.1\times1.6\times10^{-19}\,\text{J}$

As $\quad hf = \dfrac{hc}{\lambda} = \Delta E$

$$\therefore \quad \lambda = \frac{hc}{\Delta E} = \frac{6.6\times10^{-34}\times3\times10^8}{2.1\times1.6\times10^{-19}} = 5893\times10^{-10}\,\text{m} = 5893\,\text{Å}$$

As $T_1 = \frac{2\pi r_1}{v_1}$, $\nu_1 = \frac{1}{T_1} = \frac{v_1}{2\pi r_1}$

Further, $v_1 = \left(\frac{c}{1}\right)\alpha = (3\times10^8\ \text{m/s})\times\frac{1}{137} = 2.2\times10^6\ \text{m/s}$

$$r_1 = 0.53\ \text{Å} = 0.53\times10^{-10}\ \text{m}$$

Thus, $\nu_1 = \frac{(2.2\times10^6\ \text{m/s})}{2(3.14)(0.53\times10^{-10}\ \text{m})} = 6.6\times10^{15}\ \text{Hz}$

Energy of an electron in the n^{th} orbit of H-atom, $E_n = -\frac{13.6}{n^2}\text{eV}$

$\therefore \quad E_1 = -13.6\text{eV},\ E_2 = -3.4\text{eV},\ E_3 = -1.51\text{eV},\ldots$

Clearly, $E_2 - E_1 = -3.4-(-13.6) = 10.2\text{eV}$

Thus the energy state $n = 2$ has an excitation energy of 10.2 eV with respect to the ground state. Hence the electron is making a transition from state $n = n_1$ to the state $n = 2$, where $n_1 > 2$.

Now, $E_{n_1} - E_2 = \frac{hc}{\lambda} = \frac{12400}{4875}\text{eV} = 2.55\text{eV}$

$\therefore \quad E_{n_1} = E_2 + 2.55\text{eV} = -3.4+2.55 = -0.85\text{eV}$

Also, $E_{n_1} = -\frac{13.6}{n_1^2}eV \ \therefore -\frac{13.6}{n_1^2} = -0.85$

or $\quad n_1 = \sqrt{\frac{13.6}{0.85}} = \sqrt{16} = 4$

9. (a) The energy E of a photon of wavelength 275 nm is given by

$$E = \frac{hc}{\lambda} = \frac{12400}{275} = 4.5\text{eV}$$

This energy corresponds to the transition B for which the energy change = 0 – (–4.5) = 4.5 eV.

(b) Energy of emitted photon, $E = \frac{hc}{\lambda} \propto \frac{1}{\lambda}$

$\therefore \quad \lambda_{\max} \propto \frac{1}{E_{\min}}$ and $\lambda_{\min} \propto \frac{1}{E_{\max}}$

(i) Transition A, for which the energy emission is minimum, corresponds to the emission of radiation of maximum wavelength,

(ii) Transition D, for which the energy emission is maximum, corresponds to the emission of radiation of minimum wavelength.

10. (a) $Z = 3$ for Li^{2+}.

Further, we know that $r_n = \frac{n^2}{Z}a_0$

Substituting $n = 3$, $Z = 3$, and $a_0 = 0.529$ Å,

We have r_3 for $Li^{+2} = \frac{(3)^2}{(3)}(0.529)$ Å $= 1.587$ Å

(b) $Z = 2$ for He^+. Also, we know that $v_n = \frac{Z}{n}v_1$

Substituting $n = 4$, $Z = 2$, and $v_1 = 2.19\times10^6\ \text{m s}^{-1}$, we get

v_4 for $He^+ = \left(\frac{2}{4}\right)(2.19\times10^6)\text{ms}^{-1} = 1.095\times10^6\ \text{ms}^{-1}$

11. $E_1 = -13.60$ eV $\Rightarrow$ $K_1 = -E_1 = 13.60$ eV

$V_1 = -2K_1 = -27.2$ eV

$E_2 = \frac{E_1}{(2)^2} = -3.40$ eV

$\Rightarrow \quad K_2 = 3.40$ eV and $U_2 = -6.80$ eV

Now, $U_1 = 0$, i.e., potential energy of each shell has been increased by 27.20 eV while kinetic energy will remain unchanged. Changed values in tabular form are as under.

Orbit	K (eV)	U (eV)	E (eV)
First	13.6	0	13.60
Second	3.40	20.40	23.80

12. The force at a distance r is $f = -dU/dr = -2ur$.

Suppose r be the radius of n^{th} orbit. Then, the necessary centripetal force is provided by the above force. Thus,

$$\frac{mv^2}{r} = 2ur \qquad \text{...(i)}$$

Further, the quantization of angular momentum gives

$$mvr = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii) for r, we get

$$r = \left(\frac{n^2h^2}{8um\pi^2}\right)^{1/4} \qquad \text{...(iii)}$$

Exercise 6.3

1. $\frac{1}{\lambda_1} = R_\infty Z^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$ ...(i)

$$\frac{1}{\lambda_2} = R_\infty Z^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] \qquad \text{...(ii)}$$

$$\frac{1}{\lambda_3} = R_\infty Z^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right] \qquad \text{...(iii)}$$

On adding (i) and (ii), we get $\frac{1}{\lambda_1}+\frac{1}{\lambda_2} = R_\infty Z^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right]$

Thus, $\frac{1}{\lambda_1}+\frac{1}{\lambda_2} = \frac{1}{\lambda_3}$

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$$

2. (a) Let n_1 be the initial state of electron. Then, $E_1 = -\frac{13.6}{n_1^2}\text{eV}$

Here, $E_1 = -0.85$ eV

$\therefore \quad -0.85 = -\frac{13.6}{n_1^2}$

or $\quad n_1 = 4$

(b) Let n_2 be the final excitation state of the electron. Since excitation energy is always measured with respect to the ground state, therefore

$$\Delta E = 13.6\left[1-\frac{1}{n_2^2}\right]$$

Here, $\Delta E = 10.2$ eV

$\therefore \quad 10.2 = 13.6\left[1-\frac{1}{n_2^2}\right]$

or $\quad n_2 = 2$

Thus, the electron jumps from $n_1 = 4$ to $n_2 = 2$.

(c) The wavelength of the photon emitted for a transition between $n_1 = 4$ to $n_2 = 2$, is given by

$$\frac{1}{\lambda} = R_\infty\left[\frac{1}{n_2^2}-\frac{1}{n_1^2}\right]$$

$$\frac{1}{\lambda} = 1.09\times10^7\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$$

$\lambda = 4860$ Å

3. (a) H_β line of Balmer series corresponds to the transition from $n = 4$ to $n = 2$ level. The corresponding wavelength for H_β line is

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 0.2056 \times 10^7 \text{ m}^{-1}$$

$$\lambda = 4.9 \times 10^{-7} \text{ m}$$

(b) $f = \dfrac{c}{\lambda} = \dfrac{3.0 \times 10^8}{4.9 \times 10^{-7}} = 6.12 \times 10^{14}$ Hz

4. The transition equation for Lyman series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), n = 2, 3, \ldots$$

The largest wavelength is corresponding to $n = 2$

$$\therefore \quad \frac{1}{\lambda_{\text{max}}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{4}\right) = 0.823 \times 10^7$$

$$\Rightarrow \quad \lambda_{\text{max}} = 1.2154 \times 10^{-7} \text{ m} = 1215 \text{ Å}$$

The shortest wavelength corresponds to $n = \infty$

$$\therefore \quad \frac{1}{\lambda_{\text{min}}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{\infty}\right)$$

or $\quad \lambda_{\text{min}} = 0.911 \times 10^{-7} \text{ m} = 911$ Å

Both of these wavelengths lie in the ultraviolet (UV) region of electromagnetic spectrum.

5. (a) We have

$$\frac{m_p v^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{4e^2}{r_n^2} \qquad \text{...(i)}$$

The quantization of angular momentum gives

$$m_p v r_n = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get $r = \dfrac{n^2h^2\varepsilon_0}{2\pi m_p}e^2$

Substituting $m_p = 100\, m$, where m = mass of electron, we get

$$r_n = \frac{n^2h^2\varepsilon_0}{400\pi me^2}$$

(b) As we know, $\quad E_1 = -13.60$ eV (For H-atom)

and $\quad E_n \propto \left(\dfrac{z^2}{n^2}\right)m$

For the given particle, $E_4 = \dfrac{(-13.60)(4)^2}{(4)^2} \times 100 = -1360$ eV

and $\quad E_2 = \dfrac{(-13.60)(4)^2}{(2)^2} \times 100 = -5440$ eV

$$\Delta E = E_4 - E_2 = 4080 \text{ eV}$$

$$\lambda(\text{in Å}) = \frac{12375}{\Delta E\,(\text{in eV})} = \frac{12375}{4080} = 3.03 \text{ Å}$$

6. Given that electrostatic potential energy = –1.7 eV.

$$\therefore \quad \text{KE} = \frac{1.7}{2} = 0.85 \text{ eV}$$

So, total energy = –1.7 + 0.85 = –0.85 eV

Now, $\quad E_n = -\dfrac{13.6}{n^2} = -0.85$

$$\therefore \quad n^2 = \frac{13.6}{0.85} \quad \text{or} \quad n = 4$$

Hence, the atom is excited to state $n = 4$.

$$\Delta E = -0.85 - (-13.6) = 12.75 \text{ eV}$$

Using Einstein's equation, we have $h\nu = W + (\text{KE})_{\text{max}}$

or $\quad (\text{KE})_{\text{max}} = h\nu - W = 12.75 - 2.3 = 10.45$ eV

The minimum de-Broglie wavelength is given by

$$\lambda_{\text{min}} = \frac{h}{(p)_{\text{max}}} = \frac{h}{\sqrt{2m(\text{KE})_{\text{max}}}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times (9.1 \times 10^{-31})(10.45 \times 1.6 \times 10^{-19})}}$$

$$= 3.8 \times 10^{-10} \text{ m} = 3.8 \text{ Å}$$

7. If mass of nucleus is considered (not infinite), then the reduced mass of nucleus–electron system can be taken as

$$\mu = \frac{mM}{m + M}$$

Here, m is the mass of electron and M is that of the nucleus. The binding energy in ground state of hydrogen atom can now be given as

$$E = \frac{2\pi^2 K^2 e^4 \mu}{h^2} = 13.6 \times \frac{\mu}{m}\text{eV} = \frac{13.6M}{m + M}\text{eV}$$

We have hydrogen atom Rydberg constant given as

$$R = \frac{2\pi^2 K^2 e^4 m}{ch^3}$$

If effect of mass of nucleus is considered, the new value of Rydberg constant can be given as

$$R_\mu = \frac{2\pi^2 K^2 e^4 \mu}{ch^3} \quad \text{or} \quad R_\mu = \frac{RM}{m + M}$$

Percentage difference in the values of R and R_μ is given as

$$\frac{\Delta R}{R} \times 100 = \frac{(R_\mu - R) \times 100}{R_\mu} = \frac{m}{M} \times 100 = 0.055\%$$

8. The energy lost by the electron in exciting the hydrogen atom equals the energy corresponding to $\lambda = 1.216 \times 10^7$ m

$$h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{1.216 \times 10^{-7}} = 16.36 \times 10^{-19} \text{ J}$$

Now, the initial energy of electron = 20 eV = 32×10^{-19} J.

Hence, the kinetic energy of the scattered electron,

$$E = 32 \times 10^{-19} \text{ J} - 16.36 \times 10^{-19} \text{ J} = 15.64 \times 10^{-19} \text{ J}$$

The velocity v of the scattered electron is given by $\dfrac{1}{2}mv^2 = E$

or $\quad v = \left(\dfrac{2E}{m}\right)^{1/2} = \left(\dfrac{2 \times 15.64 \times 10^{-19}}{9.11 \times 10^{-31}}\right)^{1/2} = 1.86 \times 10^6 \text{ ms}^{-1}$

9. The orbital frequency of nth orbit is given by

$$\nu_n = \frac{\text{electron speed}}{\text{orbital circumference}}$$

$$= \frac{e^2}{2nh\,\varepsilon_0} \times \frac{1}{2\pi} \times \frac{\pi me^2}{n^2\,\varepsilon_0 h^2} = \frac{me^4}{4\,\varepsilon_0^2 n^3 h^3} \qquad \left[\text{As } K = \frac{1}{4\,\pi\,\varepsilon_0}\right]$$

From the laws of electromagnetic theory, the frequency of the radiation emitted by this electron will also be ν_n.

According to Bohr's theory, the frequency of the radiation for the adjoining orbits is given by

$$\nu_n = \frac{E_n - E_{n-1}}{h} = \frac{me^4}{8\,\varepsilon_0^2 h^2}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4}{8\,\varepsilon_0^2 h^2}\frac{(2n-1)}{(n-1)^2 \times n^2}$$

A comparison of this expression with the classical expression above show that the difference in their predictions will be large for small n, i.e., for $n = 2$, $\frac{1}{n^3} = \frac{1}{8}$ and $\frac{(2n-1)}{2n^2(n-1)^2} = \frac{3}{8}$, so that the frequency given by quantum theory is 3 times that calculated from classical theory. However, for very large n, the quantum expression reduces to that obtained from classical theory because for $n \to \infty$.

$$\frac{2n-1}{2n^2(n-1)^2} \cong \frac{2n}{2n^2 n^2} = \frac{1}{n^3}$$

Consequently, the predictions of Bohr's theory agrees with that of the classical theory in the limit of very large quantum numbers. This correspondence is called as Bohr's correspondence principle.

Here, $\lambda = 275$ nm $= 275 \times 10^{-9}$ m

Therefore, energy of the emitting photon,

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{275\times10^{-9}}$$

$$= 7.22 \times 10^{-19}\text{J} = \frac{7.22\times10^{-19}}{1.6\times10^{-19}} = 4.5 \text{ eV}$$

The photon of energy 4.5 eV (or of wavelength 275 nm) will be emitted, corresponding to the transition B.

(a) The second excited state corresponds to $n = 3$,

Now, $E_3 = -\frac{13.6}{n^2} = -\frac{13.6}{3^2} = -\frac{13.6}{9} = 1.51$ eV

The kinetic energy of an electron in the second excited state $= -(-1.51) = 1.51$ eV

(b) Energy emitted, when the electron jumps from the second excited state to the ground state,

$$E = -1.51 - (-13.6) = 12.09 \text{ eV} = 12.09 \times 1.6 \times 10^{-19} \text{ J}$$

The wavelength of the spectral line emitted,

$$\lambda = \frac{hc}{E} = \frac{6.62\times10^{-34}\times3\times10^{8}}{12.09\times1.6\times10^{-19}} = 1.027 \times 10^{-7}\text{m} = 1.027 \text{ Å}$$

12. (a) Since $E_n \propto \frac{Z^2}{n^2}$ in a hydrogen-like atom, the energy levels will be $29^2 = 841$ times the corresponding energies for hydrogen.

$$\therefore \quad E_n = 841 \times \left(\frac{-13.6}{n^2}\right)$$

or $\quad E_1 = -11.44$ keV; $E_2 = -2.86$ keV; $E_3 = -1.27$ keV

(b) Making use of $\Delta E = E_n - E_1$

$\Rightarrow \quad \lambda_1 = 1.44$ Å, $\lambda_2 = 1.22$ Å, and $\lambda_3 = 1.15$ Å

Ionization potential is the potential corresponding to energy $+|E_1|$.

$$V_\infty = \frac{|E_1|}{e} = 11.44 \text{ kV}$$

Exercise 6.4

1. (a) True. According to the theory of electromagnetism, electromagnetic waves are generated by the acceleration or retardation of charges.

(b) True. Line spectrum of electromagnetic waves are produced due to the transition of electron from the higher level to the lower level.

(c) False. The cut-off wavelength depends only on the kinetic energy of the striking electrons.

(d) True. The K_α transition is more probable than the K_β transition.

(e) False. Frequency of K_β is more than that of K_α because $\lambda_B > \lambda_A$ as shown:

$$\frac{1}{\lambda_\beta} = R_\infty (Z-1)^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8}{9} R_\infty (Z-1)^2$$

and $$\frac{1}{\lambda_\alpha} = R_\infty (Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4} R_\infty (Z-1)^2$$

2. We know that $\lambda_{\text{min}} = \frac{hc}{\text{eV}} = \frac{12400}{\text{V}} = \frac{12400}{40\times10^3} = 0.31$ Å

3. We know that for K_α-line $\frac{1}{\lambda_\alpha} = R_\infty (Z-1)^2\left[1 - \frac{1}{2^2}\right] = \frac{3}{4} R_\infty (Z-1)^2$

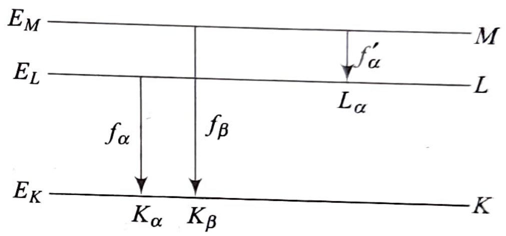
or $$\lambda_\alpha = \frac{4}{3R_\infty (Z-1)^2}$$

Here, $R_\infty = 1.096 \times 10^7$ m^{-1} and $Z = 74$, therefore

$$\lambda_\alpha = \frac{4}{3(1.096\times10^7)(74-1)^2} = 2.28 \times 10^{-11} \text{ m}$$

or $\quad \lambda_\alpha = 0.228$ Å

4. From the energy diagram as shown in the following figure, we know that



Frequency of K_α-line is $f_\alpha = \frac{E_L - E_K}{h}$

Frequency of K_β-line is $f_\beta = \frac{E_M - E_K}{h}$

Frequency of L_α-line is $f'_\alpha = \frac{E_M - E_L}{h}$

Since $E_M - E_K = (E_M - E_L) + (E_L - E_K)$

$\therefore \quad f_\beta = f_\alpha + f'_\alpha$

5. When an electron of charge e is accelerated through a potential difference V, it acquires energy eV. If m be the mass of the electron and v_{max} the maximum speed of the electron, then

$$\frac{1}{2} m v_{\text{max}}^2 = eV \quad \text{or} \quad v_{\text{max}} = \sqrt{\left(\frac{2eV}{m}\right)}$$

Substituting the given values, we get

$$v_{\text{max}} = \sqrt{\left(\frac{2\times(1.6\times10^{-19})\times20{,}000}{9\times10^{-31}}\right)} = 8.4 \times 10^{7} \text{ ms}^{-1}$$

6. (a) $\lambda_{\text{min}} = 0.45$ Å

$$E_{\text{max}} = h\nu_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} = \frac{12431}{0.45} = 27624.44 \text{ eV} = 27.624 \text{ keV}$$

(b) The minimum accelerating voltage for electrons is

$$\frac{27.6 \text{ keV}}{e} = 27.6 \text{ kV, i.e., of the order of 30 kV.}$$

7. According to Moseley's law, the frequency for K series is proportional to $V(Z-1)^2$

or $$\frac{c}{\lambda} \propto (Z-1)^2$$

or $$\frac{1}{\lambda} = k(Z-1)^2 \qquad \text{...(i)}$$

where k is a constant. Let λ' be the wavelength of K_α line emitted from molybdenum, then

$$\frac{1}{\lambda'} = k(Z-1)^2 \qquad \text{...(ii)}$$

Dividing (i) and (ii), we get

$$\lambda' = \left(\frac{Z-1}{Z-1}\right)^2 \lambda = \left(\frac{30-1}{42-1}\right)^2 \times 1.415\ \text{Å} = 0.708\ \text{Å}$$

8. The short series limit of the Balmer series is corresponding to transition $n = \infty$ to $n = 2$ which is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$$

or $$R = \frac{4}{\lambda} = \frac{4}{3644}\ (\text{Å})^{-1}$$

The shortest wavelength corresponds to transition from $n = \infty$ to $n = 1$ which is given as

$$\frac{1}{\lambda_c} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$$

or $$(Z-1)^2 = \frac{1}{\lambda_c R} = \frac{1}{1\text{Å} \times \frac{4}{3644}(\text{Å})^{-1}} = \frac{3644}{4} = 911$$

or $$Z - 1 = 30.2$$

or $$Z = 31.2 \simeq 31$$

Thus, the atomic number of the element is 31 which is gallium.

9. The binding energy for K shell in eV is

$$E_k = \frac{hc}{\lambda_k} = \frac{12431}{0.2}\ \text{eV} = 62.155\ \text{keV}$$

The energy of the incident photon in eV is

$$E = \frac{hc}{\lambda} = \frac{12431}{0.15} = 82.873\ \text{keV}$$

Therefore, the maximum energy of the photoelectrons emitted from the K shell is

$$E_{\max} = E - E_k = (82.873 - 62.155)\ \text{keV} = 20.718\ \text{keV}$$

10. Tungsten is a multielectron atom. Due to the shielding of the nuclear charge by the negative charge of the inner core electrons, each electron is subject to an effective nuclear charge Z_{eff}, which is different for different shells.

For an electron in the K shell, $\sigma = 1$.

Thus, $Z_{\text{eff}} = Z - \sigma = Z - 1$

Here, as electron drops from M shell ($n = 3$) to K shell ($n = 1$), we call the radiated emission K_β X-ray and from Moseley's law the wavelength emitted of K_β X-ray is given as

$$\frac{1}{\lambda_{k_\beta}} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

or $$\frac{1}{\lambda_{k_\beta}} = 10967800 \times (74-1)^2\left[\frac{8}{9}\right]$$

or $$\lambda_{k_\beta} = 0.192\ \text{Å}$$

11. The X-rays produced under an accelerating potential V will have varying wavelengths with the minimum due to the entire energy of the accelerated electron being lost in a single collision with the target atoms. Here, the shortest wavelength generated of X-rays can be given by

$$\lambda_c = \frac{12431}{V}\text{Å} = \frac{12431}{20000} = 0.6215\ \text{Å}$$

12. K_α X-rays are produced when an electron makes a transition from $n = 2$ to $n = 1$ to fill a vacancy in K shell. The wavelength of X-ray lines is given by

$$\frac{1}{\lambda_{K_\alpha}} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R(Z-1)^2$$

or $$(Z-1)^2 = \frac{4}{3R\lambda_{K_\alpha}} = \frac{4}{3 \times (1.097 \times 10^7) \times (0.76 \times 10^{-10})}$$

$$= 1599.25$$

or $$(Z-1)^2 \simeq 1600 \quad \text{or} \quad Z - 1 = 40$$

or $$Z = 41$$

Exercises

Single Correct Answer Type

1. (4) $$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

or $$\frac{f}{c} = R\left[\frac{1}{4} - \frac{1}{16}\right]$$

or $$f = cR\left[\frac{1}{4} - \frac{1}{16}\right] = 3 \times 10^8 \times 10^7 \times \frac{3}{16} = \frac{9}{16} \times 10^{15}\ \text{Hz}$$

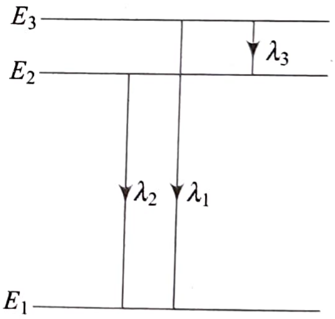
2. (3) Volume occupied by one mole of gold $= \dfrac{197\ \text{g}}{19.7\ \text{g cm}^{-3}} = 10\ \text{cm}^3$

Volume of one atom $= \dfrac{10}{6 \times 10^{-23}} = \dfrac{5}{3} \times 10^{23}\ \text{cm}^3$

Let r be the radius of the atom. Therefore, $\dfrac{4}{3}\pi r^3 = \dfrac{5}{3} \times 10^{23}$

or $$r \cong 1.5 \times 10^{-10}\ \text{m}$$

3. (3) Let the three energy levels be E_1, E_2, and E_3. The wavelengths λ_1, λ_2, and λ_3 of the spectral lines corresponding to the three energy transitions are depicted as shown in figure.



$$E = hf = \frac{h}{\lambda} \quad \text{or} \quad E \propto \frac{1}{\lambda}$$

(given $\lambda_1 < \lambda_2 < \lambda_3$)

Thus, for the three wavelengths, we have

$$E_3 - E_2 = \frac{h}{\lambda_3} \qquad \text{...(i)}$$

$$E_2 - E_1 = \frac{h}{\lambda_2} \qquad \text{...(ii)}$$

$$E_3 - E_1 = \frac{h}{\lambda_1} \qquad \text{...(iii)}$$

Now, $E_3 - E_1 = (E_3 - E_2) + (E_2 - E_1)$

$$\Rightarrow \quad \frac{h}{\lambda_1} = \frac{h}{\lambda_3} + \frac{h}{\lambda_2} \quad \Rightarrow \quad \frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$$

(4) $v_n = k\dfrac{2\pi e^2}{nh}$

We know that in cgs system $k = 1$

$$\therefore \quad v_n = \frac{2\pi e^2}{nh} \Rightarrow v_1 = \frac{2\pi e^2}{h}$$

$$\text{So} \quad \frac{v_1}{c} = \frac{2\pi e^2}{ch}$$

(1) Barrier height $= \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r_e}\,\text{J} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e}{r_e}\,\text{eV}$

$$= \frac{9\times10^9\times1.6\times10^{-19}}{10^{-14}}\,\text{eV} = 1.44\times10^5\ \text{eV}$$

(1) The recoil momentum of atom is same as that of photon but in opposite direction.

Hence, recoil momentum,

$$P = \frac{E}{c} = \frac{12.09\times1.6\times10^{-19}}{3\times10^8}\,\text{Ns} = 6.45\times10^{-27}\ \text{Ns}$$

Note that almost whole of the energy will be carried away by the photon because it is very light in comparison to H atom.

(2) $f = cZ^2R\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

$$\Rightarrow \quad 2.7\times10^{15} = cz^2R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$f' = cZ^2R\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

Divide and solve to get: $f = 3.2\times10^{15}$ Hz

(4) $E_p = -\dfrac{ke^2}{r}$, $E = -\dfrac{ke^2}{2r}$

So, $E_p = 2E = 2(-13.6)\ \text{eV} = -27.2\ \text{eV}$.

Potential energy of electron in the ground state of Li^{2+} ion $= -3^2\times27.2$ eV or -244.8 eV.

9. (3) Required energy $= \left[\left(\dfrac{-13.6}{9}\right) - \left(\dfrac{-13.6}{1}\right)\right]\times 9$

$$= \left[13.6 - \frac{13.6}{9}\right]9 = 8\times13.6\ \text{eV}$$

$$\text{Wavelength} = \frac{12375}{8\times13.6} = 113.7\ \text{Å}$$

10. (4) $F = \dfrac{mv^2}{r}$

But $v \propto \dfrac{1}{n}$ and $r \propto n^2$

$$\Rightarrow \quad F \propto \frac{1}{n^4}$$

11. (2) $U = -\dfrac{ke^2}{2R^3}$, $F = -\dfrac{dU}{dR} = -\dfrac{3ke^2}{2R^4}$

But, $F = \dfrac{mv^2}{R} \Rightarrow \dfrac{mv^2}{R} = \dfrac{3ke^2}{2R^4}$

Also, $mvR = \dfrac{nh}{2\pi}$

Solve to get $R = \dfrac{6\pi^2ke^2m}{n^2h^2}$

12. (1) Frequency of electron revolution, $f = \dfrac{mZ^2e^4}{4\varepsilon_0^2n^3h^3}$,

Put the various values to get $f = 6.62\times10^{15}\dfrac{Z^2}{n^3}$

Now, put $Z = 1$ and $n = 1$ to get $f = 6.62\times10^{15}$ Hz

13. (2) $r_n \propto n^2$

$$\frac{n'^2}{n^2} = \frac{21.2\times10^{-11}}{5.3\times10^{-11}} \quad \text{or} \quad \frac{n'^2}{n^2} = 4$$

$$\text{or} \quad \frac{n'^2}{1^2} = 4 \quad \text{or} \quad n' = 2$$

14. (3) $\dfrac{1}{\lambda} = R\left[\dfrac{1}{2^2} - \dfrac{1}{3^2}\right]$

$$\text{or} \quad R = \frac{36}{5\lambda} = \frac{36}{5\times6563\times10^{-10}}\,\text{m}^{-1}$$

$$= \frac{36000}{5\times6563}\times10^7\ \text{m}^{-1} = 1.097\times10^7\ \text{m}^{-1}$$

15. (2) l will be same for both because it does not depend upon Z. But for energy

$$(E_n)_{\text{Li}} = -\frac{Z^2\times13.6}{n^2} \quad \text{and} \quad (E_n)_{\text{H}} = -\frac{13.6}{n^2}$$

Clearly, $|E_H| < |E_{Li}|$

16. (3) $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

For longest wavelength, $n_1 = 2$, $n_2 = 3$

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \quad \text{or} \quad \frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\Rightarrow \quad \lambda = \frac{36}{5R} \text{ for electron,}$$

$$\text{But} \quad \lambda \propto \frac{1}{m}$$

$$\text{So} \quad \lambda' = \frac{1}{2}\times\frac{36}{5R} = \frac{18}{5R}$$

17. (1) $|F| = \left|\dfrac{-dU}{dr}\right| = \dfrac{mv^2}{r} \Rightarrow v = \sqrt{\dfrac{U_0}{m}}$, which is a constant.

So, $mv_nr_n = \dfrac{nh}{2\pi} \Rightarrow r_n \propto n$

18. (2) $\lambda = \dfrac{912\ \text{Å}}{Z^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]}$

For singly ionized helium atom $Z = 2$.

For the wavelength to be longest, $n_1 = 1$, $n_2 = 2$

$$\therefore \quad \lambda = \frac{912\ \text{Å}}{(2^2)\left[1 - \frac{1}{4}\right]} = \frac{912}{3} = 304\ \text{Å}$$

19. (4) We know that frequency of orbital motion:

$$f \propto \frac{1}{n^3} \quad \text{and} \quad \text{given } f_1 = \frac{1}{27}f_2$$

$$\Rightarrow \quad \left(\frac{n_2}{n_1}\right)^3 = \frac{f_1}{f_2} \Rightarrow \frac{n_2}{n_1} = \left(\frac{1}{27}\right)^{1/3} = \frac{1}{3}$$

Among given options, option (4) is correct.

20. (4) Total energy received by the atom will be 25.2 eV. 13.6 eV energy is needed to remove the electron from the attraction of the nucleus. Rest of the energy will be almost available in the form of KE of electron. One electron of 11.6 eV will be detected.

21. (1) As magnetic moment, $\mu_B = \frac{1}{2}evr$

$$L = mvr \quad \Rightarrow \quad vr = \frac{L}{m}$$

$$\therefore \quad \mu_B = \frac{1}{2}\frac{eL}{m} \quad \Rightarrow \quad \frac{\mu_B}{L} = \frac{e}{2m}$$

22. (3) $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad \lambda = \frac{36}{5R} = \frac{36 \times 10^{-7}}{5 \times 1.097} = 6.566 \times 10^{-7}$$

$$\lambda = 6566 \text{ Å}$$

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{6.566 \times 10^{-7}} = 3.03 \times 10^{-19} \text{ J}$$

$$\therefore \quad E = 1.89 \text{ eV}$$

23. (2) For shortest wavelength in Balmer series, $n_1 = 2$; $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{\infty}\right] \quad \text{or } \lambda = \frac{4}{R}$$

For shortest wavelength in Brackett series, $n_1 = 4$; $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda'} = R\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right]$$

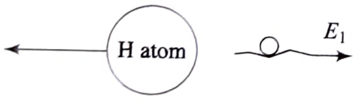
$$\text{or} \quad \lambda' = \frac{16}{R} = 4 \times \frac{4}{R} = 4\lambda$$

24. (3) First excitation energy is $Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = Rhc\frac{3}{4}$

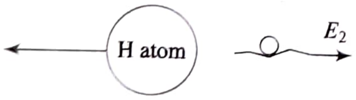
$$\frac{3}{4}Rhc = V \text{ eV}$$

$$\therefore \quad Rhc = \frac{4V}{3} \text{ eV}$$

25. (2)



Case I



Case II

In the first case, KE of H atom increases due to recoil whereas in the second case KE decreases due to recoil.

$$\therefore \quad E_2 > E_1$$

26. (1) Linear momentum, $mv \propto \frac{1}{n}$

Angular momentum, $mvr \propto n$

Therefore, product of linear momentum and angular momentum $\propto n^0$

27. (3) Energy of photon is given by mc^2. Now, the maximum energy of photon is equal to the maximum energy of electrons = eV

Hence, $mc^2 = eV$

$$\Rightarrow \quad m = \frac{eV}{c^2} = \frac{1.6 \times 10^{-19} \times 18 \times 10^3}{(3 \times 10^8)^2} = 3.2 \times 10^{-32} \text{ kg}$$

28. (4) Using $\frac{1}{\lambda} = R(Z-1)^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$

For K_α line; $n_1 = 2$, $n_2 = 1$

For metal A; $\frac{1875R}{4} = R(Z_1 - 1)^2\left(\frac{3}{4}\right)$

$$\Rightarrow \quad Z_1 = 26$$

For metal B; $675R = R(Z-1)^2\left(\frac{3}{4}\right)$

$$\Rightarrow \quad Z_2 = 31$$

Therefore, 4 elements lie between A and B.

29. (4) For second line of Balmer series in hydrogen spectrum, corresponds to 4 → 2 transition of hydrogen atom.

It is equivalent to $4 \times 3 \rightarrow 2 \times 3$ i.e., $12 \rightarrow 6$ transition of Li^{2+}

30. (4) Energy of n^{th} state in hydrogen is same as energy of $3n^{th}$ state in Li^{++}.

∴ $3 \rightarrow 1$ transition in H would give same energy as the $3 \times 3 \rightarrow 1 \times 3$ i.e., $9 \rightarrow 3$ transition in Li^{++}.

31. (1) $\frac{1}{\lambda_\alpha} = \frac{3R}{4}(Z-1)^2$

$$(Z-1) = \sqrt{\frac{4}{3R\lambda_\alpha}} = \sqrt{\frac{4}{3 \times 1.1 \times 10^7 \times 1.8 \times 10^{-10}}}$$

$$= \frac{200}{3}\sqrt{\frac{5}{33}} = \frac{78}{3} = 26 \Rightarrow Z = 27$$

32. (2) λ_m will increase to $3\lambda_m$ due to decrease in the energy of bombarding electrons. Hence, no characteristic X-rays will be visible, only continuous X-ray will be produced.

33. (4) $B = \frac{\mu_0 I}{2r}$ and $I = \frac{e}{T}$

$$B = \frac{\mu_0 e}{2rT} \qquad [r \propto n^2, T \propto n^3]$$

$$B \propto \frac{1}{n^5}$$

34. (3) $i = \frac{q}{T}$

Now $T^2 \propto r^3 \propto n^6 \quad \Rightarrow \quad i \propto \frac{1}{n^3}$

35. (2) The cut-off wavelength when $V = V_1 = 10$ kV is

$$\lambda_1 = \frac{hc}{eV_1} = 1243.125 \times 10^{-13} \text{ m}$$

The cut-off wavelength when $V = V_2 = 20$ kV is,

$$\lambda_2 = \frac{hc}{eV_2} = 621.56 \times 10^{-13} \text{ m}$$

The wavelength corresponding to K_α line is

$$\frac{1}{\lambda} = \frac{3R}{4}(Z-1)^2$$

From given information, $(\lambda - \lambda_2) = 3(\lambda - \lambda_1)$

Solving above equation, we get $Z = 29$

36. (2) In the emission spectrum 10 lines are observed, so the energy level (n) to which the sample has been excited after absorbing the radiation is given by

$$\frac{n(n-1)}{2} = 10$$

which gives $n = 5$

$$\text{So,} \quad \frac{hc}{\lambda} = 13.6\left(1 - \frac{1}{5^2}\right) \text{eV}$$

$$\frac{1242}{\lambda}\text{ eV-nm} = 13.6\times\frac{24}{25}\text{ eV}$$

$\therefore\quad \lambda = 95$ nm

(3) Let n be the number of electrons per second striking the target.

$p = VI$

$$16\text{W} = (5\times10^3\text{V})\times ne$$

$$\therefore\quad n = \frac{16}{5\times10^3\times1.6\times10^{-19}} = 2\times10^{16}$$

(3) In first case, transition is from n^{th} state to 2nd (1st excited) state.

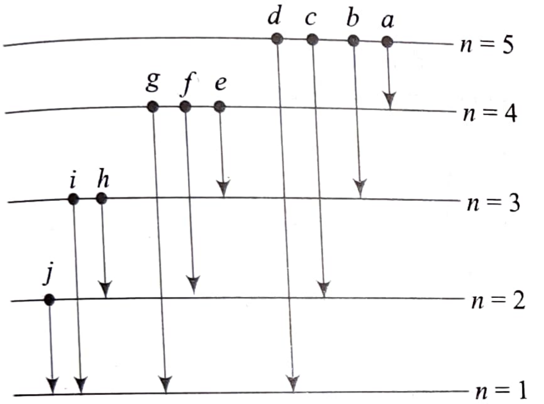
$$\therefore\quad (10.2+17.0)\text{ eV} = 13.6\times Z^2\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$$

In 2nd case, transition is from n^{th} state to 3rd state

$$\therefore\quad (4.25+5.95)\text{ eV} = 13.6\,Z^2\left[\frac{1}{3^2}-\frac{1}{n^2}\right]$$

Solving above equations, we get $n = 6$ and $Z = 3$.

(2) The wavelengths present in emission spectra are shown in figure.



Transitions a, e, h and j can be performed by a single atom also. This is also true about transitions b and i, other transitions require one atom each.

(3) Through filter, only those photons will pass through whose energy is less than $E = \dfrac{hc}{800\text{ nm}} = 1.55$ eV.

From hydrogen's energy level diagram, we can easily identify that when electron jumps from 3rd excited state (4th energy level) to 2nd excited state (3rd energy level), then photons of 1.55 eV energy have been emitted. So, the required initial energy level is the 3rd excited state.

41. (2) Option (1) explains the production of X-rays on the basis of electromagnetic theory of light, which is not able to explain the characteristic X-rays and cut-off wavelength, so option (1) is wrong.

Option (2) correctly explains the production of characteristic X-rays.

Option (3) is wrong as X-ray spectra is a continuous spectra having some peaks representing characteristic X-rays.

42. (2) The energy taken by hydrogen atom corresponds to its transition from $n = 1$ to $n = 3$ state.

$$\Delta E = 13.6\left(1-\frac{1}{9}\right) = 13.6\times\frac{8}{9} = 12.1\text{ eV}$$

43. (2) ΔE_1 (for 4th to 3rd excited states) $= 13.6\times3^2\left[\dfrac{1}{4^2}-\dfrac{1}{5^2}\right] = 2.75$ eV

ΔE_2 (for 3rd to 2nd excited states) $= 13.6\times3^2\left[\dfrac{1}{3^2}-\dfrac{1}{4^2}\right] = 5.95$ eV

For shorter wavelength, i.e., for ΔE_2, $V_{01} = 3.95$ volt

From $eV_{01} = hc - \phi$, $3.95 = 5.95 - \phi$

$\therefore\quad \phi = 2$ eV

For longer wavelength, $eV_{02} = 2.75 - 2 = 0.75$ eV

So, $\quad V_{02} = 0.75$ V

44. (4) In transition of electron from higher energy level to lower energy level, the wavelength is given by $\lambda = hc/\Delta E$, where ΔE is the energy difference between two levels.

For minimum λ, ΔE should be maximum, so (4) is the correct option.

45. (1) For a collision of neutron with hydrogen atom in ground state to be inelastic (partial or complete), the minimum KE of striking neutron must be 20.4 eV. [This condition is derived in theory.]

As the energy of the given incident neutron is less than 2.4 eV, the collision must be elastic.

46. (2) Time period for n^{th} energy level electron is,

$$T = \frac{2\pi r_n}{v_n} = \frac{4\pi^2 m}{h}\times\frac{r_n^2}{n}$$

$$r_n = n^2 a_0$$

$$T = \frac{4\pi^2 m}{h}\times n^3 a_0^2$$

Required number of revolutions, $N = \dfrac{10^{-8}}{T}$

After substituting $n = 2$, $m = 9.1\times10^{-31}$ kg,

and $h = 6.63\times10^{-34}$ J-s, we get $N = 8\times10^6$.

47. (3) As the collision is inelastic, it means a part of kinetic energy is transformed into some other form due to collision. In this case, the kinetic energy of incident electron can be absorbed by H atom and it can absorb only 10.2 eV out of 11.2 eV, so that it can reach to 1st excited state and the electron leaves with remaining energy, i.e., 1.0 eV.

48. (2) $E = R_\infty ch\times\left[1-\dfrac{1}{2^2}\right] = \dfrac{3}{4}R_\infty\times hc$

Momentum of photon emitted is, $p = \dfrac{E}{c} = \dfrac{3R_\infty h}{4}$

Recoiling speed of hydrogen atom is given by $v = p/m$, where m is the mass of hydrogen atom.

$$v = \frac{3R_\infty h}{4m} = \frac{3\times1.1\times10^7\times6.63\times10^{-34}}{4\times1.67\times10^{-27}} = 3.3\text{ m s}^{-1}$$

49. (4) As the electron beam is having energy of 13 eV, it can excite the atom to the states whose energy is less than or equal to 0.6 eV (13.6 – 13). $E_5 = 0.544$ eV and $E_4 = 0.85$ eV. So, the electron beam can excite the hydrogen gas maximum to 4th energy state, hence the transit electron can come back to ground state from either of three excited states, thus emitting Lyman, Balmer and Paschen series.

50. (1) $\dfrac{hc}{\lambda} = 13.6\left[\dfrac{1}{1^2}-\dfrac{1}{10^2}\right] = 13.6\times0.99$

$$\lambda = \frac{1242}{13.6\times0.99}\text{ nm} = 92.25\text{ nm}$$

The line belongs to UV part of electromagnetic spectrum.

51. (3) As angular momentum of electron is $4h/2\pi$, it means electron is in the 4th orbit.

TE of atom in 4th orbit is –0.85 eV

KE of electron = |TE| = 0.85 eV

52. (4) We know, $r_n \propto n^2$

So, $(n+1)^2 - n^2 = (n-1)^2 \;\Rightarrow\; n = 4$

53. (1) From Moseley's law, $\sqrt{\nu} = a(Z-1)$ for K_α X-ray

i.e., $\dfrac{1}{\sqrt{\lambda}} = a(Z-1)$

Given, $\dfrac{1}{\sqrt{\lambda}} = a(11-1)$ and $\dfrac{1}{\sqrt{4\lambda}} = a(Z-1)$

Dividing, we get $Z = 6$.

54. (3) The protons move toward each other till their relative velocity becomes equal to zero. At the closest distance of approach, both the protons will be moving with the same velocity.

As coulombian repulsive force is internal for the system of protons, we can apply the law of conservation of momentum.

$\therefore \quad mv_0 = 2mv$

Change in KE $= \dfrac{1}{2}mv_0^2 - 2\times\dfrac{1}{2}m\left(\dfrac{v_0}{2}\right)^2$

This change in energy is equal to the electrical potential energy

$$\frac{mv_0^2}{2} - m\left(\frac{v_0}{2}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\therefore \quad r = \frac{e^2}{\pi\varepsilon_0 m v_0^2}$$

55. (2) Kinetic energy gained by a charge q after being accelerated through a potential difference V volt, is given by

$$qV = \frac{1}{2}mv^2$$

$$mV = \sqrt{2MqV}$$

As $\quad \lambda_b = \dfrac{h}{mV} = \dfrac{h}{\sqrt{2qmV}}$

For cut-off wavelength of X-rays, we have $qV = \dfrac{hc}{\lambda_m}$

or $\quad \lambda_m = \dfrac{hc}{qV}$

Now, $\quad \dfrac{\lambda_b}{\lambda_m} = \dfrac{\sqrt{\dfrac{qV}{2m}}}{c}$

As $\quad \dfrac{q}{m} = 1.8\times10^{11}$ C kg^{-1} for electron,

we have $\quad \dfrac{\lambda_b}{\lambda_m} = \dfrac{\sqrt{1.8\times10^{11}\times10\times10^3/2}}{3\times10^8} = 0.1$

Therefore, the answer is (2).

6. (2) $P = VI$

Therefore, total power drawn by Coolidge tube $P_T = VI = 200$ W.

As 0.5% of the energy is carried by electron,

Power carried by X-rays is 0.5% of $P_T = \dfrac{0.5}{100}\times200 = 1$ W

The answer is (2).

7. (4) As $\quad \lambda_0 = \dfrac{hc}{E} = 1.55\times10^{-11}$ m

$\therefore \quad \lambda_0 = 0.155$ Å

This is the minimum wavelength of continuous X-rays which carry energy equivalent to energy of incident electrons.

Now, as the energy of incident radiation is more than that of K-shell electrons, the characteristic X-rays appear as peaks on the continuous spectrum.

Therefore, (4) is the answer.

58. (1) From conservation of momentum, two identical photons travel in opposite directions with equal magnitude of momentum and energy hc/λ.

From conservation of energy, we have $\dfrac{hc}{\lambda} + \dfrac{hc}{\lambda} = m_0c^2 + m_0c^2$

$$\lambda = \frac{h}{m_0 c}$$

Therefore, the answer is (1).

59. (1) For the incident electron, $\dfrac{1}{2}mv^2 = Ve$

$$p^2 = 2meV$$

de-Broglie wavelength of incident electron, $\lambda_1 = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mVe}}$

Shortest X-ray wavelength, $\lambda_2 = \dfrac{hc}{Ve}$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{Ve}{c\sqrt{2mVe}}$$

$$= \frac{1}{c}\sqrt{\left(\frac{V}{2}\right)\frac{e}{m}} = \frac{\sqrt{\dfrac{10^4}{2}\times1.8\times10^{11}}}{3\times10^8} = 0.1 = \frac{1}{10}$$

or $\quad \lambda_1 : \lambda_2 = 1 : 10$

60. (4) The K, L, and M lines have different intercepts. The intercept of K is more than that of L, which in turn is more than that of M.

61. (3) $L_1 = (1)\dfrac{h}{2\pi}$ ...(i)

(Using Bohr's quantization rule)

In the first excited state of Li,

$$L_2 = (2)\frac{h}{2\pi} \quad \text{...(ii)}$$

$$\therefore \quad \frac{L_2}{L_1} = 2$$

62. (4) $\dfrac{0.001}{2}\times10^{-3} = (0.5\times10^{-10})n^2$

$$\left\{\begin{array}{l}\text{Because radius of } n^{\text{th}} \text{ orbit is equal to} \\ r_n = n^2 r_0, \quad \text{where} \quad r_0 = 0.529 \text{ Å}\end{array}\right\}$$

$\therefore \quad n^2 = 1000$

or $\quad n = 31$

63. (4) For each principal quantum number n, number of electrons permitted equals the number of elements corresponding to the quantum number.

Total number of electrons $= \sum 2n^2 = \dfrac{n(n+1)(2n+1)}{3}$

64. (2) $v_n = \alpha\left(\dfrac{cZ}{n}\right)$, where $\alpha = \dfrac{e^2}{2h\varepsilon_0 c}$ is the fine structure constant $\left(\alpha = \dfrac{1}{137}\right)$

$$v_{\text{He}^+} = \alpha\frac{c(2)}{2} = \alpha c \text{ and } v_{\text{H}} = \alpha\frac{c(1)}{1} = \alpha c = v_{\text{He}^+}$$

65. (3) $E = R_\infty hc\left(1 - \dfrac{1}{25}\right)$

Momentum of photon emitted is $p = \dfrac{E}{c} = R_\infty h\left(\dfrac{24}{25}\right)$

Recoil momentum of H atom will also be p.

$$mv = p$$

$$v = \frac{p}{m} = \frac{(1.097\times10^7)(6.626\times10^{-34})24}{(25)(1.67\times10^{-27})} = 4.178 \text{ m s}^{-1}$$

(2) Since $E = -\frac{13.6}{n^2}$ eV

$$E_1 = -13.6 \text{ eV}$$
$$E_2 = -3.4 \text{ eV}$$
$$E_3 = -1.50 \text{ eV}$$
$$E_4 = -0.85 \text{ eV}$$

From above, we can see that $E_3 - E_1 = 12.1$ eV

i.e., the electron must be making a transition from $n = 3$ to $n = 1$ level.

$$\Delta L = (3-1)\frac{h}{2\pi} = \frac{h}{\pi} = 2.11 \times 10^{-34} \text{ Js}$$

(2) For an elastic collision to take place, there must be no loss in the energy of electron. The hydrogen atom will absorb energy from the colliding electron only if it can go from ground state to first excited state, i.e., from $n = 1$ to $n = 2$ state. For this, hydrogen atom must absorb energy $E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV.

So, if the electron possesses energy less than 10.2 eV, it would never lose it and hence collision would be elastic.

(2) $\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For K_α line, $n_1 = 1$ and $n_2 = 2$

$$\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{3}{4}\right)$$

$$Z = \sqrt{\frac{4}{3\lambda R_\infty}} = 39.9 \approx 40$$

(3) Making potential energy zero increases the value of total energy by $13.6 - (-13.6) = 27.2$ eV

Now, actual energy in second orbit = -3.4 eV

Hence, new value is $(-3.4 + 27.2)$ eV = 23.8 eV

(2) $\text{KE}_{\lambda_1} = \frac{hc}{\lambda_1} - \psi = e\Delta V$

$$\text{KE}_{\lambda_2} = \frac{hc}{\lambda_2} - \psi = 2e\Delta V$$

$$\Rightarrow \quad 3\left(\frac{hc}{\lambda_1} - \psi\right) = \frac{hc}{\lambda_2} - \psi$$

$$\Rightarrow \quad \psi = hc\left(\frac{3}{2\lambda_1} - \frac{1}{2\lambda_2}\right)$$

$$\Rightarrow \quad \text{KE}_{\lambda_3} = \frac{hc}{\lambda_3} - hc\left[\frac{3}{2\lambda_1} - \frac{1}{2\lambda_2}\right] = hc\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$$

$$e\Delta V = hc\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$$

$$\Delta V = \frac{hc}{e}\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$$

71. (4) $E = \frac{Z^2}{n^2} E_0 = 13.6 \times (11^2)$ eV

72. (3) $\lambda_{\text{min}} = \frac{hc}{eV_{\text{max}}}$

73. (2) Use Moseley's law

74. (4) $\lambda_{\text{min}} = \frac{hc}{eV_{\text{max}}}$

75. (3) Assuming that ionization occurs as a result of a completely inelastic collision, we can write

$$mv - 0 = (m + m_H)\, u$$

where m is the mass of incident particle, m_H the mass of hydrogen atom, v_0 the initial velocity of incident particle, and u the final common velocity of the particle after collision. Prior to collision, the KE of the incident particle was

$$E_0 = \frac{mv_0^2}{2}$$

The total kinetic energy after collision

$$E = \frac{(m + m_H)u^2}{2} = \frac{m^2 v_0^2}{2(m + m_H)}$$

The decrease in kinetic energy must be equal to ionization energy. Therefore,

$$E_1 = E_0 - E = \left(\frac{m_H}{m + m_H}\right) E_0$$

i.e., $\frac{E_1}{E_0} = \frac{1}{1 + \frac{m}{m_H}}$

i.e., the greater the mass m, the smaller the fraction of initial kinetic energy that be used for ionization.

76. (2) $a = \frac{v^2}{r}$

$\therefore \quad a \propto \frac{(z)^2}{(1/z)}$ (for $n = 1$)

or $\quad a \propto z^3$

$$\therefore \quad \frac{a_1}{a_2} = \left(\frac{2}{1}\right)^3 = 8$$

77. (1) Shortest wavelength of Brackett series corresponds to the transition of electron between $n_1 = 4$ and $n_2 = \infty$ and the shortest wavelength of Balmer series corresponds to the transition of electron between $n_1 = 2$ and $n_2 = \infty$. So,

$$(Z^2)\left(\frac{13.6}{16}\right) = \left(\frac{13.6}{4}\right)$$

$\therefore \quad Z^2 = 4$

or $\quad Z = 2$

78. (2) Let v = speed of neutron before collision,

v_1 = speed of neutron after collision,

v_2 = speed of proton or hydrogen atom after collision,

and ΔE = energy of excitation

From conservation of linear momentum,

$$mv = mv_1 + mv_2 \qquad \text{...(i)}$$

From conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E \qquad \text{...(ii)}$$

From Eq. (i), $v^2 = v_1^2 + v_2^2 + 2v_1v_2$

From Eq. (ii), $v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$

$$\therefore \quad 2v_1v_2 = \frac{2\Delta E}{m}$$

$$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$$

$$\Rightarrow \quad (v_1 - v_2)^2 = v^2 - 4\frac{\Delta E}{m}$$

As $v_1 - v_2$ must be real, therefore $v^2 - 4\dfrac{\Delta E}{m} \geq 0$

or $\quad \dfrac{1}{2}mv^2 \geq 2\Delta E$

The minimum energy that can be absorbed by hydrogen atom in ground state to go into excited state is 10.2 eV. Therefore,

$$\frac{1}{2}mv_{\min}^2 = 2 \times 10.2 \text{ eV} = 20.4 \text{ eV}$$

79. (4) $r = \dfrac{\varepsilon_0 n^2 h^2}{e^2 \pi m} = \dfrac{\varepsilon_0 (2\pi L)^2}{e^2 \pi m} \qquad \left(L = n\dfrac{h}{2\pi} \text{ or } nh = 2\pi L\right)$

$\therefore \quad Lr^{-\frac{1}{2}} = \text{constant}$

80. (4) $\dfrac{E_{4n} - E_{2n}}{E_{2n} - E_n} = \dfrac{\dfrac{E_1}{16n^2} - \dfrac{E_1}{4n^2}}{\dfrac{E_1}{4n^2} - \dfrac{E_1}{n^2}} = \dfrac{1}{4} = \text{constant}$

81. (4) $E_n \propto \dfrac{1}{n^2}$ and $r_n \propto n^2$

Therefore, $E_n r_n$ is independent of n.

Hence, $E_1 r_1 = (13.6 \text{ eV})\,(0.53 \text{ Å}) = 7.2$ eV-Å = constant

82. (4) $\Delta\lambda = \lambda_{K_\alpha} - \lambda_{\min}$

When V is halved, $\lambda_{\min}$ becomes two times but λ_{K_α} remains the same.

$\therefore \quad \Delta\lambda' = \lambda_{K_\alpha} - 2\lambda_{\min} = 2\,(\Delta\lambda) - \lambda_{K_\alpha}$

$\therefore \quad \Delta\lambda' < 2\,(\Delta\lambda)$

83. (2) Potential energy $= -2 \times$ Kinetic energy $= -2E$

Total energy $= -2E + E = -3.4 \text{ eV} = -E$

or $\quad E = 3.4$ eV

p = momentum, m = mass of electron

$$E = \frac{p^2}{2m}$$

or $\quad p = \sqrt{2mE}$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}} \approx 10^{-24}$$

de Brogile wavelength $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = 6.6 \times 10^{-10}$ m

84. (2) de Broglie wavelength of electron in hydrogen atom

$= \dfrac{h}{mv} = \dfrac{2\pi r_n}{n}$

For second Bohr orbit, $\lambda = \dfrac{600 \times 10^{-9}}{2} = 3000 \times 10^{-9}$ m

$$\lambda = \sqrt{\frac{150}{V}} \text{ Å} = 300 \text{ Å}$$

$\therefore \quad V = \dfrac{150}{(3000)^2} = \dfrac{5}{3} \times 10^{-5}$ V

85. (3) Using Moseley's law, $v^{1/2} = a(Z - b)$

$$\left(\frac{c}{\lambda_1}\right)^{1/2} = a(Z_1 - b) \text{ and } \left(\frac{c}{\lambda_2}\right)^{1/2} = a(Z_2 - b)$$

$$\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}} = \frac{a(Z_1 - b)}{a(Z_2 - b)} \Rightarrow \left(\frac{7.12}{15.42}\right)^{\frac{1}{2}} = \frac{(29 - b)}{(42 - b)}$$

$(42 - b) = 1.47\,(29 - b) \Rightarrow b = 1.44$

$$\left(\frac{\lambda_1}{\lambda}\right)^{\frac{1}{2}} = \frac{(Z - 1.44)}{(Z_1 - 1.44)}$$

$$\left(\frac{15.42}{22.85}\right)^{\frac{1}{2}}(27.56) = Z - 1.44 \Rightarrow Z = 24$$

86. (4) Angular momentum, $L = 4.2176 \times 10^{-34} = \dfrac{n_2 h}{2\pi}$

$\Rightarrow \quad n_2 = 4$

For the transition from $n_2 = 4$ to $n_1 = 3$, the wavelength of spectral line $= \lambda$

$$\frac{1}{\lambda} = \frac{13.6}{hc}\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{13.6 \text{ eV}}{1240 \text{ eV nm}}\left(\frac{7}{9 \times 16}\right)$$

$$\lambda = \frac{1240 \times 144}{13.6 \times 7} = 1876 \text{ nm} = 18760 \text{ Å} = 1.876 \times 10^4 \text{ Å}$$

87. (3) The angular momentum is $mvr = \dfrac{nh}{2\pi} \Rightarrow n = 1$

Centripetal force, $\dfrac{mv^2}{r} = \dfrac{Ze^2}{4\pi\varepsilon_0 r^2}$

$$r = \frac{\varepsilon_0 n^2 h^2}{\pi m_\pi e^2 Z} = \left(\frac{\varepsilon_0 h^2}{\pi m_e e^2}\right)\left(\frac{m_e}{m_\pi}\right)\frac{1}{Z}$$

$$= \frac{0.53 \times 10^{-10}}{264\,Z} = \frac{200 \times 10^{-15}}{Z} \qquad \left[\because \frac{m_\pi}{m_e} = 264\right]$$

Since r cannot be less than nuclear radius, $r > 1.6\,Z^{\frac{1}{3}} \times 10^{-15}$ m

or $\quad \dfrac{200 \times 10^{-15}}{Z} > 1.6 \times 10^{-15}\, Z^{\frac{1}{3}}$

$\Rightarrow \quad Z < \left(\dfrac{200}{1.6}\right)^{\frac{3}{4}} < 37$

88. (1) For the first line of Balmer series of hydrogen

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \Rightarrow \lambda = \frac{36}{5R}$$

For singly ionized helium ($Z = 2$), $\dfrac{1}{\lambda'} = 4R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

Given $\lambda' = \lambda$

For $\quad n_1 = 6$ to $n_2 = 4$

$$\frac{1}{\lambda'} = 4R\left(\frac{1}{4^2} - \frac{1}{6^2}\right) = \frac{20\,R}{144} = \frac{5R}{36}$$

It corresponds to transition from $n_1 = 6$ to $n_2 = 4$.

89. (4) The energy of the K_α X-ray photon

$E_{K_\alpha} = E_i - E_f = (Z - 1)^2\,(-3.4 \text{ eV} + 13.6 \text{ eV}) = (Z - 1)^2\,(10.2 \text{ eV})$

Given $E_{K_\alpha} = 7.46$ keV

$\therefore \quad 7.46 \times 10^3 \text{ eV} = (Z - 1)^2\,(10.2 \text{ eV})$

or $\quad (Z - 1)^2 = \dfrac{7.46 \times 10^3}{10.2} = 731.4$

or $\quad (Z - 1) = 27 \Rightarrow Z = 28$

90. (1) The maximum number of electrons in an orbit is $2n^2$. Since $n > 4$ is not possible, therefore the maximum number of electrons that can be in the first four orbits are

$$2(1)^2 + 2(2)^2 + 2(3)^2 + 2(4)^2 = 2 + 8 + 18 + 32 = 60$$

Therefore, possible elements are 60.

91. (4) We know that $\dfrac{1}{\lambda} = RZ^2\left[\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right] \Rightarrow \dfrac{1}{\lambda} \propto Z^2$

λ is shortest when $1/\lambda$ is largest, i.e., when Z is big. Z is highest for lithium.

(4) For hydrogen and hydrogen-like atoms, $E_n = -13.6\frac{(Z)^2}{(n)^2}$ eV

Therefore, ground state energy of doubly ionized lithium atom ($Z = 3, n = 1$) will be

$$E_1 = (-13.6)\frac{(3)^2}{(2)^2} = -122.4 \text{ eV}$$

Therefore, ionization energy of an electron in ground state of doubly ionized lithium atom will be 122.4 eV.

(4) For an atom following Bohr's model, the radius is given by

$$r_m = \frac{r_0 m^2}{Z}$$

where r_0 = Bohr's radius and m = orbit number.

For Fm, $m = 5$ (i.e. fifth orbit in which the outermost electron is present)

$$\therefore \quad r_m = \frac{r_0 5^2}{100} = r_0 n \text{ (given)} \Rightarrow n = \frac{1}{4}$$

(1) The cut-off wavelength is given by

$$\lambda_0 = \frac{hc}{eV} \qquad \text{...(i)}$$

According to de Broglie equation,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} \Rightarrow \lambda^2 = \frac{h^2}{2meV}$$

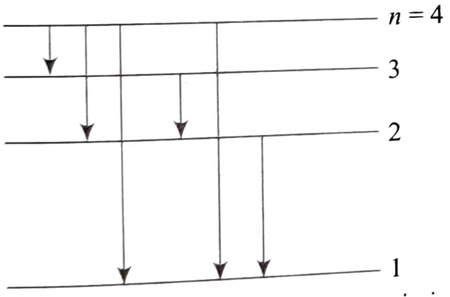
$$\Rightarrow \quad V = \frac{h^2}{2me\lambda^2} \qquad \text{...(ii)}$$

From (i) and (ii), $\lambda_0 = \frac{hc \times 2me\lambda^2}{eh^2} = \frac{2mc\lambda^2}{h}$

ltiple Correct Answers Type

(1), (2), (4)

$$\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$$



If the initial state were $n = 3$, in the emission spectrum, no wavelengths shorter than λ_0 would have occurred.

This is possible if initial state were $n = 2$.

(2), (3)

$$E_0 z^2\left(1 - \frac{1}{9}\right) - E_0 z^2\left(\frac{1}{4} - \frac{1}{9}\right) = 3E_0$$

$$\Rightarrow \quad \frac{27}{36}E_0 Z^2 = 3E_0$$

$$\Rightarrow \quad Z = 2$$

$$\frac{\lambda_1}{\lambda_2} = 3$$

$$\text{KE}_1 = E_0\left(1 - \frac{1}{9}\right) - \phi; \text{KE}_2 = E_0 Z^2\left(1 - \frac{1}{4}\right) - \phi$$

$$\text{KE} \propto \frac{1}{\lambda^2} = 8.5 \text{ eV}$$

3. (1), (3), (4)

(1) As $v \propto 1/n$, so momentum $= mv \propto 1/n$

(2) is not true as radius $r \propto n^2$

(3) $\text{KE} = \frac{1}{2}mv^2$,

As $v \propto 1/n$, so $\text{KE} \propto \frac{1}{n^2}$

(4) is true.

4. (2), (3), (4)

Ground state $n = 1$

First excited state $n = 2$

$$\text{KE} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}(Z = 1)$$

$$\text{KE} = \frac{14.4 \times 10^{-10}}{2r} \text{ eV}$$

Now $r = 0.53\, n^2$ Å ($Z = 1$)

$$(\text{KE})_1 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10}} \text{ eV} = 13.58 \text{ eV}$$

$$(\text{KE})_2 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10} \times 4} \text{ eV} = 3.39 \text{ eV}$$

KE decreases by 10.2 eV

$$\text{Now, PE} = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r} = \frac{-14.4 \times 10^{-10}}{r} \text{ eV}$$

$$(\text{PE})_1 = \frac{-14.4 \times 10^{-10}}{0.53 \times 10^{-10}} \text{ eV} = -27.1 \text{ eV}$$

$$(\text{PE})_2 = \frac{-14.4 \times 10^{-10}}{0.53 \times 10^{-10} \times 4} = -6.79 \text{ eV}$$

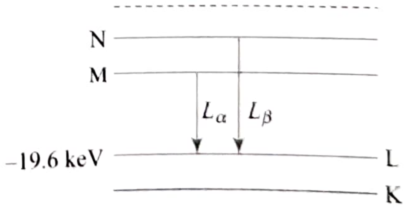
PE increases by 20.4 eV

Now, angular momentum, $L = mvr = \frac{nh}{2\pi}$

$$L_2 - L_1 = \frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{6.28} = 1.05 \times 10^{-34} \text{ J s}$$

5. (1), (2), (3)

$$\lambda_{\min} = \frac{12400}{V_0} \text{ Å} = \frac{12400}{20,000} = 62 \text{ Å}$$



6. (1), (2)

$$|F| = \frac{dU}{dt} = \frac{Ke^2}{r^4} \qquad \text{...(i)}$$

$$\frac{Ke^2}{r^4} = \frac{mv^2}{r} \qquad \text{...(ii)}$$

$$\text{and } mvr = \frac{nh}{2\pi} \qquad \text{...(iii)}$$

By (ii) and (iii),

$$r = \frac{Ke^2 4\pi^2}{h^2}\frac{m}{n^2} = K_1\frac{m}{n^2} \qquad \text{...(iv)}$$

where $K_1 = \dfrac{Ke^2 4\pi^2}{h^2}$

Total energy $= \dfrac{1}{2}$ (potential energy)

$$= \frac{Ke^2}{6r^3} = \frac{-Ke^2}{6\left(\frac{K_1 m}{n^2}\right)^3} = \frac{-Ke^2 n^6}{6K_1^3 m^3}$$

Total energy $\propto n^6$

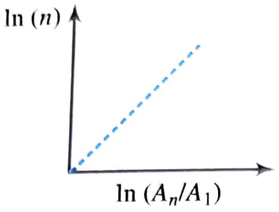
Total energy $\propto m^{-3}$

Therefore, (1) and (2) are correct.

7. (1), (2)

$r_n = n^2 r_1$

$$\ln\left(\frac{A_n}{A_1}\right) = \ln\left(\frac{\pi r_n^2}{\pi r_1^2}\right) = \ln n^4 = 4 \ln (n)$$



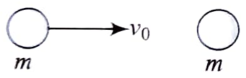
8. (2), (4)

Line emission spectra can be obtained for atoms and molecules both. An atom or molecule in an excited state emits photons by making a transition from excited state to ground state thus constituting line emission spectra.

The wavelengths emitted by the molecular energy levels which are generally grouped into several bunches are also grouped and each group is well separated from the other. The spectrum in this case looks like a band spectrum.

9. (2), (3), (4)

Let collision between two atoms be an inelastic one. From momentum conservation, $mv_0 = mv_1 + mv_2$



Before collision

After collision

From energy conservation, $\dfrac{mv_1^2}{2} + \dfrac{mv_2^2}{2} - \dfrac{mv_0^2}{2} = -\Delta E$

where ΔE is the energy absorbed by the initially stationary atom to change its state.

Solving above equations, we get $(v_1 - v_2)^2 = v_0^2 - \dfrac{4\Delta E}{m}$

For collision to be inelastic, $(v_1 - v_2)^2 \geq 0$: a real quantity [equal to sign for perfect inelastic collision].

The minimum value of ΔE is 10.2 eV, so for collision to be inelastic, $E \geq 20.4$ eV.

For perfectly inelastic collision, $v_1 = v_2$ and hence $E = 20.4$ eV.

For $E = 18$ eV, the collision is elastic one and as masses are the same, velocities would be interchanged during collision.

10. (1), (3), (4)

(1) $r_n = \dfrac{n^2 h^2}{4\pi^2 mke^2}$, i.e., $r_n \propto n^2$

(3) Bohr's 2nd postulate, $mvr = \dfrac{nh}{2\pi}$

(4) $K_n = \dfrac{kZe^2}{2r_n}$, $U_n = \dfrac{kZe^2}{r_n}$

11. (2), (3)

Lyman series lies in the ultraviolet region, Balmer series in visible region, and Paschen series in infrared region.

$$\lambda_R > \lambda_Y > \lambda_B > \lambda_V$$

12. (1), (2)

Continuous spectrum is obtained from the bulk state of matter, it has no relation with the atomic or molecular state. It is produced by thermal vibrations of atoms in the macroscopic matter (not by the transition between the energy states of atoms). Every vibrating atom emits light of frequency of its vibrations. In a white hot matter, atoms vibrating with all frequencies within a definite range are present, so this matter emits frequencies of continuous range.

13. (1), (2), (3)

$E_n^H = E_1^H + \Delta E = -13.6$ eV $+ 12.75$ eV $= -0.85$ eV i.e., hydrogen atoms are excited to $n = 4$ level, i.e., transitions $4 \to 1, 3 \to 1, 2 \to 1$ are possible which correspond to Lyman series, then transitions $4 \to 2$ and $3 \to 2$ are possible which correspond to Balmer series, and then transition $4 \to 3$ is also possible which correspond to Paschen series.

14. (1), (2), (3)

(1) $U_1 = E$, then total energy in the orbit $= \dfrac{U_1}{2} = \dfrac{E}{2}$

(2) $\text{IE} = (\text{TE})_{n=\infty} - (\text{TE})_{n=1} = 0 - (\text{TE})_{n=1} = -\left(\dfrac{E}{2}\right)$

(3) $(\text{KE})_{n=1} = -(\text{TE})_{n=1} = -\left(\dfrac{E}{2}\right)$

15. (1), (3), (4)

$$p_n = mv_n,\ p_n \propto v_n \propto \frac{1}{n},\ r_n \propto n^2 \text{ and } K_n = \frac{Ke^2}{2r_n}$$

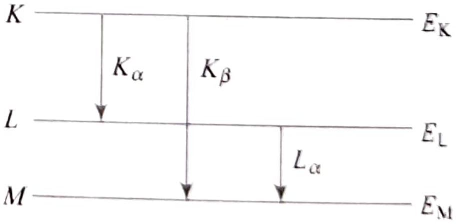
i.e., $K_n \propto \dfrac{1}{n^2}$ and $L \propto n$

16. (2), (3)

Any transition causing a photon to be emitted in the Balmer series must end at $n = 2$. This must be followed by the transition from $n = 2$ to $n = 1$, emitting a photon of energy 10.2 eV, which corresponds to a wavelength of about 122 nm. This belongs to the Lyman series.

17. (1), (3)

$$E_K - E_L = \frac{hc}{\lambda_\alpha} \quad \text{...(i)}$$



$$E_K - E_M = \frac{hc}{\lambda_\beta} \quad \text{...(ii)}$$

$$E_L - E_M = \frac{hc}{\lambda'_\alpha} \quad \text{...(iii)}$$

(ii) – (i) ⇒ $E_L - E_M = \dfrac{hc}{\lambda'_\alpha} = \dfrac{hc}{\lambda_\beta} - \dfrac{hc}{\lambda_\alpha}$

$$\frac{1}{\lambda_\beta} = \frac{1}{\lambda_\alpha} + \frac{1}{\lambda'_\alpha}$$

Also, $(E_K - E_M) > (E_K - E_L) > (E_L - E_M)$

$$\frac{hc}{\lambda_\beta} > \frac{hc}{\lambda_\alpha} > \frac{hc}{\lambda'_\alpha}$$

18. (2), (4)

When potential difference between filament and target is increased, then KE of striking electrons gets increased. Since most energetic electrons now strike the target, therefore more energetic electrons are emitted. It means, frequency of X-ray photons increases. It means, penetration power of X-rays gets increased.

To increase the photon flux, rate of collision of electrons with the target must be increased. This can be achieved only when rate of emission of electrons from the filament is increased. To achieve this, filament current must be increased. Therefore, options (2) and (4) are correct.

19. (1), (3)

First line of Lyman series is obtained during transition of hydrogen atom from $n = 2$ to $n = 1$. Hence, its energy is equal to $E_2 - E_1 = (13.6)\text{ eV} - (-3.4\text{ eV}) = -1.2\text{ eV}$.

$\therefore$ Wavelength of the first line of Lyman series is equal to

$$\frac{12375}{10.2}\text{ Å} = 1215\text{ Å}$$

Therefore, it lies in ultraviolet region.

Since energy of all the other lines of Lyman series is greater than that of first line, therefore all the lines of Lyman series lie in ultraviolet region. Hence, option (1) is correct.

First line of Balmer series is obtained during transition of hydrogen atom from $n = 3$ to $n = 2$. Hence, its energy is equal to $E_3 - E_2 = 1.89$ eV.

$\therefore$ Wavelength of first line of Balmer series is equal to 12375/1.89 Å = 6563 Å. It lies in visible region.

Energy of last line of Balmer series is equal to 3.4 eV. Therefore, its wavelength is equal to12375/3.4 Å = 3640 Å.

Since it is less than 4000 Å, therefore it lies in ultraviolet region. Hence, option (2) is wrong. Energy of last line of Paschen series is equal to 1.51 eV. It lies in infrared region. Since energy of all the other lines of Paschen series is less than its energy, therefore all the lines of Paschen series will lie in infrared region. Hence, option (3) is also correct.

20. (2), (3), (4)

Statement (1) is false. The shortest wavelength of the X-rays emitted depends on the energy of the electrons incident on the target. This, in turn, depends on the potential through which they have fallen. In fact, $\lambda_{\text{min}} = \dfrac{hc}{eV}$

Statement (2) is true. X-ray spectra of all heavy elements are similar in character.

Statement (3) is also true. The short wavelength of the X-rays (compared to the grating constant of optical grating) makes it difficult to observe X-ray diffraction with ordinary gratings.

Statement (4) is also true. The sharp limit on the short wavelength side is dependent on the voltage applied to the incident electrons and is given by $\lambda_{\text{min}} = \dfrac{hc}{eV}$

21. (3), (4)

$$V = 6.6\text{ kV} = 6600\text{ V}$$

Now, $\nu_{\text{max}} = \dfrac{eV}{h} = \dfrac{1.6 \times 10^{-19} \times 6600}{6.6 \times 10^{-34}} = 1.6 \times 10^{18}\text{ Hz}$

Thus, the frequency of the X-rays cannot exceed 1.6×10^{18} Hz. Hence, the correct choices are (3) and (4).

22. (1), (3)

Statement (1) is correct. The angular momentum of the earth ($= mr^2\,\omega$) has to be equal to $nh/2\pi$. This gives, $n = 2\pi mr^2\,\omega/h$. Putting the numerical values of the Earth's mass, radius, and the angular velocity, we get the given value of n.

Statement (2) is incorrect. The maximum number of electrons allowed in an orbit being $2n^2$, the required number is $2(1^2 + 2^2 + 3^2 + 4^2) = 60$.

Statement (3) is correct. The 'reduced mass' of the electron $[\mu = mM/(m + M)]$ being dependent on the mass of the nucleus (M), the Rydberg constant also varies with the mass number of the given element.

Statement (4) is incorrect. The ratio is not exactly equal to 4 but slightly different from 4 because of the dependence of the Rydberg constant on the mass number of the element concerned.

23. (1), (2)

The correct choices are (1) and (2). The last two statements are incorrect because they violate the principle of conservation of charge. We always have either an electron–positron 'pair production' or an electron–positron 'pair annihilation'. It is only then that the total charge remains zero both before and after reaction.

24. (1), (3)

Moseley's law: $\lambda \propto \dfrac{1}{(z-1)^2}$; $\dfrac{\lambda_z}{\lambda_1} = \dfrac{(z_1-1)^2}{(z-1)^2} = 4$

$z_1 - 1 = (z - 1)\,2;\ z_1 = 2z - 1;$

Similarly, $\dfrac{\lambda_z}{\lambda_2} = \dfrac{1}{4} = \left(\dfrac{z_2 - 1}{z - 1}\right)^2;\ z_2 = \dfrac{z+1}{2}$

25. (1), (3), (4)

In Bohr model of hydrogen atom,

$$R \propto n^2,\ V \propto \frac{1}{n},\ T \propto n^3 \text{ and } E \propto \frac{1}{n^2}$$

$$\Rightarrow\ VR \propto n;\ TE \propto n;\ \frac{T}{R} \propto n \text{ and } \frac{V}{E} \propto n$$

26. (1), (3)

Power loss increases the temperature.

27. (1), (2), (3)

Energy of K absorption edge

$$E_{\text{K}} = \frac{1242\text{ eVnm}}{0.0172\text{ nm}} = 72.21 \times 10^3\text{ eV} = 72.21\text{ ke V}$$

Energy of K_α line is $E_{K_\alpha} = \dfrac{he}{e\lambda_\alpha} = \dfrac{1242\text{ eVnm}}{0.021\text{ nm}} = 59.14\text{ keV}$

Similarly, $E_{K_\beta} = \dfrac{1242}{0.0192} = 64.69\text{ keV}$

$$E_{K_\gamma} = \frac{1242}{0.0180} = 69\text{ keV}$$

Energy of K shell $= (E_{K_\alpha} - E_{\text{K}})$

$= (59.14 - 72.21)\text{ keV} = -13.04\text{ keV}$

Energy of L shell $= E_{K_\beta} - 72.21\text{ keV}$

$= 64.69\text{ keV} - 72.21\text{ keV} = -7.52\text{ keV}$

Energy of M shell $= E_{K_\gamma} - E_K = \dfrac{1242\text{eVnm}}{0.018\text{ nm}} - 72.21\text{ keV}$

$= 69\text{ keV} - 72.21\text{ keV} = -3.21\text{ keV}$

28. (1), (2)

For the third line of Balmer series, $n_1 = 2,\ n_2 = 5$

$$\therefore\ \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = RZ^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = \frac{21RZ^2}{100}$$

$E = -13.6$ eV

$$Z^2 \times \frac{21}{100} = \frac{hc}{\lambda} = \frac{1242 \text{ eVnm}}{108.5 \text{ nm}}$$

$$Z^2 = \frac{1242 \times 100}{108.5 \times 21 \times 13.6} = 4 \Rightarrow Z = 2$$

Binding energy of an electron in the ground state of hydrogen-like ion $= 13.6Z^2/n^2 = 54.4$ eV ($n = 1$).

29. (1), (2), (4)

$$28.3 = 13.6Z^2\left[\frac{1}{n_1^2} - \frac{1}{n^2}\right] \quad \text{...(i)}$$

$$302.2 = 13.6Z^2\left[1 - \frac{1}{n_1^2}\right] \quad \text{...(ii)}$$

For K_α, $\frac{1}{\lambda} = \frac{3R}{4}(Z-1)^2$

$$\frac{10^9}{7.64} = \frac{3\times 1.09677 \times 10^7}{4}(Z-1)^2$$

$$(Z-1)^2 = \frac{4\times 100}{7.6 \times 3 \times 1.09}$$

$$Z = 5 \quad \text{...(iii)}$$

Solving Eqs. (ii) and (iii)

$$302.2 = 13.6(5)^2\left[1 - \frac{1}{n_1^2}\right]$$

$$n_1 = 3 \quad \text{...(iv)}$$

From Eqs. (i), (iii) and (iv)

$n = 6$

30. (2), (3), (4)

Since Rydberg constant $R \propto$ mass of electron

$$\frac{1}{\lambda} = 3R\left(\frac{1}{n^2} - \frac{1}{3^2}\right) \text{ where } n = 1, 2$$

So, the possible emitted wavelength are $\frac{3}{8R}, \frac{4}{9R}$ and $\frac{12}{5R}$

31. (3), (4)

In the case of hydrogen, atomic number = mass number.

In other atoms, atomic number < mass number.

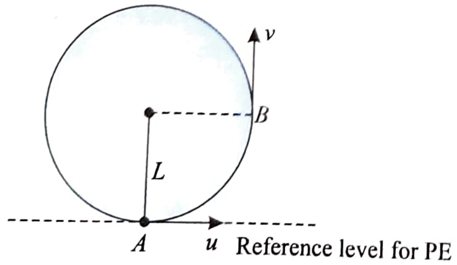
Therefore, (3) and (4) are the correct options.

32. (1), (4)

The time period of the electron in a Bohr orbit is given by $T = 2\pi r/v$

Since for the n^{th} Bohr orbit, $mvr = n(h/2\pi)$, the time period becomes

$$T = \frac{2\pi r}{nh/(2\pi m r)} = \left(\frac{4\pi^2 m}{nh}\right) r^2$$



Since the radius of the orbit r depends on n, we replace r.

The expression of Bohr radius of a hydrogen atom is

$$r = n^2\left(\frac{h^2\varepsilon_0}{\pi m e^2}\right)$$

Hence, $T = \left(\frac{4\pi^2 m}{nh}\right)\left(\frac{n^4 h^4 \varepsilon_0^2}{\pi^2 m^2 e^4}\right) = n^3\left(\frac{4h^3\varepsilon_0^2}{me^4}\right)$

For two orbits, $\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3$

It is given that $T_1/T_2 = 8$. Hence, $n_1/n_2 = 2$.

Linked Comprehension Type

For Problems 1–3

1. (2) On other planet: $mvr = 2n\frac{h}{2\pi} \Rightarrow v = \frac{nh}{\pi m r}$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \Rightarrow \frac{mn^2h^2}{\pi^2 m^2 r^3} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

Putting $n = 1$, we get $r = \frac{4h^2\varepsilon_0}{m\pi e^2}$

2. (2) On our planet: $v_0 = \frac{e^2}{2\varepsilon_0 nh}$

On other planet: $v = \frac{e^2}{2\varepsilon_0(2n)h} = \frac{v_0}{2}$

3. (2) On our planet: $E_n = -\frac{13.6}{n^2}$

On other planet: $E'_n = -\frac{13.6}{(2n)^2}$

$$\Rightarrow \quad E'_n = \frac{E_n}{4} = -3.4 \text{ eV}$$

For Problems 4–6

4. (1) Internal forces act between electron and proton, then how can the atom get an acceleration.

5. (3) Change in angular momentum $= I\omega$

$$\Rightarrow \quad \frac{mh}{2\pi} - \frac{nh}{2\pi} = I\omega \Rightarrow \omega = \frac{(m-n)h}{6.8I}$$

6. (4) Since no external force and torque act on the atom, still it gets an acceleration and angular acceleration. So, none is true.

For Problems 7–12

7. (3) Given that $E_1 = -15.6$ eV, $E_\infty = 0$ eV.

Ionization energy of the atom: $E_\infty - E_1 = 0 - (-15.6 \text{ eV}) = 15.6$ eV

So, ionization potential = 15.6 V

8. (2) For short wavelength limit of the series terminating at $n = 2$, a transition must take place from $n = \infty$ state to $n = 2$ state. For this, $\Delta E = 5.30$ eV

$$\lambda = \frac{12400}{\Delta E\,(\text{eV})} \text{ Å} = \frac{12400}{5.30} \text{ Å} = 2339 \text{ Å}$$

9. (3) The excitation energy for the $n = 3$ state is

$$\Delta E = E_3 - E_1 = 15.6 - 3.08 = 12.52 \text{ eV}$$

Excitation potential = 12.52 V

10. (2) $\lambda = \frac{12400}{E_3 - E_1} = \frac{12400}{12.52} \text{ Å} = 990 \text{ Å}$

Wave number $= \frac{1}{\lambda} = \frac{1}{990 \times 10^{-10} \text{ m}} = 1.009 \times 10^7 \text{ m}^{-1}$

11. (3) Because excitation energy for the second shell is more than 6 eV, hence electron having initial kinetic energy of 6 eV will not interact with the atom. Because it cannot transfer its energy to the electron in the atom.

(1) As $E_2 - E_1 = 15.6 - 5.3 = 10.3$ eV

hence energy of the electron after interaction is

$$11 - 10.3 \text{ eV} = 0.7 \text{ eV}$$

Problems 13–14

(3) We can use the equation:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\lambda = \frac{4}{3R} = \frac{4}{3(1.097 \times 10^7)} = 1.215 \times 10^{-7} \text{ m} = 121.5$$

The frequency of the photon is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8}{1.215 \times 10^{-7}} = 2.47 \times 10^{15} \text{ Hz}$$

The energy of the photon is $E = hf$

$$= (4.136 \times 10^{-15})(2.47 \times 10^{15}) = 10.2 \text{ eV}$$

(2) From conservation of momentum, as the total momentum before emission is zero, the total momentum after emission must also be zero. The photon and atom therefore move off in opposite directions, with $mv = E_{\text{photon}}/c$, where m and v are the mass and recoil speed of the hydrogen atom, E_{photon} is the actual energy of the photon (less than 10.2 eV), and c is the speed of light. The energy difference between the $n = 2$ and $n = 1$ levels, E is the source of both the photon energy and the recoil kinetic energy of the atom.

From energy conservation, we have $E = E_{\text{photon}} + \frac{1}{2}mv^2$

Atom being massive, we can assume that its recoil speed v and kinetic energy are so small that $E = E_{\text{photon}}$. Substituting $E_{\text{photon}} = 10.2$ eV into the expression for mv yields.

$$mv = 10.2 \text{ eV}/c$$

The recoil kinetic energy of the hydrogen atom can now be calculated.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = (0.5)\frac{(10.2)^2}{mc^2}$$

$$= \frac{(0.5)(10.2)^2}{938.8 \times 10^6} = 5.54 \times 10^{-8} \text{ eV}$$

Thus, the fraction of the energy difference between the $n = 2$ and $n = 1$ levels that goes into atomic recoil energy is very small.

$$\frac{K}{E} = \frac{5.54 \times 10^{-8}}{10.2} = 5.43 \times 10^{-9}$$

That is why by equating the photon's energy to the atomic energy level separation yields accurate answers because little energy is needed to conserve momentum.

or Problems 15–16

5. (2) As we know, energy of a photon is given by $E = \dfrac{hc}{\lambda}$

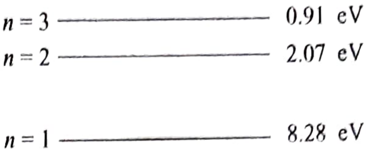
From the given condition, $\lambda = \dfrac{1500p^2}{p^2 - 1}$

Hence, $E = \dfrac{hc}{\lambda} = \dfrac{hc}{1500}\left(1 - \dfrac{1}{p^2}\right) \times 10^{10}$ J

$$= \frac{hc}{(1500)(1.6 \times 10^{-19})}\left(1 - \frac{1}{p^2}\right) \times 10^{10} \text{ eV}$$

$$= 8.28\left(1 - \frac{1}{p^2}\right) \text{ eV}$$

Hence, energy of nth state is given by $E_n = -\dfrac{8.28}{n^2}$ eV



Maximum energy is released for transition from $p = \infty$ to $p = 1$; hence, wavelength of most energetic photon is 1500 Å.

16. (4) The ionization potential corresponds to energy required to liberate an electron from its ground state, i.e. ionization energy = 8.28 eV.

Hence, ionisation potential = 8.28 V

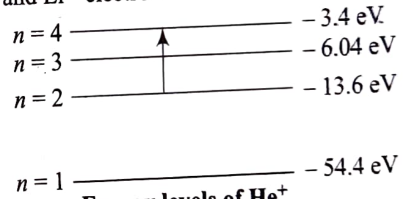
For Problems 17–20

17. (2), 18. (1), 19. (3), 20. (2)

Consider energy level diagram of hydrogen atom. After absorbing photons of energy 10.2 eV, it would reach the first excited state of –3.4 eV, since energy difference corresponding to $n = 1$ and $n = 2$ is 10.2 eV. When this excited hydrogen atom de-excites, it would release 10.2 eV energy, which is absorbed by He^+ and Li^{2+}.

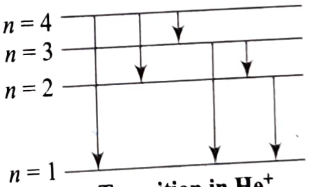
Energy of the n^{th} state of a hydrogen-like atom with atomic number Z is given by $E_n = \dfrac{13.6Z^2}{n^2}$ eV

After absorbing 10.2 eV, He^+ electron moves from $n = 2$ to $n = 4$ and Li^{2+} electron moves from $n = 3$ to $n = 6$.



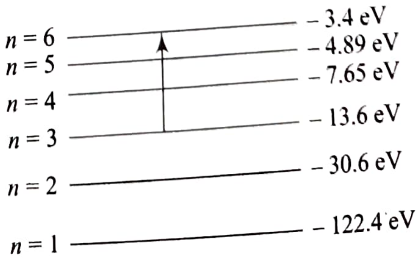
Energy levels of He⁺

In the spectrum of He^+, there would be ${}^4C_2 = 6$ lines.



Transition in He⁺

$$\lambda_{\min\, n=4\to n=1} = \frac{hc}{\Delta E} = \frac{1242}{13.6\left[4 - \frac{4}{16}\right]} = 24.4 \text{ nm}$$



Similarly, in spectrum of Li^{2+} there will be ${}^6C_2 = 15$ lines.

$$\lambda_{\min\, n=6\to n=1} = \frac{1242}{13.6\left[9 - \frac{9}{36}\right]} = 10.4 \text{ nm}$$

For Problems 21–22

21. (3) For a conservative force field, $-\dfrac{dU}{dr} = F$

Since $U = k \log r$, $-\dfrac{dU}{dr} = -\dfrac{k}{r} = F$

This force $F = -k/r$ provides the centripetal force for circular motion of electron.

$$\frac{mv^2}{r} = \frac{k}{r} \qquad \text{...(i)}$$

Applying Bohr's quantization rule,

$$mvr = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get $r = \dfrac{nh}{2\pi\sqrt{mk}}$

22. (1) From Eq. (i), KE of electron $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}k$

Total energy of electron = KE + PE

PE of electron $= k \log r = \dfrac{1}{2}k + k \log r$

$$E = \frac{k}{2}\left[1 + \log\frac{n^2h^2}{4\pi^2 mk}\right]$$

For Problems 23–25

23. (4) For Balmer series, $n_1 = 2$; $n_2 = 3, 4, \ldots$

(lower) (higher)

Therefore, in transition (VI), proton of Balmer series is absorbed.

24. (3) In transition II, $E_2 = -3.4$ eV, $E_4 = -0.85$ eV

$\Delta E = 2.55$ eV

$$\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = 487 \text{ nm}$$

25. (4) Wavelength of radiation = 103 nm = 1030 Å

$$\Delta E = \frac{12400}{1030 \text{ Å}} = 12.0 \text{ eV}$$

So, difference of energy should be 12.0 eV (approx)

Hence $n_1 = 1$ and $n_2 = 3$

(−13.6) eV (−1.51) eV

Therefore, transition is V.

For Problems 26–28

26. (3), 27. (2), 28. (2)

Let n be the lowest energy level of the given series of lines, then maximum wavelength is given by

$$\frac{hc}{\lambda_{\max}} = 13.6 \times Z^2\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$\frac{hc}{\lambda_{\min}} = 136 \times Z^2\left[\frac{1}{n^2} - \frac{1}{\infty^2}\right]$$

where $\lambda_{\max} = 41.02$ nm and $\lambda_{\min}$ 22.79 nm.

Solving above equations, we get $n = 2$, which corresponds to Balmer series.

$Z = 4$

For next to longest wavelength, transition takes place from $n_1 = 4$ to $n_2 = 2$.

So, $$\frac{hc}{\lambda} = 13.6 \times 4^2\left[\frac{1}{4} - \frac{1}{16}\right]$$

$\therefore$ $\lambda = 30.47$ nm

For Problems 29–31

29. (1) The minimum energy required to remove the electron from K-level is 3×10^{-15} J.

Let V be the potential difference required, then $eV = 3 \times 10^{-15}$ J

$$V = \frac{3 \times 10^{-15}}{1.6 \times 10^{-19}} = 18750 \text{ V}$$

30. (2) The energy released due to transition of electron from L-level to K-level is

$$\Delta E = 3 \times 10^{-15} - 2 \times 10^{-16} = 28 \times 10^{-16} \text{ J}$$

The wavelength of the electromagnetic wave is

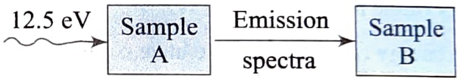
$$\lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{28 \times 10^{-16}} \text{ m} = 7.104 \times 10^{-11} \text{ m}$$

31. (4) The minimum energy required to eject the electron from M–level is 3×10^{-17} J. The energy used up in ejecting the electron from M–level is, 28×10^{-16} J. So, the kinetic energy of emitted electrons from M–level is $K = (28 \times 10^{-16} - 3 \times 10^{-17})$J or $K = 277 \times 10^{-17}$ J

For Problems 32–34

32. (4), 33. (4), 34. (4)

As photon energy of incident light is not equal to energy difference between two energy states, that is why sample A does not absorb any photon and therefore remains in ground state, and hence, no emission spectra. Thus, sample B remains as it is, but it will de-excite itself to ground state by emitting radiations of energy 10.2 eV.



When photon energy is replaced by electron beam, then atoms of sample A can absorb either 10.2 eV (to reach 1st excited state) or 12.1 eV (to reach 2nd excited state). In emission spectra of A, we will have 3 lines corresponding to $n = 2$ to 1, $n = 3$ to 2 and $n = 3$ to 1, having energies equal to 10.2 eV, 1.9 eV, and 12.1 eV, respectively.

As least energy of this emission spectra is corresponding to transition from $n = 2$ to 3 and sample B is in 1st excited state, so B can be excited to some higher energy level and if B absorbs energy corresponding to $n = 1$ to 2 and $n = 1$ to 3, then it may ionize.

For Problems 35–37

35. (1) $\Delta E = -13.6Z^2\left[\dfrac{1}{2^2} - \dfrac{1}{3^2}\right]$

$$47.2 = 13.6Z^2\left[\frac{1}{4} - \frac{1}{9}\right] \Rightarrow Z = 5$$

36. (2) $r_n = \dfrac{0.529}{Z} \times 10^{-10} \text{ m} = \dfrac{0.529}{5} \times 10^{-10} = 0.106 \times 10^{-10}$ m

37. (1) $L = mvr = \dfrac{nh}{2\pi}$

For first Bohr orbit, $L = 1 \times \dfrac{h}{2\pi} = 1.05 \times 10^{-13}$ Js

For Problems 38–39

38. (3) Characteristic X-ray arises due to transition of electrons.

39. (1) When the accelerated electron beam strikes the nucleus of target atom, continuous X-rays are produced.

Problems 40–42

(1) In photoelectric effect

$$E = W + K_{max} = 0.73 + 1.82 = 2.55 \text{ eV}$$

(1) $E_n = \dfrac{-13.6 z^2}{n^2}$ eV

$\therefore \quad E_1 = -13.6$ eV; $E_2 = -3.4$ eV; $E_3 = -1.51$ eV; $E_4 = -0.85$ eV

$$E_4 - E_2 = -(0.85) - (-3.4) = 2.55 \text{ eV}$$

(3) $L_4 - L_2 = \dfrac{4h}{2\pi} - \dfrac{2h}{2\pi} = \dfrac{2h}{2\pi} = \dfrac{h}{\pi}$ $\qquad \left[L = \dfrac{nh}{2\pi}\right]$

Problems 43–45

(1) Least intensity $2 \to 1 \Rightarrow K_\alpha$, X–rays

(3) $4 \to 1 \Rightarrow K_\gamma$, X–rays

(4) Calculate from energy of Li^{++}.

Problems 46–47

(2), **47. (4)**

$$\Delta E = \left\{13.6\, Z_B^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) - 13.6\, Z_A^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)\right\} \text{eV}$$

$$81.6 \text{ eV} = \frac{1.36 \times 3}{4}(Z_B^2 - Z_A^2) \text{ eV}$$

$$Z_B^2 - Z_A^2 = 8 \qquad \text{...(i)}$$

Using conservation of momentum,

For A, $m_A u = MV_1 - m_A \dfrac{u}{2}$

$$\frac{3}{2} m_A u = MV_1$$

For B, $-m_B u = MV_2$

But, $MV_2 = 3MV_1$

$\therefore \quad m_B = 3m_A$

Since both A and B carry same number of protons and neutrons, we have

$$Z_B = 3Z_A \qquad \text{...(ii)}$$

But $\quad Z_B^2 - Z_A^2 = 8$

$$9Z_A^2 - Z_A^2 = 8$$

$$Z_A = 1, Z_B = 3$$

Hence, A is ${}_1^2$H and B is ${}_4^6$Li

Now, the difference in energy between the first Balmer lines emitted by A and B

$$\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)Z_B^2 - 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)Z_A^2$$

$$= 13.6 \times \frac{5}{36} \times (Z_B^2 - Z_A^2) = \frac{13.6 \times 5 \times 8}{36} = 15.1 \text{ eV}$$

For Problems 48–49

48. (3) Number of atoms in the second excited state

$$N_2 = \frac{N_{av}}{M} \times m \times \frac{15}{100} = \frac{6.023 \times 10^{23}}{1.008} \times 1.8 \times \frac{15}{100} = 1.61 \times 10^{23}$$

In the first excited state, $N_1 = \dfrac{N_{av}}{M} m \times \dfrac{27}{100} = 2.9 \times 10^{23}$

49. (1) Amount of energy that would be evolved $= U_i - U_f$

$$U_i = N_1 E_1 + N_2 E_2$$

$$= -1.61 \times 10^{23} \times \frac{21 \times 10^{-12}}{3^2} - \frac{2.9 \times 10^{23} \times 21.7 \times 10^{-12}}{2^2}$$

$$= -3.88 \times 10^{11} - 15.75 \times 10^{11} = -19.63 \times 10^{11} \text{ erg}$$

$$U_f = (N_1 + N_2)E_0$$

$$= -(1.61 \times 10^{23} + 2.9 \times 10^{23})\, 21.7 \times 10^{-12} = -97.87 \times 10^{11} \text{ erg}$$

Energy evolved $= U_i - U_f = (-19.63 \times 10^{11} + 97.87 \times 10^{11})$

$$= 78.237 \times 10^{11} = 782 \text{ kJ}$$

For Problems 50–52

50. (4) As emission spectrum of the atom shows 10 different lines, so the highest excited state of the electron is $n = 5$.

51. (2) As some lines have energy less then $\dfrac{64}{225}E_0$ and some have more than $\dfrac{64}{225}E_0$ and some have equal to it. So initial state of electron can be $n = 2$ or $n = 3$.

Again if initial level is $n = 2$, then

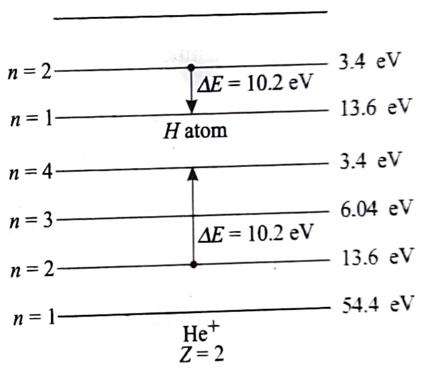
$$Z^2\left(-\frac{E_0}{25}\right) - Z^2\left(-\frac{E_0}{4}\right) = \frac{64}{225}E_0,$$

Z is in fraction (not possible)

52. (1) If $n = 3$, then $Z^2\left(-\dfrac{E_0}{25}\right) - Z^2\left(-\dfrac{E_0}{9}\right) = \dfrac{64}{225}E_0$, $Z = 2$

For Problems 53–55

53. (3)



Energy given by H atom in transition from $n = 2$ to $n = 1$ is equal to energy taken by He⁺ atom in transition from $n = 2$ to $n = 4$.

54. (3) Visible light lies in the range, $\lambda_1 = 4000$ Å to $\lambda_2 = 7000$ Å. Energy of photons corresponding to these wavelengths (in eV) would be

$$E_1 = \frac{12375}{4000} = 3.09 \text{ eV, and } E_2 = \frac{900}{11R} = 1.77 \text{ eV}$$

From energy level diagram of He⁺ atom, we can see that in transition from $n = 4$ to $n = 3$, energy of photon released will lie between E_1 and E_2.

$$\Delta E_{43} = -3.4 - (-6.04) = 2.64 \text{ eV}$$

Wavelength of photon corresponding to this energy,

$$\lambda = \frac{12375}{264} \text{Å} = 4687.5 \text{ Å} = 4.68 \times 10^{-7} \text{ m}$$

55. (1) Kinetic energy, $K \propto Z^2$

$$\frac{K_H}{K_{He^+}} = \left(\frac{1}{2}\right) = \frac{1}{4}$$

Matrix Match Type

1. i. → a., c.; ii. → b., d.; iii. → b., d.; iv. → b., c.

(i) $f = \dfrac{mZ^2 e^4}{4\varepsilon_0^2 h^3 n^3} \quad \Rightarrow \quad f \propto \dfrac{Z^2}{n^3}$

(ii) $L = \dfrac{nh}{2\pi} \quad \Rightarrow \quad L \propto n$

(iii) Magnetic moment, $M = IA = \dfrac{e}{2\pi r} v\pi r^2$

$$= \frac{e}{2} vr = \frac{e}{2m}(mvr) = \frac{e}{2m} L$$

$$\Rightarrow \quad M = \frac{e}{2m}\left(\frac{nh}{2\pi}\right) \quad \Rightarrow \quad M \propto n$$

(iv) $I = \dfrac{ev}{2\pi r} = \dfrac{e}{2\pi}\left(\dfrac{\pi m Z e^2}{n^2 h^2 \varepsilon_0}\right)\dfrac{Ze^2}{2\varepsilon_0 nh} \quad \Rightarrow \quad I \propto \dfrac{Z^2}{n^3}$

2. i. → d.; ii. → a.; iii. → d.; iv. → c.

(i) Photoelectrons can have KE anywhere from 0 to $\text{KE}_{\text{max}} = hc/\lambda - \phi$. So, average KE is unpredictable.

(ii) Minimum KE can be zero.

(iii),(iv) In continuous X-rays, wavelength can vary from $\lambda_{\text{min}} = hc/eV$ to ∞.

3. i. → b., c.; ii. → a., c.; iii. → a., c.; iv. → d.

(i) KE of electrons striking the target: KE = eV.

So, if V increases, KE increases.

Cut-off wavelength, $\lambda_{\text{min}} = \dfrac{12400}{\text{V}}$ Å

So, if V increases, λ_{min} decreases.

(ii) If work function of target is increased in photoelectric effect, the KE of photoelectrons emitted decreases from

$$\text{KE} = hf - W_0$$

Now, $hf_0 = W_0 \Rightarrow \dfrac{hc}{\lambda_0} = W_0 \Rightarrow \lambda_0 = \dfrac{hc}{W_0}$

If work function (W_0) increases, then cut-off wavelength (λ_0) decreases.

(iii) eV_0 = KE, where V_0 is stopping potential. If V_0 decreases, then KE decreases. V_0 is also decreased by increasing W_0, hence λ_0 decreases as explained above.

(iv) $f_{K_\alpha} = \dfrac{3}{4}Rc(Z-1)^2$. If f_{K_α} increases, then f_{K_α} decreases and hence Z decreases.

4. i. → c.; ii. → a., b., c., d.; iii. → b., iv. → d.

(i) Moseley's law is about characteristic X-rays.

(ii) In photoelectric effect, from electromagnetic radiation, electrons are ejected. In X-rays, the process is reverse; here, electromagnetic radiation (X-ray) is produced from fast moving electrons.

Also X-rays are produced using high potential difference.

(iii) Cut-off wavelength is related to potential difference applied in continuous X-rays.

(iv) In continuous X-rays, electromagnetic radiations are emitted.

5. i. → c., d.; ii. → b., c., d.; iii. → c., d.; iv. → b., c., d.

From Moseley's law: $f = \dfrac{1}{\lambda} = R(Z-\sigma)^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

For Al, Z is highest and for K_β, $n_1 = 1$ and $n_2 = 3$; and for K_α, $n_1 = 1$, and $n_2 = 2$

Hence, order of frequency:

$$f_{K_\beta}(\text{Al}) > f_{K_\alpha}(\text{Al}) > f_{K_\alpha}(\text{Na}) > f_{K_\beta}(\text{Be}) > f_{K_\alpha}(\text{Li})$$

Speed will be same (iii) for all photons of any frequency.

6. i. → b., c., d.; ii. → a., b., c., d.; iii. → b., c., d.; iv. → b., c., d.

(i) Emission spectra is discrete as only those wavelengths are emitted which correspond to energy difference of two energy levels.

It is due to electronic transition, i.e., due to transition of electron from one energy level (higher) to the another energy level (lower). It is explained by quantum theory of light.

(ii) For energies of incident light less than ionization energy, the absorption spectra is discrete. For energies of incident photon greater than ionization energy, the absorption spectra is continuous. It occurs due to electronic transition which is explained by quantum theory of light.

(iii) Continuous X-rays constitute continuous spectra and characteristic X-rays arise due to electronic transition.

(iv) Thermal radiation spectra is a continuous spectra and is explained by atomic transition and quantum theory.

7. i. → d.; ii. → e.; iii. → b.; iv. → a.

$$r_n = \frac{0.529\, n^2}{Z} \text{ Å}$$

So, (i) → (d)

Magnetic field, $B = \dfrac{12.5\, Z^3}{n^5}$ T

So, (ii) → (e)

$$B \propto \frac{1}{n^5}$$

$$V_n = \frac{2.2 \times 10^6\, Z}{n}$$

$$V_n \propto \frac{1}{n}$$

$$n\uparrow V_n \uparrow$$

So, (iii) → (b)

Total energy, $E_n = \dfrac{-13.6\, Z^2}{n^2}$ eV

$$n \uparrow E_n \uparrow$$

So, (iv) → (a)

8. i. → b.; ii. → c.; iii. → d.; iv. → a.

$$r_n = \frac{0.529\, n^2}{Z} \text{ Å}$$

So, (i) → (b)

$$V_n = \frac{2.2 \times 10^6\, Z}{n} \text{ m s}^{-1}$$

So, (iv) → (a)

$$I = \frac{1.06\, Z^2}{n^3} \text{ mA}$$

So, (ii) → (c)

Magnetic field, $B = \dfrac{12.5\, Z^3}{n^5}$ T

So, (iii) → (d).

9. i. → d.; ii. → a.; iii. → b.; iv. → c.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Wavelength range from 950 Å to 1350 Å corresponds to ultraviolet region of electromagnetic spectrum, i.e., Lyman series.

For transition 4 → 1: $\dfrac{1}{\lambda_1} = 1.1 \times 10^7 \left(\dfrac{15}{16}\right)$

$$\therefore\ \lambda_1 = \frac{16}{15 \times 1.1 \times 10^7} = 970\ \text{Å}$$

For transition 3 → 1: $\lambda_2 = \dfrac{9}{8 \times 1.1 \times 10^7} = 1023\ \text{Å}$

For transition 2 → 1: $\lambda_3 = \dfrac{4}{3 \times 1.1 \times 10^7} = 1212\ \text{Å}$

For Li^2 atom ($Z = 3$), for transition 2 → 1: $\dfrac{1}{\lambda'} = RZ^2\left(1 - \dfrac{1}{2^2}\right)$

$$\lambda' = \frac{1}{1.1 \times 10^7 \times 9 \times 0.75} = 134\ \text{Å}$$

For He^+ atom ($Z = 2$) for transition 2 → 1: $\dfrac{1}{\lambda'} = RZ^2\left(1 - \dfrac{1}{2^2}\right)$

$$\lambda' = \frac{1}{1.1 \times 10^7 \times 4 \times 0.75} = 303\ \text{Å}$$

merical Value Type

(4) $E_{\text{photon}} = 13.6\left(1 - \dfrac{1}{25}\right)\text{eV} = 13.0\ \text{eV}$

$E/c = mv$ (momentum conserved)

$$v = \frac{E}{mc} = \frac{(13 \times 1.6 \times 10^{-19})}{(1.67 \times 10^{-27} \times 3 \times 10^8)} = 4\ \text{m/s}$$

(6) $\dfrac{\lambda_1}{\lambda_2} = \dfrac{(Z_2 - 1)^2}{(Z_1 - 1)^2}$ $\left(\text{since, } \dfrac{1}{\lambda} \propto (Z-1)^2\right)$

$\dfrac{1}{4} = \dfrac{(Z_2 - 1)^2}{(11 - 1)^2}$; on solving, $Z_2 = 6$

(6) When electron jumps from n^{th} state to ground state, number of possible emission lines = $n(n-1)/2$. But here transition takes place from n^{th} state to state $n_1 = 2$.

Here, number of possible emission lines = $n(n-1)(n-2)/2$ = 10 (given).

On solving, $n = 6$.

(2) The shortest wavelength of Brackett series is corresponding to transition of electron between $n_1 = 4$ and $n_2 = \infty$.

Similarly, the shortest wavelength of Balmer series is corresponding to transition of electron between $n_1 = 2$ and $n_2 = \infty$.

$$(Z)^2\left(\frac{13.6}{16}\right) = \left(\frac{13.6}{4}\right) \quad \text{or} \quad Z = 2$$

5. (1) Heat produced/sec = 200 W

$$\Rightarrow \quad i = \frac{200}{V} = \frac{200}{20 \times 10^3} = 10\ \text{mA}$$

6. (4) For the first line of Balmer series of hydrogen

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \Rightarrow \lambda = \frac{36}{5R}$$

For singly ionized helium $Z = 2$

$$\frac{1}{\lambda'} = 4R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

Given $\lambda' = \lambda$ $\Rightarrow$ $4R\left(\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right) = \dfrac{5R}{36}$

$\Rightarrow \quad n_2 = 4$

7. (5) $E_n = -\dfrac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2}$, so $hf = +\dfrac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left[\dfrac{1}{16} - \dfrac{1}{25}\right]$

$$\therefore \quad f = \frac{mZ^2e^4}{8\varepsilon_0^2 h^3}\left[\frac{9}{16 \times 25}\right] \qquad \text{...(i)}$$

and frequency $f_4 = \dfrac{Z^2e^4m}{4\varepsilon_0^2 n^3 h^3} = \dfrac{Z^2e^4m}{4\varepsilon_0^2 (4)^3 h^3}$ ...(ii)

$\therefore \quad f/f_4 = 18/25$, so $m = 5$

8. (9) Number of revolutions before transition = frequency × time

Also, frequency $\propto \dfrac{1}{n^3}$

So required ratio = $\dfrac{(1/2)^3 12.8 \times 10^{-8}}{(1/3)^3 4.8 \times 10^{-8}} = 9$

9. (2) $\dfrac{T_1}{T_2} = \dfrac{n_1^3}{n_2^3} \Rightarrow \dfrac{8}{1} = \dfrac{n_1^3}{n_2^3} \Rightarrow \dfrac{n_1}{n_2} = \dfrac{2}{1}$

10. (6) According to conservation of energy

$$\frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = RchZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$n = 6$

11. (823)

The smallest frequency and largest wavelength in ultra-violet region will be for transition of electron from orbit 2 to orbit 1. Therefore,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{122 \times 10^{-9}\ \text{m}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[1 - \frac{1}{4}\right] = \frac{3R}{4}$$

$$\Rightarrow \quad R = \frac{4}{3 \times 122 \times 10^{-9}}\ \text{m}^{-1}$$

The highest frequency and smallest wavelength for infrared region will be for transition of electron from ∞ to 3rd orbit.

$$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{4}{3 \times 122 \times 10^{-9}}\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$$

$$\therefore \quad \lambda = \frac{3 \times 122 \times 9 \times 10^{-9}}{4} = 823.5\ \text{nm}$$

12. (2.25)

Let T_1 be the initial temperature and T_2 be the increased temperature of the black body. According to Stefan's law,

$$\left(\frac{T_2}{T_1}\right)^4 = 81 = (3)^4$$

$$T_2 = 3T_1 \qquad \text{...(i)}$$

Also, $\lambda_1 T_1 = \lambda_2 T_2$

$$\lambda_2 = \frac{\lambda_1 \times T_1}{T_2} = \frac{9000 \times T_1}{3T_1} = 3000\ \text{Å}$$

Now, $\dfrac{hf}{\lambda_2} - W = eV_0$ or $\dfrac{hf}{\lambda_2} - W = 13.6\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right)$

Solving, we get, $W = 2.25$ eV

13. (6) According to Moseley's law, $\sqrt{f} = a(Z-b) \Rightarrow f = a^2(Z-b)^2$

$$\Rightarrow \quad \frac{c}{\lambda} = a^2(Z-b)^2 \qquad \text{...(i)}$$

For K_α line, $b = 1$

From (i), $\frac{\lambda_2}{\lambda_1} = \frac{(Z_1-1)^2}{(Z_2-1)^2} \Rightarrow \frac{4\lambda}{\lambda} = \frac{(11-1)^2}{(Z_2-1)^2}$

$\Rightarrow \quad Z_2 - 1 = \frac{10}{2} \Rightarrow Z_2 = 6$

14. (15) $v = v_0 \frac{Z^2}{n^3} \quad \Rightarrow \quad \frac{Z^2}{n^3} = 2$ ....(i)

$E = E_0 \frac{Z^2}{n^2} \Rightarrow \frac{Z^2}{n^2} = 4$ ....(ii)

Solving (i) and (ii), $n = 2$, $Z = 4$

$L = mvr = \frac{nh}{2\pi}, \ \Delta L = \tau \Delta t = \Delta n \frac{h}{2\pi}$

$\tau = \frac{\Delta n}{\Delta t} \times \frac{h}{2\pi} = \frac{1}{7\times10^{-9}} \times \frac{1}{2} \times 2.1\times10^{-34} = \frac{2.1}{14}\times10^{-25}$

$\tau \times 10^{27} = \frac{2.1}{14}\times10^{-25}\times10^{27} = 15$

15. (2) $\frac{1}{\lambda} = RZ\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$\frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{1}{9} - \frac{1}{n^2}\right)}{8/9}$,

where n is the principal quantum number of the initial excited state.

Angular width $\theta = \frac{2\lambda}{d}$

$\frac{\theta_2}{\theta_1} = \frac{\lambda_2}{\lambda_1} = \frac{2}{25} = \frac{n^2-9}{8n^2} \quad \Rightarrow \quad n = 5$

$\frac{2\lambda_1}{d} = \theta_1, \quad \frac{1}{\lambda_1} = \frac{2}{d\times\theta_1} = \frac{2}{6.4}\times10^7 \text{ m}^{-1}$

$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{9} - \frac{1}{n^2}\right)$

$\frac{2}{6.4}\times10^7 = 1.097\times10^7 Z^2\left(\frac{1}{9} - \frac{1}{25}\right)$

$Z^2 = \frac{2}{6.4}\times\frac{225}{16\times1.097} \quad \Rightarrow \quad Z = 2$

16. (2) $E = -(13.6 \text{ eV})Z^2\left(\frac{1}{(Z+2)^2} - \frac{1}{Z^2}\right)$

$= -13.6\times\left(\frac{Z^2-(Z+2)^2}{(Z+2)^2}\right) = \frac{4(Z+1)\times13.6}{(Z+2)^2}\text{eV}$ ...(i)

Now energy of electron is $K = \frac{h^2}{2\lambda^2 m}$

Solving $K = 6$ eV

So, $\frac{4(Z+1)\times13.6}{(Z+2)^2} = 6 + 4.2 = 10.2$ eV

$\frac{Z+1}{(Z+2)^2} = \frac{3}{16} \quad \Rightarrow \quad (Z-2)(3Z+2) = 0$

So, the value of $Z = 2$ (neglecting the negative/fractional value)

7. (2) Using COM: $mu = 2mv$

$v = \frac{u}{2}, \ \Delta E = \frac{1}{2}mu^2 - 2\times\frac{1}{2}m\left(\frac{u}{2}\right)^2$

$\Rightarrow \quad \Delta E = \frac{1}{4}mu^2 = \frac{1}{4}\times1.66\times10^{-27}\times(6.24\times10^4)^2$

$13.6\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = \frac{1.66\times10^{-27}\times(6.24\times10^4)^2}{4\times1.6\times10^{-19}}$

$\Rightarrow \quad n = 2$

18. (3) We have,

$10.2 = W + K_{\text{max},1}$ ...(i)

and $10.2\,Z^2 = W + K_{\text{max},2}$ ...(ii)

$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = 2.3 \Rightarrow K_2 = 5.25\,K_1$ ...(iii)

Also, $10.2\,Z^2$ = energy corresponding to longest wavelength of the Lyman series = 3 × 13.6

$\Rightarrow \quad Z = 2$

From Eqs. (i), (ii) and (iii), $W = 3$ eV

19. (30) $\lambda_{C_1} = \frac{h}{eV} = \frac{12375}{10\times10^3}\text{Å} = 1.2375\text{Å}$ ...(i)

$\lambda_{C_2} = 0.61875$ Å ...(ii)

$\frac{1}{\lambda_{K\alpha}} = (Z-1)^2\left(\frac{1}{1} - \frac{1}{4}\right)\times10^7$ ...(iii)

It is given, $3\times\left(\lambda_{K\alpha} - 1.2375\right) = \lambda_{K\alpha} - 0.61875$

$\Rightarrow \quad \lambda_{K\alpha} = 1.54338$ Å ...(iv)

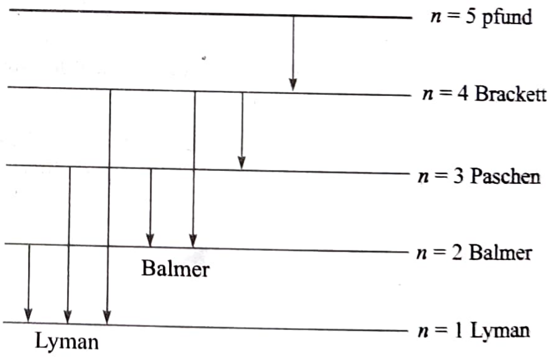
Putting this value in (iii), $Z - 1 = 29 \Rightarrow Z = 30$

Archives

JEE Main

Single Correct Answer Type

1. (4)



The transition 4 → 3 belongs to Paschen series. This is not in the ultraviolet region but this is in the infrared region.

The transition 5 → 4 also belongs to Brackett series and it is in the infrared region.

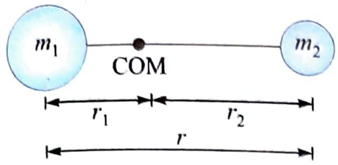
2. (3) $E_1 = -\frac{13.6(3)^2}{(1)^2}, \ E_2 = -\frac{13.6(3)^2}{(3)^2}$

$\therefore \quad \Delta E = E_2 - E_1 = 13.6(3)^2\left[1 - \frac{1}{9}\right] = \frac{13.6\times9\times8}{9} = 108.8\,\text{eV}$

3. (4) If $n = 4$, lines $= \frac{n(n-1)}{2} = 6$

4 (3) Let the masses of two atoms of the molecule be m_1 and m_2 at a distance r apart. The distances of the atoms from the centre of mass of the molecule be r_1 and r_2. The moment of inertia of this molecule about an axis passing through its

centre of mass and perpendicular to a line joining the atoms,
$I = m_1 r_1^2 + m_2 r_2^2$



Also we can write, $m_1 r_1 = m_2 r_2$ or $r_1 = \frac{m_2}{m_1} r_2$

But $r_1 + r_2 = r$

Hence $r_1 = \frac{m_2}{m_1}(r - r_1) \Rightarrow r_1 = \frac{m_2 r}{m_1 + m_2}$

Similarly, $r_2 = \frac{m_1 r}{m_1 + m_2}$

Therefore, the moment of inertia can be written as

$$I = m_1\left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1 + m_2}\right)^2$$

$$= \frac{m_1 m_2}{m_1 + m_2} r^2 \qquad \text{...(i)}$$

According to Bohr's quantization condition

$$L = \frac{nh}{2\pi} \text{ or } L^2 = \frac{n^2 h^2}{4\pi^2} \qquad \text{...(ii)}$$

Rotational energy, $E = \frac{L^2}{2I}$

$$E = \frac{n^2 h^2}{8\pi^2 I} \qquad \text{(Using (ii))}$$

$$= \frac{n^2 h^2 (m_1 + m_2)}{8\pi^2 (m_1 m_2) r^2} \qquad \text{(Using (i))}$$

$$= \frac{n^2 \hbar^2 (m_1 + m_2)}{2 m_1 m_2 r^2} \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

(3) When an electron makes a transition from an energy level with n to $n - 1$, for a hydrogen like atom, the frequency of emitted radiation is

$$f = RcZ^2\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = RcZ^2\left[\frac{n^2 - (n-1)^2}{(n^2)(n-1)^2}\right] = \frac{RcZ^2(2n-1)}{n^2(n-1)^2}$$

As $n >> 1$

$$\therefore \quad f = \frac{RcZ^2 2n}{n^4} = \frac{2RcZ^2}{n^3} \text{ or } f \propto \frac{1}{n^3}$$

(4) Radius of a charged particle rotating in a constant magnetic field is given by $R = \frac{mv}{qB}$

or $$R^2 = \frac{m^2 v^2}{q^2 B^2} = \frac{2m\left(\frac{1}{2}mv^2\right)}{q^2 B^2} = \frac{2m(K)}{q^2 B^2}$$

Hence kinetic energy of the charged particle, $K = \frac{q^2 B^2 R^2}{2m}$

$$\Rightarrow \quad K_{\max} = \frac{q^2 B^2 R^2_{\max}}{2m} = 0.80 \text{ eV}$$

Energy of photon corresponding transition from orbit 3 → 2 in hydrogen atom.

$$E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89 \text{ eV}$$

Using Einstein photoelectric equation.

$$E = K_{\max} + \phi$$

$$\Rightarrow \quad 1.89 = 0.8 + \phi$$

$$\Rightarrow \quad \phi = 1.09 \approx 1.1 \text{ eV}$$

7. (1) $\text{PE} = -27.2\frac{Z^2}{n^2} \text{ eV}; \ \text{TE} = -\frac{13.6Z^2}{n^2} \text{ eV}; \ \text{KE} = \frac{13.6Z^2}{n^2} \text{ eV}$

$\text{KE} = \frac{13.6}{n^2}$ eV, as n decreases, KE ↑

$\text{PE} = -\frac{27.2}{n^2}$ eV, as n decreases, PE ↓

$\text{TE} = -\frac{13.6}{n^2}$ eV, as n decreases, TE ↓

8. (3) In X-ray tube, $\frac{hc}{\lambda_{\min}} = eV \quad \Rightarrow \quad \frac{1}{\lambda_{\min}} = \frac{eV}{hc}$ ...(i)

Taking log on both sides

$$-\ln(\lambda_{\min}) = \ln V + \ln\frac{e}{hc} \quad \text{or} \quad \ln(\lambda_{\min}) = -\ln V - \ln\left(\frac{e}{hc}\right)$$

It is a straight line with –ve slope.

9. (2) Using $\Delta E = \frac{hc}{\lambda}$

For λ_1, $E - (-2E) = \frac{hc}{\lambda_1}$

$$\Rightarrow \quad \lambda_1 = \frac{hc}{E} \qquad \text{...(i)}$$

For λ_2, $E - \left(-\frac{4E}{3}\right) = \frac{hc}{\lambda_2}$

$$\Rightarrow \quad \lambda_2 = \frac{3hc}{E} \qquad \text{...(ii)}$$

From (i) and (ii), $\frac{\lambda_1}{\lambda_2} = r = \frac{1}{3}$

10. (2) de Broglie wavelength of the electron in the n^{th} state

$$\lambda_n = \frac{h}{mu} = \frac{h}{\sqrt{2mK_n}} \Rightarrow K_n = \frac{h^2}{2m\lambda_n^2} = -E_n$$

Similarly for ground state $K_g = \frac{h^2}{2m\lambda_g^2} = -E_g$

$$\Rightarrow K_g - K_n = \frac{h^2}{2m}\left[\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right]$$

For emitted photon,

$$\frac{hc}{\Lambda_n} = E_n - E_g = K_g - K_n \Rightarrow \Lambda_n = \frac{hc}{K_g - K_n}$$

$$\Lambda_n = \frac{hc}{\frac{h^2}{2m}\left[\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right]} = \frac{2mc}{h\left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2 \lambda_n^2}\right)} = \frac{2mc\lambda_g^2\lambda_n^2}{h(\lambda_n^2 - \lambda_g^2)}$$

$$\Lambda_n = \frac{2mc\lambda_g^2\lambda_n^2}{h\left(\lambda_n^2 - \lambda_g^2\right)} = \frac{2mc\lambda_g^2\lambda_n^2}{h\lambda_n^2\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)} = \frac{2mc\lambda_g^2}{h}\left[1 - \left(\frac{\lambda_g}{\lambda_n}\right)^2\right]^{-1}$$

$$\Lambda_n = \frac{2mc\lambda_g^2}{h}\left[1 + \left(\frac{\lambda_g}{\lambda_n}\right)^2 + \text{higher power of } \frac{\lambda_g}{\lambda_n}\right]$$

But $\lambda_n >> \lambda_g$

$$\Lambda_n \approx \frac{2mc\lambda_g^2}{h}\left(1+\left(\frac{\lambda_g}{\lambda_n}\right)^2\right) \approx \left(\frac{2mc\lambda_g^2}{h}+\frac{2mc\lambda_g^4}{h}\left(\frac{1}{\lambda_n^2}\right)\right)$$

$$\Lambda_n \approx A+\frac{B}{\lambda_n^2}$$

where $A=\frac{2mc\lambda_g^2}{h}$ and $B=\frac{2mc\lambda_g^4}{h}$

11. (1) The energy of a photon in transition of an electron in atom

$$h\nu = E_0\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

for Lyman series for series limits $n_2=\infty$, $n_1=1$

$$h\nu_L = E_0[1] \qquad \text{...(i)}$$

for P fund series limits $n_2=\infty$, $n_1=5$

$$h\nu_P = E_0\left[\frac{1}{25}\right] \qquad \text{...(ii)}$$

By dividing Eqs. (i) and (ii)

$$\frac{\nu_L}{\nu_P}=\frac{25}{1} \Rightarrow \nu_P=\frac{\nu_L}{25}$$

JEE Advanced

Single Correct Answer Type

1. (2) $$\frac{1}{\lambda_{H_2}} = RZ_H^2\left[\frac{1}{4}-\frac{1}{9}\right]=R(1)^2\left[\frac{5}{36}\right]$$

$$\frac{1}{\lambda_{He}} = RZ_{He}^2\left[\frac{1}{4}-\frac{1}{16}\right]=R(4)^2\left[\frac{3}{16}\right]$$

$$\frac{\lambda_{H_2}}{\lambda_{He}} = \frac{1}{4}\left[\frac{16}{3}\times\frac{5}{36}\right]=\frac{5}{27}$$

$$\lambda_{He}=\frac{5}{27}\times 6561 = 1215\ \text{Å}$$

Multiple Correct Answers Type

1. (1), (2), (4)

As radius $r \propto \frac{n^2}{Z}$

$$\Rightarrow \frac{\Delta r}{r}=\frac{\left(\frac{n+1}{Z}\right)^2-\left(\frac{n}{Z}\right)^2}{\left(\frac{n}{Z}\right)^2}=\frac{2n+1}{n^2}\approx\frac{2}{n}\propto\frac{1}{n}$$

As energy $E \propto \frac{Z^2}{n^2}$

$$\Rightarrow \frac{\Delta E}{E}=\frac{\frac{Z^2}{n^2}-\frac{Z^2}{(n+1)^2}}{\frac{Z^2}{(n+1)^2}}=\frac{(n+1)^2-n^2}{n^2.(n+1)^2}.(n+1)^2$$

$$\Rightarrow \frac{\Delta E}{E}=\frac{2n+1}{n^2}\approx\frac{2n}{n^2}\propto\frac{1}{n}$$

As angular momentum $L=\frac{nh}{2\pi}$

$$\Rightarrow \frac{\Delta L}{L}=\frac{\frac{(n+1)h}{2\pi}-\frac{nh}{2\pi}}{\frac{nh}{2\pi}}=\frac{1}{n}\propto\frac{1}{n}$$

Linked Comprehension Type

For Problem 1–3

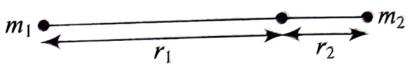
1. (4) $L=\frac{nh}{2\pi}$

$$\text{KE}=\frac{L^2}{2I}=\left(\frac{nh}{2\pi}\right)^2\frac{1}{2I}$$

2. (1) $hf=\text{KE}_{n=2}-\text{KE}_{n=1}$

$I=1.87\times10^{-46}$ kg m^2

3. (3) $r_1=\frac{m_2 d}{m_1+m_2}$ and $r_2=\frac{m_1 d}{m_1+m_2}$



$$I=m_1r_1^2+m_2r_2^2$$

$\therefore$ $d=1.3\times10^{-10}$ m

Numerical Value Type

1. (6) $E=\frac{hc}{\lambda}=\frac{12370}{970}=12.7$ eV

The incident light excite electron to n quantum state

$$\Rightarrow \quad -13.6+12.7=-\frac{13.6}{n^2}$$

$n^2=16$ or $n=4$

Number of lines $={}^nC_2=\frac{n(n-1)}{2}=6$

2. (5) $PE=V=-27.2\frac{Z^2}{n^2}$ $\quad[Z=1$ for H$]$

$$V=\frac{-27.2}{n^2} \Rightarrow V\propto\frac{1}{n^2}$$

So, $\frac{V_i}{V_f}=\left(\frac{n_f}{n_i}\right)^2=6.25 \quad \therefore \frac{n_f}{n_i}=2.5=\frac{5}{2}$

Minimum integral value of n_f is 5.

3. (3) In transition from $n=2$ to $n=1$

$$\Delta E_{2-1}=13.6\times Z^2\left[1-\frac{1}{4}\right]=13.6\times Z^2\left[\frac{3}{4}\right]$$

In transition from $n=3$ to $n=2$

$$\Delta E_{3-2}=13.6\times Z^2\left[\frac{1}{4}-\frac{1}{9}\right]=13.6\times Z^2\left[\frac{5}{36}\right]$$

We are given $\Delta E_{2-1}=\Delta E_{3-2}+74.8$

$$13.6\times Z^2\left[\frac{3}{4}\right]=13.6\times Z^2\left[\frac{5}{36}\right]+74.8$$

$$13.6\times Z^2\left[\frac{3}{4}-\frac{5}{36}\right]=74.8$$

$\Rightarrow$ $Z^2=9$ which gives $Z=+3$

Chapter 7

Concept Application Exercises

ercise 7.1

1. Number of protons in nucleus = atomic number = 11

Number of electrons in an atom = Number of protons = 11

Number of neutrons = mass number (A) – atomic number (Z)
$= 24 - 11 = 13$

2. In order to compute binding energy, let us first find the total mass of all protons and neutrons in Nb and subtract mass of the Nb:

Given: $m_p = 1.007276$ u and $m_n = 1.008665$ u

Number of protons: $N_p = 41$

Number of neutrons: $N_n = 93 - 41 = 52$

Mass difference: $\Delta m = 41m_p + 52m_n - m_{Nb}$

$$= 41(1.007825\text{ u}) + 52(1.008665\text{ u}) - (92.9063768\text{ u})$$
$$= 0.865028\text{ u}$$

Thus, binding energy per nuclear is

$$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.865028\text{ u})(931.5\text{MeV/u})}{93}$$
$$= 8.66\text{ MeV nucleon}^{-1}$$

3. Nuclei are stable because of the presence of another short-range (about 2 fm) force, the nuclear force. This is an attractive force that acts between all nuclear particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the Coulomb force. The attractive nuclear force also acts between pairs of neutrons and between neutrons and protons.

The nuclear attractive force is stronger than the Coulomb repulsive force within the nucleus (at short ranges). If this were not the case, stable nuclei would not exist.

4. $\left(\frac{\text{BE}}{A}\right)_P = \frac{100}{10} = 10$, $\left(\frac{\text{BE}}{A}\right)_Q = \frac{60}{5} = 12$, and $\left(\frac{\text{BE}}{A}\right)_R = \frac{66}{6} = 11$

Therefore, stability order is $Q > R > P$.

5. After releasing 10 MeV, it will become more stable and its binding energy will increase.

New binding energy = 100 + 10 = 110 MeV

6. $(B_3 + B_4) - (B_1 + B_2)$

7. The alpha particle contains 2 protons and 2 neutrons. The binding energy is

$$B = (2 \times 1.007826\text{ u} + 2 \times 1.008665\text{ u} - 4.00260\text{ u})c^2$$
$$= (0.03038\text{ u})c^2 = 0.03038 \times 931\text{ MeV} = 28.3\text{ MeV}$$

8. The number of protons in ${}^{56}_{26}\text{Fe}$ = 26 and the number of neutrons = 56 – 26 = 30. The binding energy of ${}^{56}_{26}\text{Fe}$

$$= [26 \times 1.00783\text{ u} + 30 \times 1.00867\text{ u} - 55.9349\text{ u}]\,c^2$$
$$= (0.52878\text{ u})c^2 = (0.52878\text{ u})\,(931\text{ MeV/u}) = 492\text{ MeV}$$

9. Mass of Avogadro number of atoms of ^{12}C is exactly 12 g. Avogadro's number, N_A, has the value 6.02×10^{23} atoms mol^{-1}. Thus, the mass of one carbon atom is

$$\frac{0.012\text{ kg}}{6.02 \times 10^{23}\text{ atom}} = 1.99 \times 10^{-26}\text{ kg}$$

Since one atom of ^{12}C is defined to have a mass of 12 u, we find that

$$1\text{ u} = \frac{1.99 \times 10^{-26}\text{ kg}}{12} = 1.66 \times 10^{-27}\text{ kg}$$

10. The nuclear density is nearly same in all atoms. The difference in densities between solid lead and gaseous oxygen arises mainly because of the difference in how closely the atoms themselves are packed together in solid and gaseous phase.

11. The binding energy of ^{8}Be is determined by the equation

$$B({}^8\text{Be}) = [4m_n + 4M({}^1\text{H}) - M({}^8\text{Be})]c^2$$

The binding energy

$$B({}^8\text{Be}) = [4 \times 1.008665 + 4 \times 1.007825 - 8.005305] \times 931.5$$
$$= 56.5\text{ MeV}$$

Now, we calculate the binding energy (B) of the decay of ^{8}Be into two α-particles.

$${}^8\text{Be} \rightarrow 2\alpha,$$
$$B = [2M({}^4\text{He}) - M({}^8\text{Be})]\,c^2$$
$$= [2 \times 4.002603 - 8.005305] \times 931.5 = -0.092\text{ MeV}$$

Because B is negative for this reaction, hence ^{8}Be is unstable against decay to two alpha particles.

Exercise 7.2

1. When an α-particle is emitted, we have the product nucleus X:

$${}_{92}\text{U}^{238} \rightarrow {}_{90}\text{X}^{234} + {}_2\text{He}^4$$

When the product nucleus emits a β-particle, the final nucleus is Y (say) given by ${}_{90}\text{X}^{234} \rightarrow {}_{91}\text{Y}^{234} + {}_{-1}\beta^0$.

Thus, atomic number of final nucleus = 91, mass number of final nucleus = 234.

2. The successive processess may be expressed as

$${}_{72}\text{A}^{180} \xrightarrow{\alpha} {}_{70}\text{A}_1^{176} \xrightarrow{\beta} {}_{71}\text{A}_2^{176} \xrightarrow{\alpha} {}_{69}\text{A}_3^{172} \xrightarrow{\gamma} {}_{69}\text{A}_4^{172}$$

Thus, the mass number of $\text{A}_4 = 172$ and the atomic number of $\text{A}_4 = 69$.

3. The successive processess may be expressed as

$${}_{92}\text{U}^{238} \xrightarrow{{}_0n^1} {}_{92}\text{U}^{239} \xrightarrow{\beta(\text{electron})} {}_{93}\text{Np}^{239} \xrightarrow{\beta(\text{electron})} {}_{94}\text{Pu}^{239}$$

Thus, the resulting plutonium has atomic number 94 and mass number 239 and is expressed as ${}_{94}\text{Pu}^{239}$.

4. The process is expressed as ${}_{92}\text{U}^{238} \rightarrow {}_{90}\text{Th}^{230} + 2({}_2\text{He}^4) + 2({}_{-1}\beta^0)$.

The processess may be expressed as

$${}_{92}\text{U}^{238} \rightarrow {}_{90}\text{Th}^{234} + {}_2\text{He}^4\ (\alpha\text{-particle})$$
$${}_{90}\text{Th}^{234} \rightarrow {}_{91}\text{Pa}^{234} + {}_{-1}\beta^0\ (\beta\text{-particle})$$
$${}_{91}\text{Pa}^{234} \rightarrow {}_{92}\text{U}^{234} + {}_{-1}\beta(\beta\text{-particle})$$
$${}_{92}\text{U}^{234} \rightarrow {}_{90}\text{Th}^{230} + {}_2\text{He}^4(\alpha\text{-particle})$$

The mass number of thorium nucleus is 230 and atomic number is 90.

5. Mass number (or atomic weight) of ${}_6\text{C}^{12}$ is 12.

Therefore, the number of atoms in 12 g of ${}_6\text{C}^{12}$

$$= \text{Avogadro number} = 6 \times 10^{23}$$

The number of electrons in 12 g of ${}_6\text{C}^{12} = 6 \times 6 \times 10^{23} = 36 \times 10^{23}$

The number of protons in 12 g of ${}_6\text{C}^{12} = 36 \times 10^{23}$

The number of neutrons in 12 g of ${}_6\text{C}^{12}$

$$= (A - Z) \times 6 \times 10^{23} = (12 - 6) \times 6 \times 10^{23} = 36 \times 10^{23}.$$

Similarly, number of electrons in 14 g of ${}_6\text{C}^{14} = 3.6 \times 10^{23}$

Number of protons in 14 g of ${}_6\text{C}^{14}$

$$= (A - Z) \times 6 \times 10^{23} = (14 - 6) \times 6 \times 10^{23} = 48 \times 10^{23}.$$

The emitted particles are α, β, β, α and α, respectively.

6. Given: Reaction

In order to balance the reaction, the total amount of nucleons (sum of A-numbers) must be the same on both sides. Same for the Z-number.

Number of nucleons (A): $7 + 4 = X + 1 \Rightarrow X = 10$

Number of protons (Z): $3 + 2 = Y + 0 \Rightarrow Y = 5$

Thus, it is B, i.e., ${}^{7}_{3}\mathrm{Li} + {}^{4}_{2}\mathrm{He} \to {}^{10}_{5}\mathrm{B} + {}^{1}_{0}\mathrm{n}$

7. Let a and b be the number of α-and β-particles emitted when ${}_{92}\mathrm{U}^{238}$ decays to ${}_{82}\mathrm{Pb}^{206}$.

We know that

(a) The emission of an α-particle (${}_{2}\mathrm{He}^{4}$) decreases the charge number by two and mass number by four.

Therefore, emission of a α-particles will reduce the charge number by $2a$ and mass number by $4a$.

(b) The emission of a β-particle increases the charge number by one and leaves the mass number unchanged.

Therefore, emission of b β-particles will increase the charge number by $b \times 1 = b$.

Thus, ${}_{92}\mathrm{U}^{238} \to {}_{82}\mathrm{Pb}^{206} + a({}_{2}\mathrm{He}^{4}) = b({}_{-1}\beta^{0})$

Applying the law of conservation of charge number and mass number before and after the decay, we have

$$92 = 82 + 2a - b \qquad \text{...(i)}$$

$$238 = 206 + 4a \qquad \text{...(ii)}$$

From Eq. (ii), $a = 8$, i.e., number of emitted α-particles $= 8$

From Eq. (i), $b = 6$, i.e., number of emitted β-particles $= 6$

8. As the net electric charge is conserved, hence number of protons before the reaction is equal to the total number after the reaction. The total number of nucleons is conserved as well, so we can equate the total number before and after the reaction.

The results are listed in the table as follows:

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	$2 + 13$	$= Z + 0$
Total number of nucleons	$4 + 27$	$= A + 1$

Thus, we get $Z = 15$ and $A = 30$. Since $Z = 15$ identifies the element as phosphorus, the nucleus produced is ${}^{30}_{15}\mathrm{P}$.

9. A deuteron is the nucleus of deuterium, the isotope of hydrogen containing one proton and one neutron, ${}^{2}_{1}\mathrm{H}$, we have the reaction

$${}^{1}_{0}\mathrm{n} + {}^{16}_{8}\mathrm{O} \to {}^{A}_{Z}\mathrm{X} + {}^{2}_{1}\mathrm{H}$$

Proceeding similar to previous problem,

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	$1 + 16$	$= Z$
Total number of nucleons	$0 + 8$	$= A$

Hence, the product nucleus must have $Z = 7$ and $A = 15$. From the periodic table we find that it is nitrogen that has $Z = 7$, so the nucleus is ${}^{15}_{7}\mathrm{N}$. The reaction can be written ${}^{16}_{8}\mathrm{O}$ (n, d) ${}^{15}_{7}\mathrm{N}$, where d represents deuterium ${}^{2}_{1}\mathrm{H}$

10. The equation for decay is ${}^{238}_{92}\mathrm{U} \to {}^{A}_{Z}\mathrm{X} + {}^{4}_{2}\mathrm{He}$

Proceed similar to previous problem, to get $Z = 90$ and $A = 234$. Thus, the nucleus is ${}^{234}_{90}\mathrm{Th}$.

11. Each α-particle carries off two protons; the total number of protons carried by the α-particles is $2N_\alpha$.

Each β-particle is an electron ${}^{0}_{-1}\mathrm{e}$ and is emitted when a neutron in the nucleus decays into a proton and an electron. Thus, the total number of protons left behind by the β^--particles is N_β.

Each α-particle carries four nucleons; therefore the total number of nucleons carried off by the α-particles is $4N_\alpha$.

With the emission of each β-particle, a neutron is replaced by a proton. But since each is a nucleon, the number of nucleons is not changed.

The overall decay process is ${}^{228}_{90}\mathrm{Th} \to {}^{212}_{83}\mathrm{Bi} + N_\alpha({}^{4}_{2}\mathrm{He}) + N_\beta({}^{0}_{-1}\mathrm{e})$

Now, we can apply conservation of charge and nucleon number.

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	90	$83 + 2N_\alpha + (-1)N_\beta$
Total number of nucleons	228	$212 + 4N_\alpha + (0)N_\beta$

On solving the two equations, we get $N_\alpha = 4$ and $N_\beta = 1$.

Exercise 7.3

1. $R_1 = \left|\dfrac{dN_1}{dt}\right| = \lambda N_1$ and $R_2 = \left|\dfrac{dN_2}{dt}\right| = \lambda N_2$

where N_1 and N_2 are the numbers of undecayed nuclei present at times t_1 and t_2 respectively

Thus, $R_1 - R_2 = \lambda(N_1 - N_2)$

Or $(N_1 - N_2) = \dfrac{(R_1 - R_2)}{\lambda} = (R_1 - R_2)\tau \quad \left(\text{as } \dfrac{1}{\lambda} = \tau\right)$

Thus, number of nuclei that have disintegrated in time interval $(t_1 - t_2)$, i.e., $(N_1 - N_2) = (R_1 - R_2)\tau$.

2. The activity is proportional to the number of undecayed atoms. In each half-life, the remaining sample decays to half of its initial value. Since $\left(\dfrac{1}{2}\right) \times \left(\dfrac{1}{2}\right) \times \left(\dfrac{1}{2}\right) = \dfrac{1}{8}$, therefore, three half–lives or 15.75 years are required for the sample to decay to 1/8th its original strength.

3. Here, $T_{1/2} = 24.1$ days, $N_0 = 100$, $N = (100 - 90) = 10$

Thus, $\dfrac{N_0}{N} = \dfrac{100}{10} = 10$

As $\dfrac{N_0}{N} = 2^{t/T_{1/2}} \Rightarrow 10 = 2^{t/T_{1/2}}$

or $\log 10 = \log 2^{t/T_{1/2}} \Rightarrow t = \dfrac{1}{0.3010} \times 24.1\,\mathrm{d} = 80\,\mathrm{d}$

4. Let t be the time taken by the activity (R) to drop to (1/50) of its present unsafe value (R_0), i.e., $\dfrac{R}{R_0} = \dfrac{1}{50}$

As $R = \lambda N$ and $R_0 = \lambda N_0$

$$\frac{N}{N_0} = \frac{R}{R_0} = \frac{1}{50}$$

or $\dfrac{N_0}{N} = 50$

As $\dfrac{N_0}{N} = 2^{t/T_{1/2}}$; $50 = 2^{t/10}$

Thus, $\log 50 = \log 2^{t/10}$

or $\log 50 = \dfrac{t}{10} \log 2$

or $t = \dfrac{10 \times \log 50}{\log 2} = \dfrac{10 \times 1.6990}{0.3010} = 56.45\,\mathrm{d}$

5. Here, $R = 1.24\times10^{4}$ dps

Number of nuclei in 1 g of ^{238}U, i.e., $N = \frac{6.023\times10^{23}}{238}$

As $R = \lambda N,$

$$\lambda = \frac{R}{N} = \frac{\left(1.24\times10^{4}\right)\times238}{6.023\times10^{23}} = 4.9\times10^{-18}\ \mathrm{s}^{-1}$$

Thus, $T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{4.9\times10^{-18}\ \mathrm{s}^{-1}} = 1.41\times10^{17}$ s

$\Rightarrow \quad T = 4.5\times10^{9}$ y $\quad$ (as 1 y $= 3.15\times10^{7}$ s)

6. We know that $\lambda = 0.693/T_{1/2}$

Here, $T_{1/2} = 3.8$ day

$$\therefore \quad \lambda = \frac{0.693}{3.8} = 0.182 \text{ per day}$$

If initially (at $t = 0$) the number of atoms present be N_0, then the number of atoms N left after a time given by

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-\lambda t} \text{ or } \frac{1}{20} = e^{-\lambda t}$$

Taking log, $\lambda t = \log_e 20 = 2.3026 \log_{10} 20$

$$\therefore \quad t = \frac{2.3026 \log_{10} 20}{\lambda} = \frac{2.3026 \log_{10} 20}{0.182} = 16.45 \text{ days}$$

7. (a) The initial number of nuclei is $N_0 = \frac{mN_A}{M}$

(b) The number of undecayed nuclei after a time t is

$$N = N_0 e^{-\lambda t} = \left(\frac{mN_A}{M}\right)e^{-\lambda t}$$

Therefore, the number of decayed nuclei is

$$N' = N_0 - N = \frac{mN_A}{M}(1 - e^{-\lambda t})$$

(c) The activity of the sample is $A = A_0 e^{-\lambda t} = \lambda N_0 e^{-\lambda t}$

$$A = \left(\frac{mN_A\lambda}{M}\right)e^{-\lambda t}$$

8. We know that $N = N_0 e^{-\lambda t}$, where $\lambda = \frac{\ln 2}{T} = \frac{1}{T_{av}}$

Here, $N = (1-\eta)N_0$

$$\Rightarrow \quad (1-\eta)N_0 = N_0 e^{-\lambda t} \quad \text{or } t = \frac{1}{\lambda}\ln\left|\frac{1}{(1-\eta)}\right|$$

(a) Thus, $t = \frac{T}{\ln 2}\left[\ln\left|\frac{1}{1-\eta}\right|\right] = \frac{T}{0.693}\ln\left|\frac{1}{(1-\eta)}\right|$

(b) $t = T_{av}\ln\left|\frac{1}{(1-\eta)}\right|$

9. $A_1 = A_0 e^{-\lambda t_1}$ and $A_2 = A_0 e^{-\lambda t_2}$

$$\therefore \quad \frac{A_1}{A_2} = e^{\lambda(t_2 - t_1)}$$

$$\Rightarrow \quad \lambda(t_2 - t_1) = \ln\left|\frac{A_1}{A_2}\right| \quad \Rightarrow \quad T = \frac{1}{\lambda} = \frac{t_2 - t_1}{\ln\left|\frac{A_1}{A_2}\right|}$$

10. The decay constant is $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730\times5.26\times10^{5}}$

$$= 2.30\times10^{-10}\ \mathrm{min}^{-1}$$

As 1 year $= 3.01\times10^{9}$ min and for 1.0 g of carbon, the number of moles is $n = 1/12$, so $N = nN_A = \left(\frac{1}{2}\right)(6.02\times10^{23}) = 5.0\times10^{22}$ nuclei g^{-1}.

The given concentration factor is $\frac{^{14}C}{^{12}C} = 1.4\times10^{-12}$

Thus, the number of ^{14}C nuclei per gram is

$$N\left(\frac{^{14}\mathrm{C}}{^{12}\mathrm{C}}\right) = (5.0\times10^{22})(1.4\times10^{-12}) = 7.0\times10^{10} \text{ nuclei g}^{-1}$$

The activity, number of decays per gram per minute, is

$$\frac{\Delta N}{\Delta t} = \lambda N = (2.30\times10^{-10})(7.0\times10^{10}) = 16 \text{ decays g}^{-1}\ \mathrm{min}^{-1}$$

11. (a) This part has already been discussed in section on successive disintegrations with the result

$$N_2(t) = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

(b) The activity of nuclide A_2 is $\lambda_2 N_2$. This is maximum when N_2 is maximum, i.e., when $dN_2/dt = 0$, activity is maximum.

$$\frac{d[N_2(t)]}{dt} = \frac{d}{dt}\left[\frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})\right]$$

$$\frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t}) = 0$$

$$e^{-(\lambda_1 - \lambda_2)}t = \frac{\lambda_2}{\lambda_1}$$

$$t = \frac{\ln(\lambda_2/\lambda_1)}{\lambda_2 - \lambda_1}$$

12. In 5730 years, half the sample will have decayed, leaving 500 radioactive $^{14}_{6}$C nuclei. In another 5730 years (for a total elapsed time of 11460 years), the number will be reduced to 250 nuclei. After another 5730 years (total time 17190 years), 125 nuclei remain.

These numbers represent ideal circumstances. Radioactive decay is an averaging process over a very large number of atoms, and the actual outcome depends on statistics. Our original sample in this example contained only 1000 nuclei, certainly not a very large number. Thus, if we were actually to count the number remaining after one half-life for this small sample, it probably would not be exactly 500.

13. First, let us convert the half-life to seconds.

$$T_{1/2} = (1.6\times10^{3} \text{ years})(3.16\times10^{7} \text{ s/years}) = 5.0\times10^{10} \text{ s}$$

$$\therefore \quad \lambda = \frac{0.693}{T^{1/2}} = \frac{0.693}{5.0\times10^{10}\,s} = 1.4\times10^{-11}\ \mathrm{s}^{-1}$$

We can calculate the activity of the sample at $t = 0$ using $R_0 = \lambda N_0$, where R_0 is the decay rate at $t = 0$ and N_0 is the number of radioactive nuclei present at $t = 0$.

$$R_0 = \lambda N_0 = (1.4\times10^{-11}\mathrm{s}^{-1})(3.0\times10^{16}) = 4.1\times10^{5} \text{ decays s}^{-1}$$

Because Ci $= 3.7\times10^{10}$ decays s^{-1}, the activity or decay rate at $t = 0$ is $R_0 = 11.1$ μCi.

Exercise 7.4

1. Energy released in the fission of a single nucleus of $^{235}_{92}$U $= 200$ MeV $= 200(1.6\times10^{-13})$J $= 3.2\times10^{-11}$ J

To produce a power of 1 kW, 1000 J of energy should be released per second.

If n is the number of fissions required per second to produce an energy of 1000 J,

$$n = \frac{1000\,\mathrm{J}}{3.2\times10^{-11}\,\mathrm{J}} = 3.125\times10^{13}$$

2. Energy output of the reactor per day

$$= 200 \times 10^6 (\text{J/s}) \times (24 \times 60 \times 60\,\text{s}) = 1728 \times 10^{10}\ \text{J}$$

$$\text{Energy input } = \frac{1728 \times 10^{10}}{(25/100)} = 6912 \times 10^{10}\ \text{J}$$

Energy released by the fusion of two ${}_1^2\text{H}$ nuclei

$$= \left[(2 \times 2.0141) - 4.0026\right] \times 931.5\ \text{MeV}$$

$$= 23.85\,\text{MeV} = 38.15 \times 10^{-13}\,\text{J}$$

$$\text{Number of } {}_1^2\text{H nuclei required } = \frac{6912 \times 10^{10}\,\text{J}}{(38.15 \times 10^{-13}\,\text{J})/2} = 362.3 \times 10^{23}$$

$$\text{Mass of fuel required } = (362.3 \times 10^{23}) \times \frac{2}{6.023 \times 10^{23}} = 120.3\,\text{g}$$

3. Energy released in one year $= \left(10^6 \times 10^3\right)\left(3.154 \times 10^7\right)$

$$\text{Number of } {}_{92}^{235}\text{U nuclei fissioned } = \frac{10^9 (3.154 \times 10^7)}{200(1.6 \times 10^{-13})} = 9.856 \times 10^{26}$$

$$\text{Mass of } {}_{92}^{235}\text{U released in the reactor} = \frac{(9.856 \times 10^{26})235}{6.023 \times 10^{23}} = 384.5\ \text{kg}$$

4. Number of atoms in 2 kg of ${}_{92}^{235}\text{U}$

$$= \frac{2000}{235} \times (6.023 \times 10^{23}) = 5.126 \times 10^{24}$$

Energy released due to fission of 2 kg of ${}_{92}^{235}\text{U}$

$$= (5.126 \times 10^{24}) \times 185\,\text{MeV} = 15.173 \times 10^{13}\ \text{J}$$

$$\text{Power output of the reactor } = \frac{15.173 \times 10^{13}}{30(24 \times 60 \times 60)} = 58.54\ \text{MW}$$

5. According to the assumption, as three alpha particles form a ${}_6^{12}\text{C}$ nucleus,

$$3\,{}_2^4\text{He} \longrightarrow {}_6^{12}\text{C}$$

Mass of three alpha particles $= 3 \times 4.002604 = 12.007812$ u

Mass of carbon nucleus = 12 u

Mass defect = 12.007812 u – 12 u = 0.007812 u

Energy released in the reaction

$$= 0.007812 \times 931.5\ \text{MeV} = 7.277\ \text{MeV}$$

6. The nuclear fission reaction is

$${}_{92}\text{U}^{235} + {}_0\text{n}^1 \rightarrow {}_{56}\text{Ba}^{141} + {}_{36}\text{Kr}^{92} + 3\,{}_0\text{n}^1$$

$$\text{Mass defect } \Delta m = \left[\left(m_\text{U} + m_\text{n}\right) - \left(m_\text{Ba} + 3m_\text{n}\right)\right]$$

$$= 256.0526 - 235.8373 = 0.2153\,\text{u}$$

Energy released $Q = 0.2153 \times 931 = 200$ MeV

$$\text{Number of atoms in } 1\,\text{g} = \frac{6.02 \times 10^{23}}{235} = 2.56 \times 10^{21}$$

Energy released in fission of 1 g of U^{235} is

$$= 200 \times 2.56 \times 10^{21} = 5.12 \times 10^{23}\,\text{MeV}$$

$$= 5.12 \times 10^{23} \times 1.6 \times 10^{-13} = 8.2 \times 10^{10}\ \text{J}$$

$$= \frac{8.2 \times 10^{10}}{3.6 \times 10^6}\,\text{kWh} = 2.28 \times 10^4\,\text{kWh}$$

7. Energy released in the nuclear fusion is $Q = \Delta mc^2 = \Delta m(931)\text{MeV}$ (where Δm is in amu)

$$Q = (2 \times 2.0141 - 4.0026) \times 931\,\text{MeV} = 23.834 \times 10^6\ \text{eV}$$

Since efficiency of reactor is 25%

So effective energy used

$$= \frac{25}{100} \times 23.834 \times 10^6 \times 1.6 \times 10^{-19}\,\text{J} = 9.534 \times 10^{-13}\,\text{J}$$

Since the two deuterium nuclei are involved in a fusion reaction, therefore, energy released per deuterium is $\dfrac{9.534 \times 10^{-13}}{2}$.

For 200 MW power per day, number of deuterium nuclei required

$$= \frac{200 \times 10^6 \times 86400}{\dfrac{9.534 \times 10^{-23}}{2}} = 3.624 \times 10^{25}$$

Since 2 g of deuterium constitute 6 × 10^{23} nuclei, therefore amount of deuterium required is

$$\frac{2 \times 3.624 \times 10^{25}}{6 \times 10^{23}} = 120.83\ \text{g/day}$$

8. Number of deuterium atoms in 2.0 kg of deuterium

$$= (6.023 \times 10^{23})\left(\frac{2.0 \times 1000\,\text{g}}{2\text{g}}\right) = 6.023 \times 10^{26}$$

Energy released when 2 deuterium nuclei fuse = 3.27 MeV

$$\text{Total energy released } = \left(\frac{3.27}{2}\right)(6.023 \times 10^{26})\,\text{MeV}$$

$$= \left(\frac{3.27}{2}\right)(6.023 \times 10^{26})(1.6 \times 10^{-31}\,\text{J})$$

$$= 15.76 \times 10^{13}\ \text{J}$$

Energy consumed by bulb per second = 100 J/s

$$\text{Time for which the bulb can be kept glowing } = \frac{15.76 \times 10^{13}\ \text{J}}{100\,\text{J/s}}$$

$$= 15.76 \times 10^{11}\text{s} = \frac{15.76 \times 10^{11}s}{3.154 \times 10^7\,\text{s/y}} = 5 \times 10^4\,\text{y}$$

9. For 1000 MW output, the total power generation is 3000 MW, of which 2000 MW is rejected as "waste" heat. Thus, the total energy released in 1 year (3 × 10^7 s) from fission is about (3 × 10^9) (3 × 10^7) ≈ 10^{17}.

If each fission releases 200 MeV of energy, the number of fissions required is

$$\frac{(10^{17})}{(2 \times 10^8)(1.6 \times 10^{-19})} \approx 3 \approx 10^{27}\ \text{fissions}$$

The mass of a single uranium atom is about (235 u) (1.66 × 10^{-27} kg/u) ≈ 4.5 × 10^{-25} kg, so that mass needed is (4.5 × 10^{-25} kg) (3 × 10^{27} fissions) ≈ 1000 kg or about a ton. Since ${}_{92}^{235}\text{U}$ is only a fraction of normal uranium, and even when enriched it is never more than 10 per cent of the total, the yearly requirement for uranium is of the order of tens of tons.

10. After decay, the mass of the daughter m_d plus the mass of the alpha particle m_α is

$$m_\text{d} + m_\alpha = 222.017\,571\ \text{u} + 4.002\,602\ \text{u} = 226.020\,173\ \text{u}$$

Thus, calling the mass of the parent nucleus M_p, we find that the mass lost during decay is

$$\Delta m = M_\text{p} - (m_\text{d} + m_\alpha)$$

$$= 226.025\,402\ \text{u} - 226.020\,173\ \text{u} = 0.005\,229\ \text{u}$$

Using the relationship that 1 u corresponds to 931.494 MeV, we find that the energy liberated is

$$E = (0.005\,229\text{u})\,(931.494\ \text{MeV/u}) = 4.87\ \text{MeV}$$

11. The mass of a ${}^{12}\text{C}$ atom is exactly 12 u. The energy released in the reaction is $3({}_2^4\text{He}) \rightarrow {}_6^{12}\text{C}$

$$[3m\,({}_2^4\text{He}) - m({}_6^{12}\text{C})]\,c^2 = [3 \times 4.002603\ \text{u} - 12\ \text{u}]$$

$$(931\ \text{MeV/u}) = 7.27\ \text{MeV}$$

Exercises

ngle Correct Answer Type

1. (4) For decay (i):

$$Q = [230.033927 - 229.033496 - 1.008665] \times 931.5$$
$$= -7.7 \text{ MeV}$$

For decay (ii):

$$Q = [230.033927 - 229.032089 - 1.007825] \times 931.5$$
$$= -5.6 \text{ MeV}$$

As Q is negative for both the decays, so none of the decays is allowed.

2. (3) Suppose an initial radionuclide I decays to a final product F with a half-life $T_{1/2}$.

At any time, $N_I = N_0 e^{-\lambda t}$

Number of product nuclei $= N_F = N_0 - N_I$

$$\frac{N_F}{N_I} = \frac{N_0 - N_I}{N_I} = \left(\frac{N_0}{N_I} - 1\right)$$

$$\frac{N_0}{N_I} = \left(1 + \frac{N_F}{N_I}\right) = 1 + 0.5 = 1.5$$

$$e^{\lambda t} = 1.5 \quad \Rightarrow \lambda t = \ln 1.5$$

$$\therefore \quad \frac{T_{1/2} \ln(1.5)}{\ln 2} = 4.5 \times 10^9 \, \frac{\ln\left(\frac{3}{2}\right)}{\ln 2} \text{ year}$$

3. (1) Given, $N_2 = \dfrac{N_0}{e} = N_0 e^{-\lambda t} \Rightarrow t = \dfrac{1}{\lambda} = 10 \text{ s}$

$$\therefore \quad T_{1/2} = \frac{\ln 2}{\lambda} = 0.693 \times 10 \approx 7 \text{ s}$$

4. (4) Since no external force is present, so momentum conservation principle is completely applicable.

$$\therefore \quad m\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\text{or} \quad (m_1 + m_2)\,\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2$$

5. (1) $\dfrac{dN_A}{dt} = (-\lambda N_A) + (-2\lambda N_A) + (-2\lambda N_A) = -6\lambda N_A$

6. (2) Let, ${}^{A}_{Z}X \to {}^{A-4}_{A-2}Y + {}^{4}_{2}\text{He}$

$$K_\alpha = \frac{m_y}{m_y + m_\alpha} Q = \frac{A-4}{A} Q$$

$$\text{or} \quad 48 = \frac{A-4}{A} \times 50 \quad \Rightarrow \quad A = 100$$

7. (1) After first half hours, $N = N_0 \dfrac{1}{2}$

For $t = \dfrac{1}{2}$ h to $t = 1\dfrac{1}{2}$ h, 1 h = four half-lives

Hence, $N = \left(N_0 \dfrac{1}{2}\right)\left[\dfrac{1}{2}\right]^4 = N_0 \left(\dfrac{1}{2}\right)^5$

For $t = \dfrac{1}{2}$ to $t = 2$ h

$$\left[\text{for both } A \text{ and } B, \ \frac{1}{t_{1/2}} = \frac{1}{t_{1/2}} + \frac{1}{t_{1/4}} = 2 + 4 = 6 \Rightarrow t_{1/2} = \frac{1}{6}\right]$$

$\dfrac{1}{2}$ h = three half-lives

$$\therefore \quad N = \left[N_0 \left(\frac{1}{2}\right)^5\right]\left(\frac{1}{2}\right)^3 = N_0 \left(\frac{1}{2}\right)^8$$

8. (2) For α-decay: ${}_{x}A^{y} \to {}_{x-2}B^{y-4} + \alpha$

For β^- decay: ${}_{x}A^{y} \to {}_{x+1}B^{y} + {}_{-1}\beta^0$

For β^+ decay: ${}_{x}A^{y} \to {}_{x-1}B^{y} + {}_{+1}\beta^0$

For k-capture, there will be no change in the number of protons. Hence, only case in which number of protons increases is β^- decay.

9. (3) $N = N_0 e^{-\lambda t}$, $N_Y = N_0(1 - e^{-\lambda t})$

Rate of formation of Y is $\dfrac{dN}{dt} = +\lambda N_0 e^{-\lambda t}$

which decreases exponentially with time.

10. (2) 90% of the sample is left undecayed after time t.

$$\therefore \quad \frac{9}{10} N_0 = N_0 e^{-\lambda t}$$

$$\lambda = \frac{1}{t} \ln\left(\frac{10}{9}\right) \qquad \text{...(i)}$$

After time $2t$,

$$N_c = N_0 e^{-\lambda(2t)} = N_0 e^{-\frac{1}{t}\left[\ln\left(\frac{10}{9}\right)\right]2t} \qquad \text{...(ii)}$$

$$N = N_0 e^{-\ln\left(\frac{10}{9}\right)^2} = N_0 \left(\frac{9}{10}\right)^2 \approx 81\% \text{ of } N_0 \qquad \text{...(iii)}$$

Therefore, 19% of initial value will decay in time $2t$.

11. (3) Let N_2 be the number of atoms of X at time $t = 0$. Then, at $t = 4$ h (two half-lives),

$$N_x = \frac{N_0}{4} \quad \text{and} \quad N_y = \frac{3N_0}{4}$$

$$\therefore \quad \frac{N_x}{N_y} = \frac{1}{3} \approx 0.33$$

At $t = 6$ h (three half-lives),

$$N_x = \frac{N_0}{8} \quad \text{and} \quad N_y = \frac{7N_0}{8} \quad \text{or} \quad \frac{N_x}{N_y} = \frac{1}{7} \approx 0.142$$

The given ratio $\dfrac{1}{4}$ lies between $\dfrac{1}{3}$ and $\dfrac{1}{7}$.

Therefore, t lies between 4 h and 6 h.

12. (2) $\dfrac{dN_2}{dt} = \lambda N_1 - 2\lambda N_2$

For N_2 to be maximum, $\dfrac{dN_2}{dt} = 0$

$$\Rightarrow \quad \lambda N_1 = 2\lambda N_2 \quad \text{or} \quad \frac{N_1}{N_2} = 2$$

13. (2) The total energy required to make the electron free from nucleus is the sum of the energy required to separate the electrons from the influence of each other and the energy required to separate the electrons from the influence of nucleus, i.e.,

Total required energy = BE of electron in He atom + ionization energy of He atom = $(24.6 + 13.6 \times 2^2)$ eV = $(24.6 + 54.4)$ eV = 79 eV

14. (1) From given information, $\dfrac{dN}{dt} = \dfrac{-0.04N}{3600}$

Comparing above equation with standard decay equation,

$$\frac{dN}{dt} = -\lambda N$$

$$\lambda = 1.1 \times 10^{-5} \text{ s}^{-1}$$

$$\therefore \quad \tau = \frac{1}{\lambda} = \frac{3600}{0.04} \text{s} = 25 \text{ h}$$

15. (3) Let n collisions are required for the given condition. Then,

$$\left(\frac{1}{2}\right)^n \times 2\text{ MeV} = 0.04 \times 10^{-6}\text{ MeV}$$

$$2^n = \frac{2}{0.04} \times 10^6 = 50 \times 10^6$$

After solving above equation, $n = 26$.

16. (3) Expected atomic mass of Cu must be less than that of zinc, but it is not so. So, it means Cu is radioactive and unstable and decays to Zn through β-decay.

17. (1) As the alpha particle decays, the daughter nucleus recoils. In such a process, the momentum conservation holds good.

So, $p_\alpha = p_D = p$

$$K_\alpha = \frac{p^2}{2M_\alpha} \quad \text{and} \quad K_D = \frac{p^2}{2M_D}$$

As $M_D > M_\alpha$, so, $K_\alpha > K_D$.

18. (3) The minimum energy needed to carry out an endothermic reaction is greater than the Q value of the reaction. This is because to conserve the momentum some extra energy has to be provided.

$\text{KE}_{\text{min}} = \left(1 + \frac{m}{M}\right) \times |Q|$, where m is the mass of the incident particle and M is the mass of target.

19. (3) The number of nuclei in 1 kg ^{235}U is

$$N = \frac{N_A}{235} \times (1 \times 10^3) = \frac{6.023 \times 10^{23}}{235} \times 10^3 = 2.56 \times 10^{24}\text{ nuclei}$$

Total energy released is $E = N \times 200\text{ MeV} = 5.12 \times 10^{26}\text{ MeV}$

20. (1) When a free neutron decays to a proton along with an electron and an antineutrino, the Q value of the reaction is positive which means the reaction is possible all by itself, while a free proton cannot convert itself into a neutron due to negative Q value.

In beta minus decay, the electron originates from nucleus only, by the transformation of neutron into a proton, with simultaneous emission of an antineutrino.

21. (1) Since scheme A releases more energy than scheme B, scheme A is more likely to occur. This is because the more the energy released, the more stable the daughter nucleus is. A heavy nucleus undergoes fission such that its products will be more stable than the parent nucleus.

22. (1) At present, $\dfrac{\text{Number of K atoms}}{\text{Number of Ar atoms}} = \dfrac{1}{7}$

Let age of rock be n half-lives of K-nuclide. Then,

$$\left(\frac{1}{2}\right)^n = \frac{\text{Number of K-atoms present now}}{\text{Number of K-atom present initially}} = \frac{1}{1+7}$$

where number of K atoms present initially = number of K atoms + number of Ar atoms present now.

$\therefore \quad n = 3$

So, age of rock is 3 half-lives of K nuclides, i.e., 4.2×10^9 years.

23. (1) Probability of survival for any nucleus at time t is

$$P = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

So, in one mean life, required probability is $e^{-\lambda \times \frac{1}{\lambda}} = \frac{1}{e}$

24. (2) The complete fission reaction is

$$^{235}_{92}\text{U} + \text{n} \rightarrow {}^{94}_{40}\text{Zr} + {}^{140}_{58}\text{Ce} + 2\text{n} + 6\text{e}^{-1}$$

$$Q = [m(^{235}\text{U}) - m(^{94}\text{Zr}) - m(^{140}\text{Ce}) - m(\text{n})]c^2 = 208\text{ MeV}$$

25. (3) The net reaction is $3(^2_1\text{H}) \rightarrow {}^4_2\text{He} + \text{n} + \text{p}$

$$Q = [3 \times m(^2\text{H}) - m(^4\text{He}) - m(\text{n}) - m(\text{p})] \times 931\text{ MeV}$$

$$= 3.87 \times 10^{-12}\text{ J}$$

This is the energy produced by the consumption of 3 deuteron atoms. So, the total energy released by 10^{40} deuterons is

$$\frac{3.87 \times 10^{-12}}{3} \times 10^{40} = 1.29 \times 10^{28}\text{J}$$

Let total supply of deuterons in star be exhausted in t seconds.

Then, $10^{16} \times t = 1.29 \times 10^{28}$

$\Rightarrow \quad t = 1.29 \times 10^{12}$ s

26. (4) $N_{x_1} = N_0 e^{-10\lambda t}$

$$N_{x_2} = N_0 e^{-\lambda t}$$

$$\frac{N_{x_1}}{N_{x_2}} = \frac{1}{e} = \frac{e^{-10\lambda t}}{e^{-\lambda t}} = e^{-9\lambda t}$$

$$9\lambda t = 1 \quad \Rightarrow \quad t = \frac{1}{9\lambda}$$

27. (3) Let N be the number of nuclei at any time t. Then,

$$\frac{dN}{dt} = 200 - \lambda N$$

$$\therefore \quad \int_0^N \frac{dN}{200 - \lambda N} = \int_0^t dt$$

$$\text{or} \quad N = \frac{200}{\lambda}(1 - e^{-\lambda t})$$

Given: $N = 100$ and $\lambda = 1\text{ s}^{-1}$

$$\therefore \quad 100 = 200\,(1 - e^{-t})$$

$$\text{or} \quad e^{-t} = \left(\frac{1}{2}\right) \quad \therefore \quad t = \ln(2)\text{ s}$$

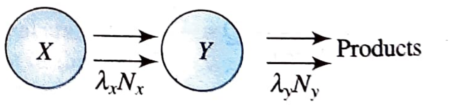
28. (1) Number of radio nuclei become constant, when rate of production becomes equal to rate of decay.

$$X = \lambda N$$

$$\text{or} \quad N = \frac{X}{\lambda}. \text{ Given, } Y = \frac{\ln 2}{\lambda}$$

$$\Rightarrow \quad N = \frac{XY}{\ln(2)}$$

29. (3)



Net rate of formation of Y at any time t is $\dfrac{dN_y}{dt} = \lambda_x N_x - \lambda_y N_y$

N_y is maximum when $\dfrac{dN_y}{dt} = 0$

or $\quad \lambda_x N_x = \lambda_y N_y$

30. (1) $\dfrac{\lambda_A}{\lambda_B} = \dfrac{1}{2}$

Probabilities of getting α- and β-particles are same. Thus, rates of disintegration are equal.

$$\therefore \quad \lambda_A N_A = \lambda_B N_B$$

$$\text{or} \quad \frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = 2$$

31. (2) Three half-lives of A is equivalent to six half-lives of B. Hence,

$$N_A\left(\frac{1}{2}\right)^3 = N_B\left(\frac{1}{2}\right)^6$$

or $\frac{N_A}{N_B} = \frac{1}{8}$

(1) Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$. Initially, rate of disintegration of A is λN_0 and that of B is $2\lambda N_0$. After one half-life of A, rate of disntigration of A will become $\lambda N_0/2$ (half-life of B = one-half the half-life of A). So, after one half-life of A or two half-lives of B,

$$\left(-\frac{dN}{dt}\right) = \left(-\frac{dN}{dt}\right)_B$$

$\therefore \quad n = 1$

(1) Maximum number of nuclei will be present when rate of decay = rate of formation

or $\lambda N = \alpha$

$\therefore \quad N = \frac{\alpha}{\lambda}$

34. (1) Fraction of nuclei which remain undecayed is

$$f = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

$$= e^{-\left(\frac{\ln 2}{T}\right)\left(\frac{T}{2}\right)} = \frac{1}{e^{\ln\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

35. (2) Let R_0 be the initial activity. Then, $R_1 = R_0 e^{-\lambda t_1}$

and $R_2 = R_0 e^{-\lambda t_2}$

$\therefore \quad \frac{R_2}{R_1} = e^{\lambda(t_1 - t_2)}$ or $R_2 = R_1 e^{\lambda(t_1 - t_2)}$

36. (4) $R_1 = \lambda N_1 \Rightarrow N_1 = \frac{R_1}{\lambda}$

and $R_2 = \lambda N_2 \Rightarrow N_2 = \frac{R_2}{\lambda}$

Therefore, number of atoms decayed $= N_1 - N_2 = \left(\frac{R_1 - R_2}{\lambda}\right)$

37. (3) Activity, $R = \lambda N$. Number of nuclei (N) per mole are equal for both the substances.

$\therefore \quad R \propto \lambda$

or $\frac{R_1}{R_2} = \frac{\lambda_1}{\lambda_2} = \frac{4}{3}$

38. (2) Activity of a radioactive substance, $R = \lambda N$

$\therefore \quad \lambda = \frac{R}{N}$

Here, $R = N_2$ particles per second and $N = N_1$.

$\therefore \quad \lambda = \frac{N_2}{N_1}$

39. (2) During fusion, binding energy of daughter nucleus is always greater than the total binding energy of the parent nuclei. The difference of binding energies is released. Hence,

$$Q = E_2 - 2E_1$$

40. (4) Energy released would be

ΔE = total binding energy of ${}_2\text{He}^4 - 2 \times$ (total binding energy of ${}_1\text{He}^4$)

$= 4 \times 7.0 - 2\,(1.1)\,(2) = 23.6$ MeV

41. (1) ${}_{92}\text{U}^{238} + {}_0\text{n}^1 \rightarrow {}_{92}\text{U}^{239} \rightarrow {}_{-1}\text{e}^0 + {}_{93}\text{Np}^{239} \rightarrow {}_{-1}\text{e}^0 + {}_{94}\text{Pu}^{239}$

42. (3) $\frac{A_0}{3} = A_0\left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$ and $A' = \frac{A_0}{3}\left(\frac{1}{2}\right)^{\frac{9}{T_{1/2}}}$

Dividing, we get $\frac{A' \times 3}{A_0} = \frac{1}{3}$ or $A' = \frac{A_0}{9}$

43. (2) $T_{1/2} = \frac{0.693}{\lambda}$ or $T_{1/2} = 0.693\left[\frac{1}{\lambda}\right]$

or $\quad T_{1/2} = 0.693\,\tau$

Clearly, $x = 0.693$.

44. (4) The emission of antineutrino is a must for the validity of different laws.

45. (1) ${}_Z\text{X} \rightarrow {}_2\text{He} + {}_{Z\ \ 2}\text{Y}$

${}^{A-4}_{Z-2}\text{Y} \rightarrow e^+ + {}^{A-4}_{Z-3}\text{Y}'$

During β^+ emission, ${}_{+1}\text{p}^1 \rightarrow {}_0\text{n}^1 + \beta^+$

The proton changes into neutron. So, charge number decreases by 1 but mass number remains unchanged.

46. (2) $\frac{N_0}{4} = \frac{N_0}{2n} \Rightarrow n = 2$

Thus, 10 days = 2 half-lives

$\therefore$ Half-life = 8 days

47. (4) α-decay decreases mass number by 4 and reduces charge number by 2. β-decay keeps mass number unchanged and increases charge by 1. Clearly, option (4) is the right choice.

48. (2) Decrease in mass number = 232 – 208 = 24

Number of α-particles emitted $= \frac{24}{4} = 6$

Due to emission of 6 particles, decrease in charge number is 12. But actual decrease in charge number is 8. Clearly, 4 β-particles are emitted.

49. (3) $A = A_0 e^{-\lambda t}$; $2100 = 16000\, e^{-12\lambda} \Rightarrow e^{12\lambda} = 7.6$

$\Rightarrow \quad 12\lambda = \log_e 7.6 = 2 \Rightarrow \lambda = \frac{2}{12} = \frac{1}{6}$

$\therefore \quad T = \frac{0.6931 \times 6}{1} = 4$

50. (2) Total mass of the products = 2.0165 amu, which is greater than the mass of the deuteron by 0.0024 amu. The extra mass must be provided by the energy of the photon so that minimum possible frequency must be given by

$$v = \frac{0.0024\ \text{u}\,c^2}{h} \quad (1\ \text{amu} = 1.66 \times 10^{-27}\ \text{kg})$$

$= 5.4 \times 10^{20}$ Hz

51. (2) The nuclear fission differs from other nuclear reactions in three respects.

(a) The nucleus is deeply divided into two large fission fragments or nuclei of roughly equal mass. The nuclei or fission fragments fly apart at great speed and thus possess large kinetic energies that carry off the greater part of the energy released.

(b) The mass decrease is appreciable and hence large energy is released.

(c) Other neutrons, called fission neutrons, are emitted in the process. Small amount of energy is released in the form of radiation.

52. (4) Disintegration of deuteron to a proton and a neutron can be represented by

$${}^2_1\text{H} + \text{Q} \rightarrow {}^1_1\text{H} + {}^1_0\text{n}$$

The energy captured is the γ-ray photon E_γ is given by

$E_\gamma + 1876 = 939 + 940$

$\Rightarrow \quad E_\gamma = (939 + 940) - 1876 = 3$ MeV.

53. (3) Decay constant, $\lambda = 10^{-6}\ \text{s}^{-1}$. The half-life $T_{1/2}$ is thus given by

$$T_{1/2}=\frac{0.639}{\lambda}=\frac{0.639}{10^{-6}}=0.639\times10^{6}\text{ s}$$
$$=192.5\text{ h}\approx 8\text{ days}=1.14\text{ week}\approx 1\text{ week}$$

54. (4) Given:

X has activity A_0 at $t=0$ and its half-life is 24 years.

Y has activity A_0 at $t=0$ and its half-life is 16 years.

At $t=48$ years, activity of X $=\frac{1}{4}A_0$ (2 half-lives have elapsed)

At $t=48$ years, activity of Y $=\frac{1}{8}A_0$ (3 half-lives have elapsed)

Thus, total activity of the mixture of X and Y at $t=48$ years is

$$\frac{1}{4}A_0+\frac{1}{8}A_0=\frac{3}{8}A_0$$

55. (1) Let $t=0$, $M_0=10$ g

$$t=2\tau=2\left(\frac{1}{\lambda}\right)$$

Then, $M=M_0e^{-\lambda t}=10e^{-\lambda\left(\frac{2}{\lambda}\right)}=10\left(\frac{1}{e}\right)^2=1.35$ g

56. (1) $N=\dfrac{N_0}{2^{t/T}}$

$$\frac{N_0}{16}=\frac{N_0}{2^{t/T}}$$
$$2^{t/T}=16=2^4$$

or $\dfrac{t}{T}=4$

or $T=\dfrac{t}{4}=\dfrac{24}{4}\text{ h}=6\text{ h}$

57. (1) $\dfrac{1}{16}=\dfrac{1}{2^{\frac{t}{100}}}$

or $\dfrac{1}{2^4}=\dfrac{1}{2^{t/100}}$ or $4=\dfrac{t}{100}$

or $t=400\ \mu$s

58. (4) The α-particle emitting radioactive gas, thoron-220, decays to radium-216 an emits an α-particle. The reaction can be represented by

$$^{220}_{90}\text{Th}\rightarrow{}^{4}_{2}\text{He}+{}^{216}_{66}\text{Ra}$$

By conservation of momentum, we have momentum of α-particle = momentum of recoiling nucleus Ra

$$\Rightarrow\quad m_\alpha v_\alpha=m_R v_R$$
$$\Rightarrow\quad \frac{v_R}{v_\alpha}=\frac{m_\alpha}{m_R}=\frac{4}{216}=\frac{1}{54}$$

The kinetic energy of Ra, E_R, is related to the kinetic energy of alpha particle E_α by

$$\frac{E_R}{E_\alpha}=\frac{\frac{1}{2}m_Rv_R^2}{\frac{1}{2}m_\alpha v_\alpha^2}=\left(\frac{m_R}{m_\alpha}\right)\left(\frac{v_R}{m_\alpha}\right)^2=\left(\frac{m_R}{m_\alpha}\right)\left(\frac{m_\alpha}{m_R}\right)^2=\frac{m_\alpha}{m_R}=\frac{1}{54}$$

$$\therefore\quad E_R=\frac{E_\alpha}{54}$$

59. (4) $\dfrac{9}{16}=\left(\dfrac{1}{2}\right)^{\frac{t}{T}}$

$$\frac{N}{N_0}=\left(\frac{1}{2}\right)^{\frac{t}{2T}}$$

$$\left(\frac{N}{N_0}\right)^2=\left(\frac{1}{2}\right)^{t/T}$$

or $\left(\dfrac{N}{N_0}\right)^2=\dfrac{9}{16}$ or $\dfrac{N}{N_0}=\dfrac{3}{4}$

Note the special technique used in the problem.

60. (1) $\dfrac{A_2}{A_1}=\dfrac{N_2}{N_1}$

$$\frac{A_2}{10^3}=\frac{1}{2}\quad\text{or}\quad A_2=\frac{1000}{2}=500\text{ s}^{-1}$$

61. (4) $N=N_0e^{-\lambda t}$

$$\Rightarrow\quad \frac{N_0}{N}=e^{\lambda t}$$

or $\lambda t=\log_e\dfrac{N_0}{N}$ or $t=\dfrac{1}{\lambda}\log_e\dfrac{N_0}{N}$

$$\therefore\quad t\propto\log_e\frac{N_0}{N}$$

$$\Rightarrow\quad 5\propto\log_e\frac{100}{90}\text{ and }20\propto\log_e\frac{N_0}{N}$$

Dividing, $\dfrac{5}{20}=\dfrac{\log_{10}\frac{100}{90}}{\log_{10}\frac{N_0}{N}}$

or $\log_{10}\dfrac{N_0}{N}=4\log_{10}\dfrac{10}{9}$

or $\dfrac{N_0}{N}=\left(\dfrac{10}{9}\right)^4\Rightarrow\dfrac{N}{N_0}=0.6561$

Percentage of substance decayed is $(1-0.6561)\times100=34.39\%$

62. (3) Let the kinetic energy of the α-particle be E_α and that of the thorium Th be E_{th}. The ratio of kinetic energies is

$$\frac{E_\alpha}{E_{\text{th}}}=\frac{\frac{1}{2}m_\alpha v_\alpha^2}{\frac{1}{2}m_{\text{th}}v_{\text{th}}^2}=\left(\frac{m_\alpha}{m_{\text{th}}}\right)\left(\frac{v_\alpha}{v_{\text{th}}}\right)^2\quad\text{...(i)}$$

By conservation of momentum, the momentum of α-particle and that of the recoiling thorium must be equal. Thus,

$$m_\alpha v_\alpha=m_{\text{th}}v_{\text{th}}$$

or $\dfrac{v_\alpha}{v_{\text{th}}}=\dfrac{m_{\text{th}}}{m_\alpha}$ ...(ii)

Substituting Eq. (ii) in Eq. (i), we have

$$\frac{E_\alpha}{E_{\text{th}}}=\left(\frac{m_\alpha}{m_{\text{th}}}\right)\left(\frac{m_{\text{th}}}{m_\alpha}\right)^2=\frac{m_{\text{th}}}{m_\alpha}=\frac{234}{4}=58.5$$

Thus, the kinetic energy of the α-particle expressed as the fraction of the total kinetic energy T is given by

$$E_\alpha=\frac{58.5}{1+58.5}T=\frac{58.5}{59.5}T=0.98\,T$$

which is slightly less than T.

63. (2) $\dfrac{A}{A_0}=\dfrac{N}{N_0}$

Let safe level activity be A. Initial activity = $64A$.

Hence, $\dfrac{N}{N_0}=\dfrac{A}{A_0}=\dfrac{A}{64A}=\dfrac{1}{64}$

or $\left(\frac{1}{2}\right)^n = \frac{1}{64}$ or $n = 6$

Hence, $\frac{t}{T} = n = 6$

$\because \quad T = 2\text{ h}$

$\therefore \quad t = 12\text{ h}$

(3) $N_1\lambda_1 = N_2\lambda_2$

$$T = \frac{0.693}{\lambda}$$

Hence, $2.8 \times 10^6 \times \frac{0.693}{T_1(\text{U})} = 1 \times \frac{0.693}{T_2(\text{Ra})}$

$$\therefore \quad T_1(\text{U}) = 1600 \times 2.8 \times 10^6 = 4.48 \times 10^9 \text{ years}$$
$$\approx 4.5 \times 10^9 \text{ years}$$

(2) Number of α-particles per second = Activity = $(-dN/dt) = N\lambda$

where $N = \frac{6.0 \times 10^{23}}{210} \times 1 \times 10^{-3}$

$$\lambda = 5.8 \times 10^{-8}\text{ s}^{-1}$$

So, $A = N\lambda = \frac{6.0 \times 10^{23}}{210} \times 1 \times 10^{-3} \times 5.8 \times 10^{-8} = 1.7 \times 10^{11}$

(1) Let $\frac{M_1}{M_2}$ (mass ratio) $= \frac{1}{2}$

2 days = 2 × 24 h = 48 h

For first substance, 4 half-life periods and for second substance 3 half-life periods are passed; the masses are reduced to

$$M_1' = M_1 \times \left(\frac{1}{2}\right)^4$$

$$M_2' = M_2 \times \left(\frac{1}{2}\right)^3$$

$$\therefore \quad \frac{M_1'}{M_2'} = \frac{M_1}{M_2} \times \frac{1}{2} = \frac{2}{1} \times \frac{1}{2} = \frac{1}{1}$$

7. (4) $R_1 = N_1\lambda, \quad R_2 = N_2\lambda$

Also, $T = \frac{\log_e 2}{\lambda}$ or $\lambda = \frac{\log_e 2}{T}$

$$\therefore \quad R_1 - R_2 = (N_1 - N_2)\lambda = (N_1 - N_2)\frac{\log_e 2}{T}$$

$$\therefore \quad (N_1 - N_2) = \frac{(R_1 - R_2)T}{\log_e 2} \text{ i.e., } (N_1 - N_2) \propto (R_1 - R_2)T$$

8. (3) We know that $N = N_0\left(\frac{1}{2}\right)^n$

For A, $N = N_0\left(\frac{1}{2}\right)^{n_A} = N_0\left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$

$$\left[\because n_A = \frac{t}{T_A} = \frac{80}{20} = 4\right]$$

For B, $N_B = N_0\left(\frac{1}{2}\right)^{n_B} = N_0\left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$

$\therefore \quad \frac{N_A}{N_B} = \frac{1}{4}$ or $N_A : N_B = 1:4$

69. (4) Let the decay constants for the first and second processes be λ_1 and λ_2 and the effective decay constant for the combined process be λ. Then,

$$\lambda_1 = \frac{\log_e 2}{t_1}, \quad \lambda_2 = \frac{\log_e 2}{t_2} \text{ and } \lambda = \frac{\log_e 2}{t}$$

Now, the probability for decay through first process in a small time interval dt is $\lambda_1\, dt$ and the probability for decay through second process in the same time interval dt is $\lambda_2\, dt$.

The probability for decay by the combined process in the same time interval dt is $\lambda_1\, dt + \lambda_2\, dt$.

But this is also equal to λdt.

$$\therefore \quad \lambda\, dt = \lambda_1\, dt + \lambda_2 dt$$

$$\therefore \quad \lambda = \lambda_1 + \lambda_2$$

or $\frac{\log_e 2}{t} = \frac{\log_e 2}{t_1} + \frac{\log_e 2}{t_2}$

or $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$ or $t = \frac{t_1 t_2}{t_1 + t_2}$

70. **(1)** According to Avogadro's hypothesis,

$$N_0 = \frac{6.02 \times 10^{23}}{226} = 2.66 \times 10^{21}$$

Half-life, $T = \frac{0.693}{\lambda} = 1620$ years

$$\therefore \quad \lambda = \frac{0.6931}{1620 \times 3.16 \times 10^7} = 1.35 \times 10^{-11}\text{ s}^{-1}$$

Because half-life is very much large as compared to its time interval, hence $N \approx N_0$. Now,

$$\frac{dN}{dt} = \lambda N \approx \lambda N_0$$

or $dN = \lambda N_0\, dt = (1.35 \times 10^{-11})(2.66 \times 10^{21}) \times 1 = 3.61 \times 10^{10}$

71. **(3)** Radius R of a nucleus changes with the nucleon number A of the nucleus as $R = 1.3 \times 10^{-15} \times A^{1/3}$ m

Hence, $\frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} = \left(\frac{128}{16}\right)^{1/3} = (8)^{1/3} = 2$

$$\therefore \quad R_2 = 2R_1 = 2(3 \times 10^{-15})\text{ m} = 6 \times 10^{-15}\text{ m}$$

72. **(2)** For nucleus of ${}_8\text{O}^{16}$: Mass = $(16)(1.67 \times 10^{-27})$ kg

Volume $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(3 \times 10^{-15})^3\text{ m}^3 = 36\pi \times 10^{-45}\text{ m}^3$

Density $= \frac{\text{mass}}{\text{volume}} = \frac{16 \times 67 \times 10^{-27}\text{ kg}}{36\pi \times 10^{-46}\text{m}^3} = 2.35 \times 10^{17}\text{ kg m}^{-3}$

73. **(1)** Mass defect, $\Delta m = 20(1.007277 + 1.00866) - 39.97545$
$= 40.31874 - 39.97545 = 0.34329$ amu

$\therefore \quad$ Binding energy $= 0.34329 \times 931 = 319.6$ MeV

When one atom of Ca-40 completely dissociates, the energy to be supplied = 319.6 MeV.

1 g of Ca-40 contains $\frac{6.023 \times 10^{23}}{40} = 1.506 \times 10^{22}$ atoms

The energy required for the dissociation of 1 g of Ca-40
$= 319.6 \times 1.506 \times 10^{22} = 4.813 \times 10^{24}$ MeV

74. **(3)** Binding energy per nucleon of fission products is 8.5 MeV
Binding energy per nucleon of reactants = 7.6 MeV
Increase in binding energy per nucleon is 8.5 – 7.6 = 0.9 MeV
Energy released per nucleon in fission is 0.9 MeV

Therefore, fractional energy released $= \frac{0.9}{931} = \frac{1}{1000}$

Percentage of mass converted into energy during fission

$$= \frac{1}{1000} \times 100 = 0.1\%$$

75. (2) $Q = (\Sigma B_r - \Sigma B_p)\, c^2$

where ΣB_r = sum of the masses of reactants

and ΣB_p = sum of the masses of the products

$\Sigma B_r = 2 \times 2.014741$ amu = 4.0294892 amu

$\Sigma B_p = (3.016977 + 1.008987)$ amu = 4.025964 amu

$\Sigma B_r - \Sigma B_p = (4.029482 - 4.025694)$ amu

= 0.003518 amu

Decrease in mass appears as equivalent energy.

∴ $Q = 0.003518 \times 931$ MeV = 3.27 MeV

76. (2) $P = 10^6$ W

Time = 1 day = $24 \times 36 \times 10^2$ s

Energy produced, $U = Pt = 10^6 \times 24 \times 36 \times 10^2 = 24 \times 36 \times 10^8$ J

Energy released per fusion reaction is 20 MeV = 32×10^{-13} J

Energy released per atom of ${}_1H^2$ is 32×10^{-13} J

Number of ${}_1H^2$ atoms used is $\frac{24 \times 36 \times 10^8}{32 \times 10^{-12}} = 22 \times 10^{21}$

Mass of 6×10^{23} atoms = 2 g

Mass of 27×10^{21} atoms = $\frac{2}{6 \times 10^{23}} \times 27 \times 10^{21} = 0.1$ g

77. (2) Power P of fission reactor, $P = 10^6$ W = 10^6 J s^{-1}

Time = t = 1 day = $24 \times 36 \times 10^2$ s

Energy produced, $U = Pt$

or $U = 10^6 \times 24 \times 36 \times 10^2 = 24 \times 36 \times 10^8$ J

Energy released per fission of U^{235} is 200 MeV = 32×10^{-12} J

Number of U^{235} atoms used is $\frac{24 \times 36 \times 10^8}{32 \times 10^{-12}} = 27 \times 10^{20}$

Mass of 6×10^{23} atoms of U^{235} = 235 g

Mass of 27×10^{20} atoms of U^{235} is

$$\left(\frac{235}{6 \times 10^{23}}\right)(27 \times 10^{20}) = 1.058 \text{ g} = 1 \text{ g}$$

78. (2) Mass of one atom of U^{235} is 235.121420 amu

Mass of one neutron = 1.008665 amu

Sum of the masses of U^{235} and neutron

= 236.130085 = 236.130 amu

Mass of one atom of U^{236} is 236.123050 amu = 236.123 amu

Mass defect = 236.136 – 236.123 = 0.007 amu

Therefore, energy required to remove one neutron is

0.007×931 MeV = 6.517 MeV = 6.5 MeV

79. (3) Total binding energy of helium atom (${}_2He^4$) is $4 \times 7 = 28$ MeV

Total binding energy of deuteron ${}_1H^2$(1p + 1n) is

$2 \times 1.1 = 2.2$ MeV

Hence, binding energy of 2 deuterons is $2 \times 2.2 = 4.4$ MeV

So, the energy released in forming helium nucleus from two deuterons is 28 – 4.4 MeV = 23.6 MeV

80. (3) Mass defect, $\Delta m = 2(2.015) - (3.017 + 1.009) = 0.004$ amu

As 1 amu = 931.5 MeV/c^2, energy released will be

0.004×931.5 MeV = 3.726 MeV

Energy released per deuteron is $\frac{3.726}{2} = 1.863$ MeV

Number of molecules in 1 kg deuterons is

$$\frac{6.02 \times 10^{26}}{2} = 3.01 \times 10^{26}$$

Therefore, energy released per kg of deuterium fusion

$= (3.01 \times 10^{26} \times 1.863) = 5.6 \times 10^{26}$ MeV $\approx 9.0 \times 10^{13}$ J

81. (2) Here, half-life of radium, t = 1500 years

Disintegration constant $\lambda = \frac{0.693}{T} = \frac{0.693}{1500}$ year^{-1}

$N_0 = 1$ g, N = 1 centigram = 10^{-2} g

Now apply $N = N_0 e^{-\lambda t}$.

⇒ $10^{-2} = e^{-\lambda t}$ or $t = 9.972 \times 10^2$ yrs

82. (2) Once the neutron gets sufficiently close to the nucleus, the strong nuclear force sucks it in. Same happens with the proton except it is electrostatically repelled by the six protons already inside the carbon nucleus. The repulsion prevents a 100 ms^{-1} proton from getting close enough to the nucleus. Therefore, the answer is (2).

83. (3) For this substance 7 days correspond to 3.5 half-lives. Over 3 half-lifes the sample reduces to $\frac{1}{2^3} = \frac{1}{8}$ of its initial mass. After 4 half-lifes, the sample has only $\frac{1}{2^4} = \frac{1}{16}$ of its initial mass.

Hence, after 3.5 half-lives the sample must contain somewhere between 1/8 and 1/16 of its initial mass.

Hence, 5 g is somewhere between 1/8 and 1/16 of the initial mass.

So, the initial mass is somewhere between 8 × 5 = 40 g and 16 × 5 = 80 g.

84. (4) As we regard the decay process as a spontaneous and statistical process, therefore the decay can start any time after $t = 0$. Therefore, the answer is (4).

85. (2) A nucleus contains protons and neutrons with no antiprotons and antineutrons. Hence, answer can be either (2) or (4). Due to conservation of spin, the answer is (2).

86. (4) Atomic mass M(H) of hydrogen and nuclear mass (M_n) are

$M(H) = 1.007825$ u and $M_n = 1.008665$ u

Mass defect, $\Delta m = [M(H) + M_n - M(D)]$

$M(D)$ = mass of deuteron = 2.016490 u – 2.014102 u

= 0.002388 u

As 1 u corresponds to 931.494 MeV energy, therefore, mass defect corresponds to energy,

$E_b = 0.002388 \times 931.5 = 2.224$ MeV

87. (4) Nuclear reactions conserve total charge, and also conserve the total approximate mass. The other particles in the reaction will have mass = 236 – 140 – 94 = 2

The other particles are two neutrons. Hence, (1) is not correct.

For nuclei, number of protons tells the charge. So, the other particles must have charge Z such that

$92 = 54 + 38 + Z$

∴ $Z = 0$

Therefore, the other particles have a total atomic mass 2 and total charge 0. Hence, only (4) is correct.

88. (3) All neutrons attract each other with the same strong nuclear force. So, the strong nuclear force holds together three protons and one neutron (${}^4_3Li$) just as vigourously as it holds together two protons and two neutrons (${}^4_2He$). Specifically, protons electrostatically repel other protons. This repulsion tries to make a nucleus fly apart. Since ${}_2^4\text{He}$ contains only two protons, the attractive strong nuclear forces overcome the repulsion of the protons. Hence, the nucleus holds together. But in ${}_3^4\text{Li}$, the mutual repulsion of the three protons overcomes the strong nuclear attractions and the nucleus falls apart (or undergoes radioactive decay into a more stable nucleus). Therefore, the answer will be (3).

(2) Number of atoms in 2 kg fuel $= \dfrac{2}{235} \times 6.02 \times 10^{26} = 5.12 \times 10^{24}$

Fission rate = Number of atoms fissioned in one second

$$= \frac{5.12 \times 10^{24}}{30 \times 24 \times 60 \times 60} = 1.975 \times 10^{18}\ \text{s}^{-1}$$

Each fission gives 185 MeV. Hence, energy obtained in one second,

$$P = 185 \times 1.975 \times 10^{18}\ \text{MeV s}^{-1}$$

$$= 185 \times 1.975 \times 10^{18} \times 1.6 \times 10^{-19}\ \text{MJ s}^{-1} = 58.46\ \text{MW}$$

(3) ${}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_1\text{He}^4 + \text{Q}$

$$\Delta m = m({}_2\text{He}^4) - 2m({}_1\text{H}^2)$$

$$\Delta m = 4.0024 - 2(2.0141)$$

$$\Delta m = -0.0258\ \text{u}$$

Now, $Q = c^2\Delta m = (0.0258)\,(931.5)\ \text{MeV} \approx 24\ \text{MeV}$

91. (3) $\dfrac{dN}{dt} = n - \lambda N$

Because the population N is simultaneously increasing at rate n and decreasing due to decay at rate λN.

$$\int_{N_0}^{N} \frac{dN}{n - \lambda N} = \int_0^t dt$$

$$\frac{1}{\lambda}\ln\left(\frac{n - \lambda N_0}{n - \lambda N}\right) = t$$

$$N = \frac{n}{\lambda} + \left(N_0 - \frac{n}{\lambda}\right)e^{-\lambda t}$$

92. (3) $t = 0,\ N = N_0$

$$t = 6.93,\ N = N_0/4$$

$N_0/4$ is the sample left after two half-lives

$$\therefore \quad 2t_{1/2} = 6.93$$

$$\Rightarrow \quad 2 \times \frac{0.693}{\lambda} = 6.93 \Rightarrow \lambda = 0.2\ \text{min}^{-1}$$

$$\Rightarrow \quad t = 60\ \text{min}$$

$$\therefore \quad N = N_0 e^{-\lambda t} = N_0 e^{-0.2 \times 60} = \frac{N_0}{e^{12}}$$

93. (4) $\left|\dfrac{dN}{dt}\right| = \lambda N$

Number of radium nuclei in m g $= \dfrac{N_A m}{226}$

Decay constant, $\lambda = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{1620 \times 3.16 \times 10^7}$

$$\left|\frac{dN}{dt}\right| = 10 = \frac{6.02 \times 10^{23} m}{226} \times \frac{0.693}{1620 \times 3.16 \times 10^7}$$

$$\therefore \quad m = \frac{10 \times 226 \times 1620 \times 3.16 \times 10^7}{6.02 \times 10^{23} \times 0.693}$$

$$= 2.77 \times 10^{-10}\ \text{g} = 2.77 \times 10^{-13}\ \text{kg}$$

94. (3) The Q value of the first reaction implies that

$${}^{14}\text{N} + \text{d} = {}^{15}\text{N} = \text{p} + 8.53\ \text{MeV}$$

where ${}^{14}\text{N}$ represents the mass of ${}^{14}\text{N}$ nucleus in energy units. This can be rearranged to give

$${}^{14}\text{N} - {}^{15}\text{N} = \text{p} - \text{d} + 8.53\ \text{MeV}$$

The second reaction similarly implies as

$${}^{15}\text{N} - {}^{13}\text{C} = \alpha - \text{d} + 7.58\ \text{MeV}$$

and the third reaction gives

$${}^{13}\text{C} - {}^{11}\text{B} = \alpha - \text{d} + 5.16\ \text{MeV}$$

Adding these three equations, we have

$${}^{14}\text{N} - {}^{11}\text{B} = \text{p} + 2\alpha - 3\text{d} + 21.27\ \text{MeV}$$

$${}^{11}\text{B}(\alpha, \text{n})^{14}\text{N} = {}^{11}\text{B} - {}^{14}\text{N} + \alpha - \text{n}$$

$$= 3\text{d} - \alpha - \text{p} - \text{n} - 21.27\ \text{MeV}$$

Now, $3\text{d} - \alpha - \text{p} - \text{n} = (3 \times 2.014 - 4.0020 - 1.0078 - 1.0087)$ amu

$$= 0.0229\ \text{amu}$$

$$\therefore \quad Q = (0.0229 \times 931 - 21.27)\ \text{MeV} = 0.05\ \text{MeV}$$

95. (4) If d is the distance of closest approach given, then the angular momentum $= mvd = 10^{-33}$ J s

$$E = \frac{1}{2}mv^2 = 1\ \text{MeV} = 1.6 \times 10^{-13}\ \text{J}$$

Momentum, $p = \sqrt{2m_n E} = \sqrt{2 \times 1.6 \times 10^{-27} \times 1.6 \times 10^{-13}}$

$$= 1.6\sqrt{2} \times 10^{-20}\ \text{kg m s}^{-1}$$

Distance of closest approach, $d = \dfrac{10^{-33}}{1.6\sqrt{2} \times 10^{-20}}$

$$= \frac{1}{1.6\sqrt{2}} \times 10^{-13} = \frac{100}{1.6\sqrt{2}}\ \text{fm} = 0.44\ \text{fm}$$

96. (3) The binding energy per nucleon is lowest for very light nuclei such as ${}_2^4\text{He}$, is greatest around $A = 60$, and then decreases with increasing A.

97. (4) Two protons and two neutrons are lost in an α-decays, so Z and N each decrease by 2. A β^+ decay changes a proton to a neutron, so Z decreases by 1 and N increases by 1. The net result is Z decreases by 3 and N decreases by 1.

$${}_Z^A\text{X} \xrightarrow{\alpha\text{-decay}} {}_{Z-2}^{A-4}\text{Y} \xrightarrow{\beta\text{-decay}} {}_{Z-3}^{A-4}\text{Z}$$

Initially, number of neutrons $N_i = (A - Z)$

Now, number of neutrons $N_f = A - 4 - Z + 3 = N_i - 1$

98. (3) $m({}^{198}\text{Au}_{79}) = 197.968225$ u, $m({}_{80}^{198}\text{Hg}) = 197.966752$ u

Mass defect, $\Delta m = 1.473 \times 10^{-3}$ u $= 1.371$ MeV

Energy of γ-photon = 0.412 MeV

Maximum kinetic energy of the electron emitted in the decay is

$$E_e = 1.371\ \text{MeV} - 0.412\ \text{MeV} = 0.959\ \text{MeV}$$

Multiple Correct Answers Type

1. (1), (2), (3), (4)

We have, $6.25\% = \dfrac{6.25}{100} = \dfrac{1}{16}$

The given time of 4 h thus equals 4 half-lives so that the half-life is 1 h.

Since half-life $= \dfrac{\ln 2}{\text{decay constant}}$ and mean life $= \dfrac{1}{\text{decay constant}}$,

after further 4 h, the amount left over would be $\dfrac{1}{2^4} \times \dfrac{1}{2^4}$, i.e., $\dfrac{1}{256}$ or $\dfrac{100}{256}$ or 0.39% of original amount.

2. (1), (2), (4)

It has been observed that total mass of nucleus is always less than the sum of the masses of its nucleons. The energy difference between the nucleus and its constituent particles due to their mass difference is termed as the binding energy of the nucleus.

In other words, we can say that to break the nucleus into its constituent particles, some energy is needed to be supplied. This energy is termed as binding energy of the nucleus.

For (1), more is the binding energy per nucleon, more is the energy required to break the nucleus and hence we can say the more stable the nucleus is.

For (2), (3) and (4), in actual the binding energy is always positive but if it were zero, then nucleus will break spontaneously.

3. (1), (3), (4)

All the statements are very conceptual statements related to different decays.

4. (1), (4)

For an exothermic reaction $X + x \rightarrow Y + y$

If K_i is the kinetic energy of incident particle x, then from energy conservation,

$$K_i + (m_x + M_X)c^2 = K_Y + K_y + (M_Y + m_y)c^2$$

$$K_Y + K_y = K_i + (m_x + M_X - M_Y - m_y)c^2$$

$$K_Y + K_y = K_i + Q$$

In any exothermic reaction, mass of the products is less than the mass of reactants, i.e., in products, the nucleons are more tightly bound and hence have greater BE per nucleon as compared to BE per nucleon of reactants. For endothermic reaction to be carried out, minimum energy given to the reactant must be greater than $|Q|$ value.

5. (1), (2), (3)

In general, fission and fusion processes are exothermic reactions, i.e., energy is released. Hence, mass of products must be less than mass of the reactant nuclide, and BE/A of reactants < BE/A of product nuclides.

6. (1), (2), (3)

If the nuclear reaction involving β-decay is $n \rightarrow p + e^{-1}$, the spins on two sides are not equal as all the three (neutron, proton and electron) have spins of $+1/2$. So, to conserve angular momentum (spin), some other particle must be emitted.

Through experiments it has been observed that direction of emitted electron and recoiling nuclei are almost never exactly opposite as required for linear momentum to be conserved.

During β-decay, the energy of electron is found to vary continuously from 0 to a maximum value (this maximum value is a characteristic of nuclide). To explain this experimental observation, we also need some other particle.

7. (1), (3)

Half-lives of both the samples would be same as half-life is the property of radioactive material and is independent of number of nuclei present or its activity. Let $R_{0B} = R_0$, then $R_{0A} = 2B_0$, where R_0 denotes initial activity.

Activity of A after 5 half-lives is $R_A = \dfrac{R_{0A}}{2^5} = \dfrac{2R_0}{2^5}$

Activity of B after 5 half-lives is $R_B = \dfrac{R_{0B}}{2^5} = \dfrac{R_0}{2^5}$

$$\therefore \quad \frac{R_A}{R_B} = \frac{2}{1}$$

8. (1), (3)

$$\lambda = (0.173\ \text{year})^{-1}$$

$$N = N_0 e^{-\lambda t}$$

As $t = \dfrac{1}{\lambda}$, hence

$$N = \frac{N_0}{e} = \frac{N_0}{2.178} = 0.37\,N_0$$

$$\Rightarrow \quad T = \frac{0.693}{\lambda} = \frac{0.693}{0.173} = 4\ \text{years}$$

9. (1), (4)

In α-decay, the entire energy is carried away by the α-particles as its kinetic energy. In β^--decay, the energy is shared between the β-particle and the anti-neutrino. Hence, the speed of the β-particle will vary, depending on the energy of the anti-neutrino.

10. (2), (3), (4)

Statement (1) is incorrect. In fact, $A = Z + N$

Statements (2), (3) and (4) are correct; they are the definitions of isobars, isotopes and isotones.

11. (1), (3)

The idea of magic number has led to the shell model and the nuclides with these number of protons or neutrons have been compared with the inert gases vis-à-vis stability in terms of closed shells.

12. (1), (2), (3)

The last statement is incorrect because the amount of energy released per unit mass of the fuel is much more for fusion than for fission. Hence, (1), (2) and (3) are correct.

13. (2), (3)

Statement (1) is incorrect. The ${}_2He^4$ nucleus (or the α-particle) is exceptionally stable and has a much higher value of BE per nucleon than that for most other light nuclei. Statement (2) is correct but the reason of decrease in binding energy is different for the cases of smaller and larger values of A. The reason for the decrease in the BE per nucleon for nuclei with large A is that with an increase in the number of protons, the Coulomb repulsion increases. On the other hand, the decrease in the BE per nucleon for nuclei with small A is due to a surface effect: the nucleons at the surface being less strongly bound than those in the interior.

Statement (3) is also correct. The energy required to remove one neutron (i.e., one nucleon) is the same as the binding energy per nucleon for a given isotope.

Statement (4) is incorrect. To ensure both charge and mass number conservation, a proton must be produced as a by-product of the reaction:

$${}_1D^2 + {}_1D^2 \rightarrow {}_1T^3 + {}_1P^1 + Q$$

14. (1), (2), (3)

Use $\dfrac{N}{N_0} = e^{-\lambda t}$

15. (1), (4)

(1) True, Cd absorbs neutrons.

(2) No, concrete reflects, does not slow down.

(3) 'Moderate the activity' is not correct. 'Moderator' in the sense of slowing the neutrons is different.

(4) True, it is a fact.

16. (1), (2), (3)

We know, $\quad N = N_0\, e^{-\lambda t} \quad$ ...(i)

where N = number of decayed nuclei in the sample at time t,
N_0 = initial number of nuclei.

Hence, total number of undecayed nuclei = $(N_0 - N)$

Substituting it in (i), we get $N_0 - N = N_0\,(1 - e^{-\lambda t})$

This shows that the total number of undecayed nuclei decays exponentially with time and total number of decayed unclei grows exponentially with time. Now,

$$R = -\lambda N = \frac{dN}{dt} \qquad (R = \text{activity})$$

Hence, activity $(R) \propto$ number of undecayed nuclei.

Therefore, (1), (2), (3) are correct answers.

7. (1), (3)

$$R = R_0 A^{1/3}$$

For O^{16}, $R = R_0(16)^{1/3}$

For ${}_{54}X^{128}$, $R' = R_0(128)^{1/3}$

$$R' = \left(\frac{128}{16}\right)^{1/3} R = 2R$$

$$\therefore \quad V' = \frac{4}{3}\pi R'^3 = 8V$$

18. (1), (4)

In nuclear fusion, two or more lighter nuclei are combined to form a relatively heavy nucleus and thus, releasing the energy.

19. (3), (4)

Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

${}^{20}_{10}\text{Ne}$ is made up of 10 protons plus 10 neutrons.

Therefore, mass of ${}^{20}_{10}\text{Ne}$ nucleus, $M_1 < 10(m_p + m_n)$

Also, heavier the nucleus, more is the mass defect.

$$20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$$

Thus, $10(m_n + m_p) > M_2 - M_1$

or $\quad M_2 < M_1 + 10(m_p + m_n)$

Now, since $M_1 < 10(m_p + m_n)$

$\therefore \quad M_2 < 2M_1$

20. (2), (4)

In fusion, two or more lighter nuclei combine to make a comparatively heavier nucleus. In fission, a heavy nucleus beaks into two or more comparatively lighter nuclei. Further, energy will be released in a nuclear process if total binding energy increases. Hence, correct options are (2) and (4).

21. (1), (3)

When their number of nuclei are equal $4N_0e^{-3\lambda t} = 2N_0e^{-2\lambda t}$

$$\Rightarrow \quad t = \frac{1}{\lambda}\ln 2$$

When their decay rate are equal $3\lambda \times 4N_0e^{-3\lambda t} = 2\lambda \times 2N_0e^{-2\lambda t}$

$$\Rightarrow \quad t = \frac{\ln 3}{\lambda}$$

At $t = \frac{1}{\lambda}\ln 4$, decay rate of $A(3\lambda)4N_0e^{-3\lambda\frac{\ln 4}{\lambda}} = \frac{3\lambda N_0}{16}$

At $t = \frac{\ln 4}{\lambda}$, decay rate of $B = 2\lambda \times 4N_0e^{-2\lambda\frac{\ln 4}{\lambda}} = \frac{\lambda N_0}{4}$

22. (1), (3)

$$\frac{K_\alpha}{K_\gamma} = \frac{m_\gamma}{m_\alpha} = \frac{M-m}{m}$$

$$r = \frac{\sqrt{2mK_\alpha}}{qB} \Rightarrow K_\alpha = \frac{q^2B^2r^2}{2m}$$

$$K_\gamma = \frac{m}{(M-m)}K_\alpha = \frac{q^2B^2r^2}{2(M-m)}$$

$$\text{Energy released} = K_\gamma + K_\alpha = \frac{q^2B^2r^2}{2m}\left(\frac{M}{M-m}\right)$$

23. (2), (3)

Here, $N_A = N_0e^{-\lambda t}$, $N_B = N_0(1 - e^{-\lambda t})$

Now $\frac{N_B}{N_A} = e^{\lambda t} - 1$

If $t << T$; then $\frac{N_B}{N_A} = 1 + \lambda t - 1$ ($e^x = 1 + x$ for small x)

$$\frac{N_B}{N_A} = \lambda t$$

Linked Comprehension Type

For Problems 1–3

1. (1), 2. (2), 3. (3)

$$\frac{dN_X}{dt} = K - \lambda N_X$$

$$N_X = \frac{1}{\lambda}[K - K - \lambda N_0)e^{-\lambda t}]$$

$$\frac{dN_Y}{dt} = \lambda N_X$$

$$N_Y = Kt + \left(\frac{K - \lambda N_0}{\lambda}\right)e^{-\lambda t} - \frac{K - \lambda N_0}{\lambda}$$

For Problems 4–6

4. (1) $\frac{dN}{dt} = q_0t - \lambda N;\ \frac{dN}{dt} + \lambda N = q_0t$

5. (1) $N = \frac{q_0t}{\lambda} - \frac{q_0}{\lambda^2} + \frac{q_0}{\lambda^2}e^{-\lambda t}$

$$P_{\text{inst}} = \lambda NE_0 = \left[q_0t - \frac{q_0}{\lambda} + \frac{q_0}{\lambda}e^{-\lambda t}\right]E_0$$

6. (1) $P_{\text{av}} = \dfrac{\int_0^t \left[q_0t - \frac{q_0}{\lambda} + \frac{q_0}{\lambda}e^{-\lambda t}\right]E_0\,dt}{\int_0^t dt}$

$$= \left[\frac{q_0t}{2} - \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} - \frac{q_0}{\lambda^2 t}e^{-\lambda t}\right]E_0$$

For Problems 7–9

7. (3) From the graph and the fact that the n/p (= no. of neutrons/no. of protons) ratio for magnesium is 27/12, which is greater than 1 (= unit slope).

8. (2) ${}^{230}_{90}\text{Th} \rightarrow {}^{214}_{83}\text{Bi} + x\,{}^{0}_{-1}\text{e}^{-} + y\,{}^{4}_{2}\text{He}^{2+} + z$ (gamma ray)

Since the sum of the atomic numbers and mass numbers on either side of the equation must be equal (matter cannot be created or destroyed), we get

$$90 = 83 - x + 2y \quad \text{and} \quad 230 = 214 + 4y$$

Solving, we get $y = 4$ and $x = 1$

The order of the reactions is irrelevant (i.e., alpha, beta,...). Since gamma rays have no atomic number or mass number, the value of z does not affect this particular calculation.

9. (4) Alpha radiation consists of the largest particles (helium nuclei with a mass number of 4, thus the greatest inertia) and are the slowest (about 1/3 times the speed of the light). Beta radiation consists of smaller (electrons, 1/1370 times lighter than a proton) and faster particles (about 4/5 times the speed of light). Gamma radiation consists of the smallest particles (photons, no mass) which travel at the greatest speed (at the speed of light)

For Problems 10–12

10. (2) In equilibrium, rate of decay = rate of production

11. (4) As rate of decay = Rate of production

$$P = \lambda N \quad \Rightarrow \quad N = \frac{P}{\lambda} = \frac{Pt_{1/2}}{\ln 2} = 1.8 \times 10^{15}$$

12. (3) As $N = \frac{Pt_{1/2}}{\ln 2}$, it is dependent upon P and $t_{1/2}$. Initial number of ^{56}Mn nuclei will make no difference as in equilibrium, rate of production equals rate of decay. Large initial number will only make equilibrium come sooner.

For Problems 13–15

13. (4) Radioactivity is independent of all external conditions. When a nucleus undergoes an α-decay, its atomic number decreases by 2 and in beta decay, atomic number increases by 1.

14. (1) $T_M = \frac{1}{\lambda}$ and $T_H = \frac{0.693}{\lambda}$

Now, $\frac{T_M}{T_H} = \frac{1}{\lambda} \times \frac{\lambda}{0.693} = \frac{1}{0.693}$

i.e., $T_M > T_H$

From the above explanation, it is clear that choice (1) is correct and other choices (2), (3) and (4) are wrong.

15. (3) $A = \frac{dN}{dt} = \lambda N; \quad n = \lambda N$

$$n = \frac{0.693}{T} N; \quad T = \frac{0.693\, N}{n}$$

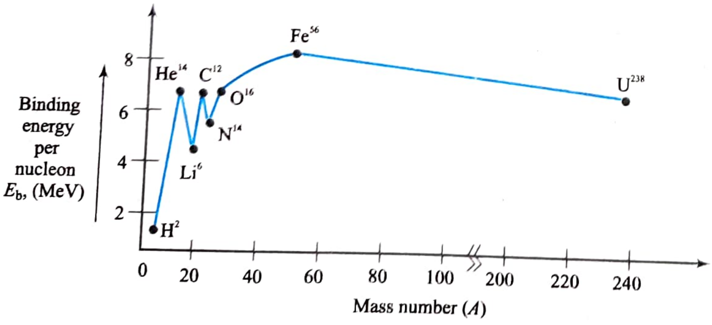
For Problems 16–18

16. (1), (2), (3), (4)

All options are basic properties of nuclear forces.

So, all options are correct.

17. (3) From graph, it is obvious that binding energy per nucleon $\left(\text{i.e., } \bar{B} = \frac{B}{A}\right)$ is maximum for middle order element and its value is around 8.8 MeV per nucleon.



18. (4) Radius (R) of nucleus is related with mass number (A) as

$$R \propto A^{1/3}$$

Now, Volume $\propto R^3 \propto A$

For Problems 19–22

19. (4) According to the passage, subatomic reactions do not conserve mass. So, we cannot find the third particle's mass by setting m_{neutron} equal to $m_{\text{proton}} + m_{\text{electron}} + m_{\text{third particle}}$.

By constrast, the total energy in this case, the sum of 'mass energy' and kinetic energy, is conserved. If E denotes total energy, then

$$E_{\text{neutron}} = E_{\text{proton}} + E_{\text{electron}} + E_{\text{third particle}}$$

The neutron has energy 949.97 MeV. The proton has energy 939.67 MeV + 0.01 MeV = 939.69 MeV. The electron has energy 0.51 MeV = 0.39 MeV = 0.90 MeV. Therefore, the third particle has energy

$$E_{\text{third particle}} = E_{\text{neutron}} - E_{\text{proton}} - E_{\text{electron}}$$

We just found the third particle's total energy, the sum of its mass energy and kinetic energy. Without more information, we cannot figure out how much of that energy is mass energy.

20. (1) As just shown, energy conservation allows us to calculate the third particle's total energy. But we don not know what percentage of that total is mass energy vs, kinetic energy.

21. (3) The term 'c^2' in $E = mc^2$ can mislead you into thinking that E varies with the square of mass. But E varies linearly with m. Indeed, we can write Einstein's equation as E = (constant) m, where the constant happens to equal c^2. By contrast, graphs (2) and (4) represent an equation such as E = (constant) m^2.

Since the particles in this ensemble all have positive kinetic energy, the total energy is positive even when the mass is zero [relativistic physics permits this, because kinetic energy no longer equals $(1/2)mv^2$]. So, the graph does not begin at the origin.

22. (2) A proton with only mass energy ($E_{\text{total}} = mc^2$) could never decay into a neutron and other particles, because a neutron has more mass energy than a proton does. Therefore, the reaction products would have more energy than the original proton, in violation of energy conservation.

But suppose the proton has additional energy, such as potential energy from its interactions with other particles in a nucleus. Then, the total energy of the proton—mc^2 plus potential energy—can equal the total energy of the products. In this case, the products (neutron and other particles) have more mass and more kinetic energy than the proton did. Therefore, during the reaction, the increase in mc^2 and the increase in kinetic energy must be offset by an equally big decrease in the system's potential energy. This happens when a nucleus undergoes β^+ decay.

For Problems 23–26

23. (4) Nuclear reactions conserve total charge and also conserve the total approximate mass (as measured by the atomic mass number). Therefore, since the uranium, xenon, and strontium nuclei have atomic masses 236, 140 and 94, the 'other particles' must have total atomic mass A such that

$$236 = 140 + 94 + A$$

So, $A = 2$. The other particles are two nucleons. This narrows down the answer to options (2), (3) and (4). For nuclei, the atomic number—i.e., the number of protons—tells us the charge. So, the other particles must have total charge Z such that

$$92 = 54 + 38 + Z \quad \text{or} \quad Z = 0$$

In summary, the other particles have total atomic mass 2 and total charge 0. Only option (4) fits this description.

(1) According to the passage, all nucleons attract each other with the same strong nuclear force. So, the strong nuclear force holds together three protons and one neutron ($^4_3$Li) just as vigorously as it holds together two protons and two neutrons ($^4_2$He). But other forces play a role, too. Specifically, protons electrostatically repel other protons. This repulsion 'tries' to make a nucleus fly apart. Since $^4_2$He contains only two protons, the attractive strong nuclear force overcomes the repulsion of the protons. Therefore, the nucleus holds together. But in $^4_3$Li, the mutual repulsion of the three protons overcomes the strong nuclear attraction, and the nucleus falls apart (or undergoes radioactive decay into a more stable nucleus).

(2) Once the neutron gets sufficiently close to the nucleus, the strong nuclear force sucks it in. The same reasoning would apply to the proton, except for one thing; the proton feels electrostatically repelled by the six protons already inside the carbon nucleus. This repulsion prevents a 100 m s^{-1} proton from getting close enough to the nucleus for the strong force to 'kick in'. Remember, the strong force becomes important only at tiny distances.

(3) Graph (1) would correctly show the binding energy per nucleons, but this differs from the total binding energy of the nucleus. The total binding energy takes into account not only the binding energy per nucleon, but also the number of nucleons.

To understand this subtle point, compare $^{94}_{38}$Sr to $^{236}_{92}$U. As the passage states, $^{94}_{38}$Sr has a higher binding energy per nucleon. Here,

$^{94}_{38}$Sr binding energy per nucleon ≈ 9 MeV

$^{236}_{92}$U binding energy per nucleon ≈ 7 MeV

But $^{236}_{92}$U still has a higher total binding energy, simply because it contains more nucleons. Indeed, since total binding energy of a nucleus = binding energy per nucleon × number of nuclei, the total binding energies are

$^{94}_{38}$Sr × (94 nucleons) ≈ 846 MeV

$^{236}_{92}$U × (236 nucleons) ≈ 1650 MeV

So, as atomic mass increases, so does the total binding energy. Uranium has more binding energy, even though it has less binding energy per nucleon.

Critically, however, the total binding energy does not increase linearly with atomic mass. It increases at a smaller and smaller rate, precisely because the binding energy per nucleon decreases. Indeed, since 'atomic mass number' specifies the number of nucleons, the binding energy per nucleon is simply the slope of a binding energy vs. atomic mass number graph. Between strontium and uranium, this slope decreases, as just explained. Thus, the answer must be (3) rather than (4).

or Problems 27–29

7. (3) If the particles are treated as point charges, $U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$

$q_1 = 2e$ (alpha particle), $q_2 = 82e$ (gold nucleus),

$r = 6.5 \times 10^{-14}$ m

$$\therefore \quad U = (8.987 \times 10^{8}\ \text{N m}^2\,\text{C}^2) \times \frac{(2\times 82)(1.602\times 10^{-19}\,\text{C})}{6.50\times 10^{-14}\,\text{m}}$$

$$= 5.82 \times 10^{-13}\ \text{J}$$

$$\text{or} \quad U = 5.82 \times 10^{-13}\ \text{J} \times \left(\frac{1\ \text{eV}}{1.602\times 10^{-19}\ \text{J}}\right)$$

$$= 3.63 \times 10^{6}\ \text{eV} = 3.63\ \text{MeV}$$

28. (3) Apply conservation of energy: $K_1 + U_1 = K_2 + U_2$

Let point 1 be the initial position of the alpha particle and point 2 be where the alpha particle momentarily comes to rest. Alpha particle is initially far from the lead nucleus implies $r_1 \approx \infty$ and $U_1 = 0$. Alpha particle stops implies $K_2 = 0$. Conservation of energy thus says

$$K_1 = U_2 = 5.82 \times 10^{-13}\ \text{J} = 3.63\ \text{MeV}$$

29. (2) $K = \dfrac{1}{2}mv^2$

$$\therefore \quad v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.82\times 10^{-13}\ \text{J})}{6.64\times 10^{-27}\,\text{kg}}} = 1.32 \times 10^{7}\ \text{m s}^{-1}$$

For Problems 30–32

30. (1), 31. (2), 32. (4)

For Problems 33–35

33. (4) $\dfrac{dN}{dt} = \lambda N(t)$

From the given data, $20000 = \lambda N_0$

$14800 = \lambda N\,(0.5\ \text{h}) = \lambda N$

$$\frac{N}{N_0} = \frac{148}{200}$$

$$N = N_0\, e^{-\lambda t}$$

$$\therefore \quad e^{-\lambda t} = \frac{148}{200}$$

$$\text{or} \quad \lambda = \frac{\left(\ln\dfrac{200}{148}\right)}{t} \approx 1.6 \times 10^{-4}\ \text{decays s}^{-1}$$

$$\text{Half-life, } T = \frac{0.693}{\lambda} = 4340\ \text{s} = 1.2\ \text{h}$$

34. (3) $\dfrac{dN}{dt} = \lambda N_0$

Number of radioactive nuclei that were present in the sample at $t = 0$,

$$N_0 = \frac{20000}{1.6\times 10^{-4}} = 1.25 \times 10^{8}$$

35. (3) $N(t) = N_0 e^{-\lambda t}$

$$\frac{dN}{dt} = \lambda N(t)$$

$$N(t) = \frac{300}{1.6\times 10^{-4}} = 1.875\times 10^{6}$$

Matrix Match Type

1. i → a.; ii. → d.; iii. → c.; iv. → b.

(i) Thermal energy of air molecules at room temperature:

$$kT = 1.38 \times 10^{-23} \times 300\ \text{J} = 0.025\ \text{eV}$$

(ii) Binding energy of heavy nuclei per nucleon ≈ 7 MeV

(iii) X-ray wavelength ≈ 1 Å

$$E = \frac{hc}{\lambda} \approx 12\ \text{keV}$$

(iv) For visible light: wavelength ≈ 6000 Å

$$E = \frac{hc}{\lambda} \approx 2\ \text{eV}$$

2. i. → a., b., c., d.; ii. → a., b., c., d.; iii. → a., b., c., d.; iv. → a., b., c., d.

In all the reactions in Column II:

Mass of products will be less than original mass of the system. The mass converts into energy, hence binding energy increases.

Basically, in all four reactions mentioned in Column II, energy is released and hence for all

$$m_{\text{products}} > m_{\text{original system}}$$

As energy is released in all 4 reactions, BE/nucleon increases in all.

Mass number and charge number are conserved in all processes.

3. i. → b., c., d.; ii. → b., c., d.; iii. → b., c., d.; iv. → a., c., d.

(i) In the given spontaneous radioactive decay, the number of protons remain constant and all conservation principles are obeyed.

(ii) In fusion reaction of two hydrogen nuclei, a proton is decreased as a position shall be emitted in the reaction. All the three conservation principles are obeyed.

(iii) In the given fission reaction, the number of protons remain constant and all conservation principles are obeyed.

(iv) In beta negative decay, a neutron transforms into a proton within the nucleus and the electron is ejected out.

4. i. → a.; ii. → b.; iii. → c.; iv → d.

For all types of waves, sound wave, light wave, string wave the term related is frequency, which is given only in one option. Other phenomenon are properly matching.

Photoelectric effect proves photon character of light.

γ-rays can only be produced from nucleus.

In case of k capture X-rays are emitted.

5. i. → a.; ii. → d.; iii. → e.; iv. → b.

Binding energy per nucleon for middle order element is maximum because middle order element is most stable. So, (i) → (b)

Nuclear force depends only on spin of nucleons. So, (ii) → (d)

For nuclear fission, $\frac{Z^2}{A}$ is greater than 15. So, (iii) → (e)

Magic numbers are explained by Shell model. So, (iv) → (b)

6. i. → c., d., e.; ii. → b.; iii. → a.; iv. → a.

Stability of nucleus is decided by

(i) Mass defect → greater → stability greater

(ii) Neutron–proton ratio, i.e., $\frac{N}{P} \simeq 1 = 1 \rightarrow$ More stable

(iii) Packing fraction = negative → more stable

(iv) Binding energy per nucleon greater → greater stability

For radioactive substance binding energy per nucleon is minimum. So, they are unstable.

For bound orbit, total energy is always negative.

Stopping potential is the particular negative potential when no electron reaches the plate (i.e., anode).

7. i. → a., b., e.; ii. → a., c.; iii. → d.; iv. → b., d.

In nuclear fusion, two lighter nuclei fuse and make big nuclei. In this, mass defect is converted into energy according to $E = mc^2$.

In nuclear fission, heavy nuclei split into two or more than two smaller nuclei. In this process, mass is converted into energy according to $E = mc^2$.

In β-decay, neutron proton ratio decreases, so nucleus becomes more stable.

Both nuclear fission and nuclear fusion are exothermic reactions.

8. i. → c.; ii. → d.; iii. → a.; iv. → b.

$$E_1 = [2m({}_1\text{H}^2) - m({}_1\text{H}^3) - m({}_1\text{H}^1)]\ 931.5\ \text{MeV} = 4\ \text{MeV}$$

$$E_2 = [-m({}_2\text{He}^4) - m({}_0\text{n}^1) + m({}_1\text{H}^3) + m_1({}_1\text{H}^2)] \times 931.5\ \text{meV} = 17.6\ \text{MeV}$$

$$E_3 = [-m({}_2\text{He}^3) - m({}_0\text{n}^1) + 2m({}_1\text{H}^2)] \times 931.5\ \text{meV} = 3.3\ \text{MeV}$$

$$E_4 = [m({}_2\text{He}^3) - m({}_1\text{H}^2) - m({}_2\text{He}^4) - m({}_1\text{H}^1)] \times 931.5\ \text{MeV} = 18.3\ \text{MeV}$$

9. (3) Before β decay, neutron is at rest. Hence $E_n = m_n c^2, p_n = 0$

After β decay, from conservation of momentum: $p_n = p_p + p_e$

Or $\quad p_p + p_e = 0 \Rightarrow |p_p| = |p_e| = p$

Also, $E_e = \left(m_e^2c^4 + p_e^2c^2\right)^{\frac{1}{2}} = \left(m_e^2c^4 + p_pc^2\right)^{\frac{1}{2}}$

From conservation of energy:

$$\left(m_p^2c^4 + p^2c^2\right)^{\frac{1}{2}} + \left(m_e^2c^4 + p^2c^2\right)^{\frac{1}{2}} = m_nc^2$$

$$m_pc^2 = 936\ \text{MeV},\ m_nc^2 = 938\ \text{MeV},\ m_ec^2 = 0.51\ \text{MeV}$$

Since the energy difference between n and p is small, pc will be small, $pc << m_pc^2$, while pc may be greater than m_ec^2.

$$\Rightarrow \quad m_pc^2 + \frac{p^2c^2}{2m_p^2c^4} \simeq m_nc^2 - pc$$

To first order $pc \simeq m_nc^2 - m_pc^2 = 938\text{MeV} - 936\text{MeV} = 2\text{MeV}$

This gives the momentum. Then,

$$E_p = \left(m_p^2c^4 + p^2c^2\right)^{\frac{1}{2}} = \sqrt{936^2 + 2^2} \simeq 936\text{MeV}$$

$$E_e = \left(m_e^2c^4 + p^2c^2\right)^{\frac{1}{2}} = \sqrt{(0.51)^2 + 2^2} \simeq 2.06\text{MeV}$$

Numerical Value Type

1. (6) We have to find the time at which $\lambda_A N_A = \lambda_B N_B$

$$\left(\frac{\ln 2}{T_A}\right)(4N_0e^{-\lambda_At}) = (N_0)\left(\frac{\ln 2}{T_B}\right)(e^{-\lambda_Bt})$$

$$e^{(\lambda_A - \lambda_B)t} = 8$$

$$(\lambda_A - \lambda_B)t = \ln 8 = 3(\ln 2)$$

$$\left(\frac{\ln 2}{1} - \frac{\ln 2}{2}\right)t = 3\ln(2) \quad \Rightarrow \quad t = 6\ \text{min}$$

2. (4) We have $\frac{t}{t_{1/2}} = \frac{40\ \text{hours}}{20\ \text{hours}} = 2$

Thus, $A = \frac{A_0}{2^{t/t_{1/2}}} = \frac{A_0}{2^2} = \frac{A_0}{4}$

So one fourth of the original activity will remain after 40 hours.

3. (2) In one half-life the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore, $8.0 \times 10^{18} \times \left(\frac{1}{2}\right) = 2 \times 10^{18}$.

4. (2) The activity of the sample at time t is given by $A = A_0e^{-\lambda t}$ where λ is the decay constant and A_0 is the activity at time $t = 0$ when the capacitor plates are connected. The charge on the capacitor at time t is given by $Q = Q_0e^{-t/CR}$

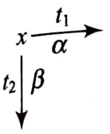
where Q_0 is the charge at $t = 0$ and $C = 100\mu\text{F}$

Thus, $\frac{Q}{A} = \frac{Q_0}{A_0}\frac{e^{-t/CR}}{e^{-\lambda t}}$

It is independent of t if $\lambda = \dfrac{1}{CR}$

Or $\quad R = \dfrac{1}{\lambda C} = \dfrac{t_{av}}{C} = \dfrac{20 \times 10^{-3}\,\text{s}}{100 \times 10^{-6}\,\text{F}} = 200\ \Omega$

(8)



$6h = 3(t_{eq})$

$$\Rightarrow \quad N = \frac{N_0}{(2)^3} \quad \Rightarrow \quad \frac{N_0}{N} = 8$$

(7) x and y are number of α-decays and β-decays respectively.

$92 - 2x + y = 85$

or $\quad 2x - y = 7$ ...(i)

Similarly, $238 - 4x = 210$ ...(ii)

$x = 7$, put in (i) we get $y = 7$

(6) Effective decay constant will be sum of all different decay constants.

So $\quad \lambda_{\text{eff}} = \lambda + 2\lambda + 3\lambda = 6\lambda$, hence $n = 6$

(12) From, $R = R_0\left(\dfrac{1}{2}\right)^n$ We have, $1 = 64\left(\dfrac{1}{2}\right)^n$

or $\quad n = 6 =$ number of half-lives

$t = n \times t_{1/2} = 6 \times 2 = 12\text{h}$

(4) Activity of $S_1 = \dfrac{1}{2}$ (activity of S_2)

or $\quad \lambda_1 N_1 = \dfrac{1}{2}(\lambda_2 N_2) \quad$ or $\quad \dfrac{\lambda_1}{\lambda_2} = \dfrac{N_2}{2N_1}$

$$\frac{T_1}{T_2} = \frac{2N_1}{N_2} \qquad \left(T = \text{half-life} = \frac{\ln 2}{\lambda}\right)$$

Given $N_1 = 2N_2$, $\dfrac{T_1}{T_2} = 4$

(8) $\dfrac{dN_x}{dt} = \dfrac{dN_y}{dt}$

$\lambda_x N_x = \lambda_y N_y$

$$\frac{\ln 2}{1}\left(8N_0 e^{-\lambda_x t}\right) = \frac{\ln 2}{2}\left(N_0 e^{-\lambda_y t}\right)$$

$$t = \frac{4\ln 2}{\lambda_x - \lambda_y} = \frac{4t_x t_y}{t_y - t_x} = 8$$

1. (12) Radioactive decay equation is

$$N = N_0 e^{-\lambda t} = N_0 e^{-\ln(2)t/T} \qquad \left(\because \lambda = \frac{\ln 2}{T}\right)$$

After decay of 99% of the initial sample, only 1% will be left, i.e., $N/N_0 = 1\%$.

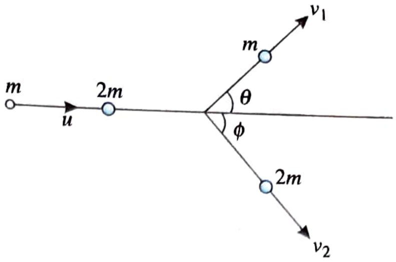
$$\therefore \quad \frac{N}{N_0} = \frac{1}{100} = e^{-\ln(2)t/T}$$

If we take the natural logarithm, we have $-\ln 100 = -\dfrac{\ln 2 \times t}{T}$

which on solving for t yields

$$t = \frac{\ln 100}{\ln 2} \times T = \frac{\log 100}{\log 2} \times T = \frac{2}{0.3010} \times 10 = 731\,\text{min} = 12.2\ \text{h}$$

12. (90)



$mu = mv_1 \cos\theta + 2mv_2 \cos\phi$ ...(i)

$mv_1 \sin\theta = 2mv_2 \sin\phi$ ...(ii)

$\dfrac{2}{3}\left(\dfrac{1}{2}mu^2\right) = \dfrac{1}{2}2mv_2^2$ ...(iii)

$\dfrac{1}{3}\left(\dfrac{1}{2}mu^2\right) = \dfrac{1}{2}mv_1^2$ ...(iv)

Solving equations, $v_1 = v_2 = u/\sqrt{3}$

$\theta + \phi = 120°$ and $\theta = 90°$

Archives

JEE Main

Single Correct Answer Type

1. (1) When two nucleons combine to form a third one, and energy is released, one has fusion reaction. If a single nucleus splits into two, one has fission. The possibility of fusion is more for light elements and fission takes place for heavy elements.

Out of the choices given for fusion, only A and B are light elements and D and E are heavy elements. Therefore $A + B \rightarrow C + \varepsilon$ is correct. The possibility of fission is only for F and not C. Therefore $F \rightarrow D + E + \varepsilon$ is the correct choice.

2. (3) After decay, the daughter nuclei will be more stable. Hence, the binding energy per nucleon will be more than that of their parent nucleus.

3. (2) Let the speed of each daughter nuclei be v_1 and v_2 respectively

Applying law of conservation of linear momentum,

$$(M + \Delta m) \times 0 = \frac{M}{2} \times v_1 - \frac{M}{2} \times v_2$$

which gives $v_1 = v_2$

Mass defect, $\Delta M = (M + \Delta m) - \left(\dfrac{m}{2} + \dfrac{m}{2}\right) = [M + \Delta m - M] = \Delta m$

Energy released, $Q = \Delta mc^2$ ...(i)

Also, $Q = \dfrac{1}{2}\left(\dfrac{M}{2}\right)v_1^2 + \dfrac{1}{2}\left(\dfrac{M}{2}\right)v_2^2 - \dfrac{1}{2}(M + \Delta m) \times (0)^2$

$\Rightarrow \quad Q = \dfrac{M}{2}v_1^2 \quad (\because v_1 = v_2)$ ...(ii)

Equating Eqs. (i) and (ii), we get

$$\left(\frac{M}{2}\right)v_1^2 = \Delta mc^2 \Rightarrow v_1^2 = \frac{2\Delta mc^2}{M}$$

which gives, $v_1 = c\sqrt{\dfrac{2\Delta m}{M}}$

4. (2) In positive beta decay, a proton is transformed into a neutron and a positron is emitted.

$$p^+ \rightarrow n^0 + e^+$$

The number of neutrons initially was $A - Z$.

The number of neutrons after decay is $(A - Z) - 3 \times 2$ (due to alpha particles) $+ 2 \times 1$ (due to positive beta decay) $= A - Z - 4$

The number of protons = $Z - 3 \times 2$ (due to alpha particles)-2 (due to positive beta particles) = $Z - 8$

Required ratio = $\frac{A-Z-4}{Z-8}$

5. (3) Number of undecayed atoms after time t_2,

$$\frac{N_0}{3} = N_0 e^{-\lambda t_2} \qquad ...(i)$$

Number of undecayed atoms after time t_1,

$$\frac{2}{3}N_0 = N_0 e^{-\lambda t_1} \qquad ...(ii)$$

Dividing (ii) by (i), we get $2 = e^{\lambda(t_2 - t_1)}$ or $\ln 2 = \lambda(t_2 - t_1)$

or $(t_2 - t_1) = \frac{\ln 2}{\lambda}$

As per question, $t_{1/2}$ = half-life time = 20 min

∴ $t_2 - t_1 = 20$ min $\left[\because t_{1/2} = \frac{\ln 2}{\lambda}\right]$

6. (None)

Mass defect, $\Delta m = m_p + m_e - m_n$

$= (1.6725 \times 10^{-27} + 9 \times 10^{-31} - 1.6725 \times 10^{-27})$ kg

$= 9 \times 10^{-31}$ kg

Energy released = $\Delta mc^2 = 9 \times 10^{-31} \times (3 \times 10^8)^2$ J

$$= \frac{9 \times 10^{-31} \times 9 \times 10^{16}}{1.6 \times 10^{-13}} \text{MeV} = 0.51 \text{ MeV}$$

∴ None of the given options is correct.

7. (1) $\frac{1}{\lambda} = Rz^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

∴ $\lambda = \frac{4}{3Rz^2}$

∴ $\lambda_1 = \frac{4}{3R}, \lambda_2 = \frac{4}{3R}, \lambda_3 = \frac{4}{12R}, \lambda_4 = \frac{4}{27R}$

⇒ $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

8. (4) Given $t = 80$ min $= 4\,T_A = 2T_B$

∴ Number of nuclei of A decayed $= N_0 - \frac{N_0}{2^4} = \frac{15N_0}{16}$

∴ Number of nuclei of B decayed $= N_0 - \frac{N_0}{2^2} = \frac{3N_0}{4}$

Hence required ratio (ratio of decayed Nuclei)

$$= \frac{15N_0}{16} \Big/ \frac{3N_0}{4} = \frac{5}{4}$$

9. (4) At time t, $\frac{N_B}{N_A} = 0.3 \Rightarrow N_B = 0.3\,N_A$

Also let initially there are total N_0 number of nuclei, so, $N_A = \frac{N_0}{1.3}$

Also as we know $N_A = N_0 e^{-\lambda t} \Rightarrow \frac{N_0}{1.3} = N_0 e^{-\lambda t}$

$$\frac{1}{1.3} = e^{-\lambda t} \Rightarrow \ln(1.3) = \lambda \text{ or } t = \frac{\ln(1.3)}{\lambda}$$

$$t = \frac{\ln(1.3)}{\frac{\ln(2)}{T}} = \frac{\ln(1.3)}{\ln(2)}T$$

JEE Advanced

Single Correct Answer Type

1. (3) $A = A_0 2^{-t/T_H}$

$$\frac{A_0}{64} = A_0 2^{-t/T_H} \Rightarrow 6 = \frac{t}{T_H} \Rightarrow t = 6T_H = 108 \text{ days}$$

2. (3) $E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$

$$n + {}^{15}_{8}\text{O} \longrightarrow {}^{15}_{7}\text{N} + {}^{1}_{1}\text{H}$$

$$Q = (M_n + M_{\text{O}^{15}_8} - M_{\text{N}^{15}_7} - M_{\text{H}^1_1})c^2$$

$= 0.003796 \times 931.5 = 3.5359$ MeV

$$\Delta E = \frac{3}{5} \times \frac{e^2}{4\pi\varepsilon_0} \times \frac{1}{R}(8 \times 7 - 7 \times 6)$$

$$= \frac{3}{5} \times (1.44 \text{ meV fm}) \times \frac{1}{R} \times 14 = 3.5359 \text{ meV}$$

$R = 3.42$ fm

Multiple Correct Answers Type

1. (1), (3)

$${}_{90}\text{Th}^{282} \longrightarrow {}_{82}\text{Pb}^{212} + x\,{}_2\text{He}^4 + y - 1e^0$$

${}^{232}_{90}$Th is converting into ${}^{212}_{82}$Pb

Change in mass number $(A) = 232 - 212 = 20$

∴ Number of α particles $x = \frac{20}{4} = 5$

Due to 5 α particles, Z will change by 10 units. Since given change is 8, place at this location. Hence $2x - y = 8$

or $2 \times 5 - y = 8$

or $y = 2$

Linked Comprehension Type

For Problems 1–3

1. (4) The very high temperature maintained in the plasma causes fusion.

2. (1) Given $k = 8.6 \times 10^{-3}$ eV/K, $\frac{e}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm

KE of two deuterons = 2×1.5 kT = 3 kT

This KE has to overcome the potential energy of the nucleus.

⇒ $3kT = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ or $T = \frac{1}{k} \cdot \frac{1}{3 \times r} \cdot \frac{e^2}{4\pi\varepsilon_0}$

∴ $T = \frac{1.44 \times 10^{-9}}{8.6 \times 10^{-5}} \times \frac{1}{3 \times 4 \times 10^{-15}}$

⇒ $T = 1.39 \times 10^9$ K

3. (2) The product of the deuteron density and confinement time t_0 should be greater than Lawson criteria, 5×10^{14} s cm^{-3}.

Deuteron density = 8.0×10^{14} cm^{-3}

Confinement time = 9.0×10^{-1} s

∴ Lawson criteria = $8.0 \times 10^{14} \times 9.0 \times 10^{-1} = 7.2 \times 10^{14}$

It is not enough if the density is very high but the confinement time also should be high enough for the reaction.

For Problems 4–5

4. (4) Total energy remains conserved. Energy is shared by antineutrino, proton and electron. Kinetic energy of electron has continuous spectrum and it is maximum when antineutrino does not share any kinetic energy. So total energy is shared with proton and electron only.

∴ $K \leq 0.8 \times 10^6$ eV

Kinetic energy of electron will be minimum or zero when total energy is shared by proton and antineutrino.

$\therefore \quad 0 \le K \le 0.8 \times 10^6$ eV

. (3) $K_p + K_{\bar{e}} + K_{\bar{v}} = 0.8 \times 10^6$ eV

When electron has zero kinetic energy is shared by antineutrino and proton.

Then, $K_p + K_{\bar{v}} = 0.8 \times 10^6$ eV

As antineutrino is very high mass in comparison to proton so it will have almost contribution in total energy.

$\therefore$ Its energy is almost 0.8×10^6 eV

or Problems 6–7

. (3) ${}^6_3\text{Li} \rightarrow {}^4_2\text{He} + {}^2_1\text{H}$

$$\frac{Q}{c^2} = 6.015123 - 4.002603 - 2.014102 = -0.001582 < 0$$

So no α-decay is possible.

$${}^{210}_{84}\text{Po} \rightarrow {}^1_1\text{H} + {}^{209}_{83}\text{Bi}$$

$$\frac{Q}{c^2} = 209.9828766 - 1.007825 - 208.980388 = -0.005337 < 0$$

So, this reaction is not possible.

$${}^2_1\text{He} + {}^4_2\text{He} \rightarrow {}^6_3\text{Li}$$

$$\frac{Q}{c^2} = 2.014102 + 4.002603 - 6.015123 = 0.001582 > 0$$

So, this reaction is possible.

$${}^{70}_{30}\text{Zn} + {}^{82}_{34}\text{Se} \rightarrow {}^{152}_{64}\text{Gd}$$

$$\frac{Q}{c^2} = 69.925325 + 81.916709 - 151.919803 = -0.077769 < 0$$

So this reaction is not possible.

7. (1) ${}^{210}_{84}\text{Po} \rightarrow {}^4_2\text{He} + {}^{206}_{82}\text{Pb}$

$Q = (209.982876 - 4.002603 - 205.97455)c^2 = 5.422$ MeV

From conservation of momentum, $\sqrt{2K_1(4)} = \sqrt{2K_2(206)}$

$$\therefore \quad K_1 = \frac{103}{2} K_2$$

$$K_1 + K_2 = 5.422 \quad \text{or} \quad K_1 + \frac{2}{103} K_1 = 5.422$$

$$\Rightarrow \quad \frac{105}{103} K_1 = 5.422$$

$$\therefore \quad K_1 = 5.319 \text{ MeV} = 5319 \text{ keV}$$

Matrix Match Type

1. **a. → p., q., t.; b. → q.; c. → s.; d. → s.**

(a) → (p): The energy of a capacitor is increased when connected to a battery.

(a) → (q): By compressing a gas adiabatically, the internal energy increases.

(a) → (t): By placing a loop of wire in a time varying magnetic field perpendicular to its plane $\varepsilon = -\left(\frac{d\phi}{dt}\right)$ an e.m.f. is generated. This increases the energy of the loop.

(b) → (q): Mechanical energy of pushing the piston, does work on a gas, increasing its energy by increasing the velocity of random motion.

(c) → (s): Internal energy of the system is converted into mechanical energy. Energy internal to system, is converted to energy of motion, temperature and photons in nuclear fission (natural radioactivity) of heavy fragments.

(d) → (s): The increase in the energy of the products of fission or radioactivity comes from a decrease in the initial mass.

2. (3)

${}^{15}_{8}\text{O} \rightarrow {}^{15}_{7}\text{N} + {}^{0}_{1}\beta$ (beta decay)

${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^4_2\text{He}$ (alpha decay)

${}^{185}_{83}\text{Bi} \rightarrow {}^{184}_{82}\text{Pb} + {}^1_1\text{H}$ (proton emission)

${}^{239}_{94}\text{Pu} \rightarrow {}^{140}_{57}\text{La} + {}^{99}_{37}\text{Rb}$ (fission)

Numerical Value Type

1. (8) $N = N_0 e^{-\lambda t}$

$\ln|dN/dt| = \ln(N_0\lambda) - \lambda t$

From graph $\lambda = \frac{1}{2}$ per year

$$\frac{t_1}{2} = \frac{0.693}{1/2} = 1.386 \text{ year}$$

4.16 year $= 3t_{1/2}$

$p = 8$

2. (1) $N = N_0 e^{-\lambda t}$

$$\frac{dN}{dt} = 10^{10} = N_0(\lambda)\, e^{-10^{-9}t}$$

At $t = 0$, $10^{10} = N_0 10^{-9}$

$N_0 = 10^{19}$

Mass of sample $= N_0 \times$ (mass of atom)

$= 10^{19} \times 10^{-25}$ kg $= 10^{-6}$ kg $= 1$ mg

3. (4) $f = (1 - e^{-\lambda t}) = 1 - e^{-\lambda t} \approx 1 - (1 - \lambda t) = \lambda t$

$f = 0.04$

Hence % decay $\approx 4\%$

4. (9) ${}^{12}_{5}\text{B} \rightarrow {}^{12}_{6}\text{C} + {}^{0}_{-1}e + \bar{v}$

Mass defect $= (12.014 - 12)\, u$

$\therefore$ Released energy $= 13.041$ Mev

$\therefore$ Energy used for excitation of ${}^{12}_{6}\text{C} = 4.041$ MeV

$\therefore$ Energy converted to KE of electron $= 13.041 - 4.041 = 9$ MeV

5. (5) Suppose total volume of blood is V ml

$\because$ 2.5 ml of blood activity $= 115$ Bq

$\therefore$ 1 ml of blood activity $= \frac{115}{2.5}$

$\therefore$ Activity of whole blood $= \left(\frac{115}{2.5}\right)V$ Bq

Activity $A = A_0 e^{-\lambda t}$

Here, $\lambda = \frac{\ln 2}{8}(\text{days})^{-1}$

$$\left(\frac{115V}{2.5}\right) = 2.4 \times 10^5 e^{-\lambda t} = 2.4 \times 10^5 e^{-\frac{\ln 2}{8\times 24}\times 11.5}$$

$$= 2.4 \times 10^5 e^{-\frac{0.7\times 11.5}{8\times 24}} = 2.4 \times 10^5 e^{-\frac{1}{24}} = 2.4 \times 10^5 \left(1 - \frac{1}{24}\right)$$

$$V = 2.4 \times 10^5 \times \frac{23}{24} \times \frac{2.5}{115} \approx 5000 \text{ ml}$$

$\therefore$ Total volume of blood $= 5$ liter

Chapter 8

Concept Application Exercises

Exercise 8.1

1. The width of forbidden energy gap is equal to energy of the radiation emitted on the combination of a hole and an electron. Therefore,

$$E_g = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{630\times10^{-9}} = 3.152\times10^{-19}\,\text{J} = 1.97\,\text{eV}$$

2. (a) In a semiconductor, the forbidden energy gap between valence and conduction bands is of the order of 1 eV. Since in the doped semiconductor, energy gap is much smaller (40 meV), the impurity levels in the doped semiconductor corresponds to donor energy level. Hence, the doped semiconductor is of *n*-type.

   (b) Here $kT = 40\,\text{meV} = 40\times10^{-3}\,\text{eV}$

$$\therefore \quad T = \frac{40\times10^{-3}}{k} = \frac{40\times10^{-3}}{8.62\times10^{-5}} = 464\,\text{K}$$

3. The minimum energy required to create a hole-electron pair,

   E = energy gap between valence and conduction band = energy of sodium light of wavelength 5890 Å

$$= \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{5{,}890\times10^{-10}} = 3.372\times10^{-19}\,\text{J}$$

$$= \frac{3.372\times10^{-19}}{1.6\times10^{-19}} = 2.11\,\text{eV}$$

4. Here, $e = 1.6\times10^{-19}\,\text{C}$;

$$\sigma = 5\Omega^{-1}\text{cm}^{-1} = 500\Omega^{-1}\text{m}^{-1};$$

$$\mu_e = 3{,}900\,\text{cm}^2\text{V}^{-1}\text{s}^{-1} = 0.39\,\text{m}^2\,\text{V}^{-1}\text{s}^{-1}$$

   Now, $\sigma = en_e\mu_e$ (neglecting contribution of holes)

$$\therefore \quad n_e = \frac{\sigma}{e\mu_e} = \frac{500}{1.6\times10^{-19}\times0.39} = 8.013\times10^{21}\,\text{m}^{-3}$$

Exercise 8.2

1. (c) In an *n*-type silicon, holes are minority carriers and pentavalent atoms are the dopants.
2. (d) A *p*-type semiconductor is obtained by doping a semiconductor with a trivalent impurity. The impurity so added produces holes. Therefore, in a *p*-type semiconductor holes are majority carries and trivalent atoms are the dopants. Hence, the option (d) is correct.
3. (c) The energy band gap is largest for carbon, less for silicon and least for germanium.

   So, the correct statement is (c), i.e., $(E_g)_C > (E_g)_{Si} > (E_g)_{Ge}$.
4. (c) In an unbiased p-n junction, holes diffuse from the p-region to n-region because hole concentration in p-region is more as compared to that in the n-region.
5. (c) When a forward bias is applied to a p-n junction, the bias opposes the potential barrier and consequently, potential barrier is lowered.
6. Considering the diode characteristic as a straight line passing through the origin, we can calculate the resistance using Ohm's law.

   (i) From the curve, at $I_D = 20\,\text{mA}, V_D = 0.8\,\text{V}$

$$\therefore \quad R_D = \frac{V_D}{I_D} = \frac{0.8\,\text{V}}{20\,\text{mA}} = \frac{0.8\,\text{V}}{20\times10^{-3}\,\text{A}} = 40\Omega$$

   (ii) From the curve, at $V_D = -10\,\text{V}, I_D = -1\,\mu\text{A}$

$$\therefore \quad R_D = \frac{V_D}{I_D} = \frac{10\text{V}}{1\mu\text{A}} = \frac{10\,\text{V}}{1\times10^{-6}\,\text{A}} = 10^7\,\Omega = 10\,\text{M}\Omega$$

7. The current through the diode should not exceed 100 mA.

$$\therefore \quad IR = V - 0.7$$

$$100\times10^{-3}R = 10 - 0.7 = 9.3$$

   or $R = \frac{9.3}{0.1} = 93\Omega$

8. As the diode is reversed biased, only drift current exists in circuit which is 20 μA.

   Potential drop across 15 Ω resistor

$$= 15\Omega\times20\mu\text{A} = 300\mu\text{V} = 0.0003\,\text{V}$$

   Potential difference across the diode $= 4.0 - 0.0003 = 3.9997 = 4.0\,\text{V}$

9. In the given circuit, the diode is reverse biased. No current flows through the upper resistance.

   ∴ Reading of ammeter $A_1 = 0$

   Reading of ammeter $= A_2 = \frac{4\text{V}}{20\Omega} = 0.2\,\text{A}$

Exercise 8.3

1. (b,c) When emitter is forward biased, electron-hole recombination takes place in the base region. So that only a small amount (about 5%) of electron-hole recombination may take place, the base is made thin and lightly doped. Hence, the option (b) is correct.

   In a transistor, emitter is always forward biased and collector is reverse biased. Hence, the option (c) is correct.
2. (c) In a transistor, the voltage gain is high at high and low frequencies and constant at mid frequencies. Hence, the option (c) is correct.
3. Here $\Delta V_{CE} = 0.2\,\text{V}$, $\Delta I_C = 0.004\,\text{mA} = 4\times10^{-6}\,\text{A}$

   Output resistance, $R_0 = \frac{\Delta V_{CE}}{\Delta I_C} = \frac{0.2}{4\times10^{-6}} = 50\times10^3\,\Omega = 50\,\text{k}\Omega$
4. Voltage gain = current gain × resistance gain

$$= \text{current gain} \times \frac{R_C}{R_i} = 30\times\frac{6}{1} = 180$$

5. Here, $\Delta I_B = 0.2\,\text{mA}, \Delta I_C = 5\,\text{mA}$

   (i) $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{5\,mA}{0.2\,mA} = 25$

   (ii) As $A_v = \beta\frac{R_L}{R_i}$

$$\therefore \quad R_L = \frac{A_v R_i}{\beta} = \frac{75\times2\text{k}\Omega}{25} = 6\,\text{k}\Omega$$

6. Current gain, $\alpha = \frac{\text{Power gain}}{\text{Voltage gain}} = \frac{800}{840} = \frac{20}{21}$

   Now $\beta = \frac{\alpha}{1-\alpha} = \frac{20/21}{1-20/21} = 20$

   But $\beta = \frac{I_C}{I_B}$

$$\therefore \quad I_C = \beta\cdot I_B = 20\times1.2 = 24\,\text{mA}$$

7. $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{1.0\times10^{-3}\,\text{A}}{50\times10^{-6}\,\text{A}} = 20$

$$\Delta I_E = \Delta I_B + \Delta I_C = 50\times10^{-6} + 1.0\times10^{-3} = 1050\times10^{-6}\,\text{A} = 1050\,\mu\text{A}$$

$$\alpha = \frac{\beta}{1+\beta} = \frac{20}{1+20} = \frac{20}{21}$$

Here, $V_{CE} = 1.5\,\text{V}$

$$R_C = 3k\Omega = 3\times10^3\,\Omega,\ \beta = 50$$

$$\therefore \quad I_C = \frac{V_{CE}}{R_C} = \frac{1.5}{3\times10^3} = 0.50\times10^{-3}\,\text{A}$$

$$I_B = \frac{I_C}{\beta} = \frac{0.50\times10^{-3}}{50} = 0.10\times10^{-3}\,\text{A}$$

$$I_E = I_C - I_B = (0.50-0.01)\times10^{-3}$$

$$= 0.49\times10^{-3}\,\text{A} = 0.49\,\text{mA}$$

Here, $\beta = 59$, $I_E = 6.0\,\text{mA}$

Now $\quad \alpha = \dfrac{\beta}{1+\beta}$

or $\quad \dfrac{I_C}{I_E} = \dfrac{59}{1+59}$

or $\quad I_C = \dfrac{59}{60} I_E = \dfrac{59}{60}\times 60 = 5.9\,\text{mA}$

$$I_B = I_E - I_C = 6.0 - 5.9 = 0.1\,\text{mA}$$

(i) Applying Kirchhoff's second law to the loop ABCDEFGHK, we get

$$V_{CC} = V_{BE} + I_B R_B$$

or $\quad V_{BE} = V_{CC} - I_B R_B = 60 - 5.0\times10^{-6}\times1.0\times10^{6} = 1.0\,\text{V}$

Applying Kirchhoff's second law to the loop EDFGHCBE, we get

$$V_{CC} = V_{CE} + I_C R_C$$

or $\quad V_{CE} = V_{CC} - I_C R_C = 60 - 5.0\times10^{-3}\times1.1\times10^{3} = 0.5\,\text{V}$

The collector is at +0.5 V w.r.t. the emitter and the base is +1.0 V w.r.t. emitter, so the base is at $(1.0-0.5) = +0.5$ V w.r.t. the collector. Thus, both the emitter base and the collector-base junctions are forward biased. Hence the circuit cannot be used as an amplifier.

(ii) When $R_C = 400\,\Omega$,

$$V_{CE} = V_{CC} - I_C R_C = 6.0 - 5.0\times10^{-3}\times400 = 4.0\,\text{V}$$

Now the collector is at +4.0 V w.r.t. the emiter and the base is at +1.0 V w.r.t. the emitter. Thus, the base-collector is reversed biased by $(1.0-4.0) = -3.0$ V. Hence the circuit can now work as an amplifier.

ercise 8.4

1. (a) Here, $y = A\cdot\overline{B} + A\cdot B$

For $A = 0, B = 0$: $y = 0.\overline{0} + 0.0 = 0.1 + 0.0 = 0 + 0 = 0$

For $A = 1, B = 0$: $y = 1.\overline{0} + 1.0 = 1.1 + 1.0 = 1 + 0 = 1$

For $A = 0, B = 1$: $y = 0.\overline{1} + 0.1 = 0.0 + 0.1 = 0 + 0 = 0$

For $A = 1, B = 1$: $y = 1.\overline{1} + 1.1 = 1.0 + 1.1 = 0 + 1 = 1$

(b) Here, $y = A.B + \overline{A.B}$

For $A = 0, B = 0$: $y = 0.0 + \overline{0.0} = 0.0 + \overline{0} = 0 + 1 = 1$

For $A = 1, B = 0$: $= 1.0 + \overline{1.0} = 1.0 + \overline{0} = 0 + 1 = 1$

For $A = 0, B = 1$: $y = 0.1 + \overline{0.1} = 0.1 + \overline{0} = 0 + 1 = 1$

For $A = 1, B = 1$: $y = 1.1 + \overline{1.1} = 1.1 + \overline{1} = 1 + 0 = 1$

2. Clearly, the output is low when both the inputs are high otherwise it is high. Hence it is a NAND gate.

3. The logic gate marked as X is NAND gate, while the gate marked as Y is NOT gate. Let y' be the output of NAND gate X. Then, output at y will be as given in table.

A	B	y′	y
1	1	0	1
0	0	1	0

4. The logic gate marked X is NAND gate, while the gate marked as Y is OR gate. Corresponding to inputs A and B, let y' be the output of NAND gate X. Then, the inputs of OR gate Y are A and y'. The output at y will be as given in table.

A	B	y′	y
1	1	0	1
0	1	1	1

5. The truth table of combination of gates is produced as below:

A	B	y'	y_1	y_2	y
0	0	1	1	1	0
1	0	1	0	1	1
0	1	1	1	0	1
1	1	0	1	1	0

≡

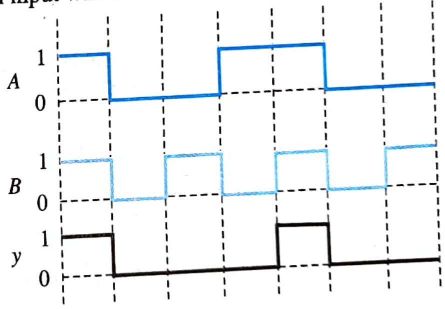
A	B	y
0	0	0
1	0	1
0	1	1
1	1	0

Since it is the truth table of XOR gate, the given combination of gates represents XOR gate.

6. The circuit symbol shown in figure represents the AND gate. Its truth table is as given by table.

A	1	0	0	1	1	0	0
B	1	0	1	0	1	0	1
y	1	0	0	0	1	0	0

Following the truth table of AND gate and the output waveform for the given input will be as shown in figure

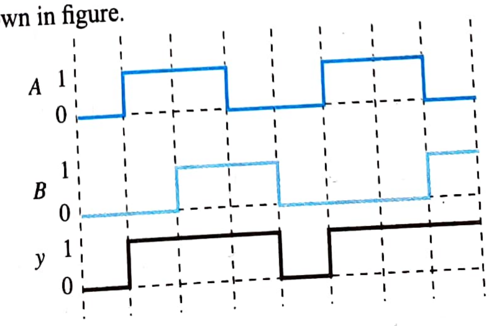


7. The truth table of OR gate is as given by table.

The input signal has been analysed in figure. At the different instants (marked by vertical dotted lines), the input waveforms A and B are high (1) and low (0) as given below:

A	0	1	1	0	0	1	0
B	0	0	1	1	0	0	1
y	0	1	1	1	0	1	1

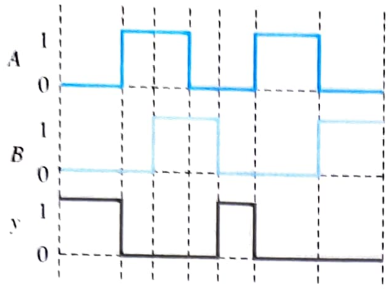
Following the truth table of OR gate, the output waveform will be as shown in figure.



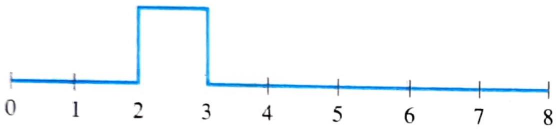
8. The circuit symbol shown in figure represents the NOR gate and its truth table is as given by table.

A	0	1	1	0	0	1	0
B	0	0	1	1	0	0	1
y	1	0	0	0	1	0	0

The input signal has been analysed as given in table. At the different instants (marked by vertical dotted lines), the input waveforms A and B are high (1) and low (0) as given below:



9. (i) The output of an AND gate is 1 when both the inputs are 1. Hence the output waveform from the AND gate for the inputs A and B is of the type shown below:



(ii) A NAND gate is obtained when the output of an AND gate is fed to a NOT gate.

Exercises

Single Correct Answer Type

1 (4) Resistance of conductors (Cu) decreases with decrease in temperature while that of semiconductors (Ge) increases with decrease in temperature.

2. (3) $n_e n_h \approx n^2$

or $\quad n_e = \frac{n^2}{n_h} = \frac{10^{16}\times 10^{16}}{4.5\times 10^{22}} = \frac{10^{32}}{4.5\times 10^{22}}\,\text{m}^{-3}$

3. (1) In n-type semiconductor, free electrons are the majority charge carriers.

4. (1) Phosphorus is pentavalent.

5. (2) In insulators, the forbidden energy gap is very large. In case of semiconductor, it is moderate and in conductors, the energy gap is zero.

6. (3) In semiconductors, forbidden energy gap is of the order of 1 eV.

7. (2) Gallium is a trivalent impurity.

8. (4) When a free electron is produced, simultaneously a hole is also produced.

9. (3) The resistance of semiconductor decreases with increase in temperature.

10. (4) At absolute zero temperature, semiconductor behaves as insulator.

11. (3) Antimony is a fifth-group impurity and is therefore a donor of electrons.

12. (4) A conductor has positive temperature coefficient of resistance, but a semiconductor has negative temperature coefficient of resistance.

13. (3) In intrinsic semiconductors, the creation or liberation of on free electron by the thermal energy has created one hole. Thus in intrinsic semiconductors, $n_e = n_h$.

14. (2) In n-type semiconductor, impurity energy level lies just belo the conduction band.

15. (1) Because electrons needed less energy to move.

16. (4) In sample X, no impurity level is seen, so it is undoped. In sample Y, impurity energy level lies below the conductio band, so it is doped with fifth group impurity. In sample Z impurity energy level lies above the valence band so it is doped with third group impurity.

17. (1) Forbidden energy gap for carbon is greater than that o silicon.

18. (4)

$$\text{Extrinsic semiconductor} \begin{cases} \rightarrow p\text{-Type}(n_p >> n_e) \\ \rightarrow n\text{-Type}(n_e >> n_p) \end{cases}$$

19. (3) Number density of atoms in silicon specimen $= 5\times 10^{28}$ atom/m
$= 5 \times 10^{22}$ atom/cm^3

Since one atom of indium is doped in 5×10^7 Si atom, so number of indium atoms doped per cm^3 of silicon.

$$n = \frac{5\times 10^{22}}{5\times 10^{7}} = 1\times 10^{15}\ \text{atom/cm}^3$$

20. (3) By using mass action law, $n_i^2 = n_e n_h$

$$\Rightarrow \quad n_h = \frac{n_i^2}{n_e} = \frac{(10^{16})^2}{10^{21}} = 10^{11}\ \text{per m}^3$$

21. (4) Diode cannot be used for the purpose of amplification.

22. (2) If pn junction diode is made from germanium or silicon, the energy released due to recombination of free electrons and holes is in the infrared region. However if the pn junction diode is made from gallium arsenide or indium phosphide, then the energy released due to recombination of free electrons and holes is in the visible region.

23. (1) The current is due to the flow of minority charge carriers.

24. (2) Because in case (i), n is connected with n. This is not a series combination of transistor.

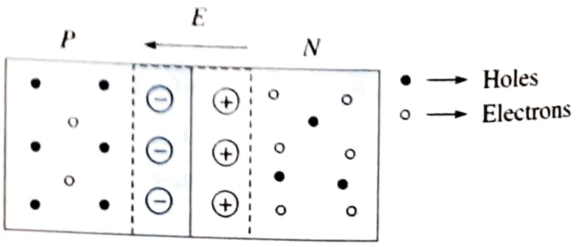
25. (2) In reverse biasing, width of depletion layer increases.

26. (2) Depletion layer is more in less doped side.

27. (2) When reverse bias is increased, the electric field at the junction also increases. At some stage the electric field breaks the covalent bond, thus the large number of charge carriers are generated. This is called Zener breakdown.

28. (4) By using $E = \frac{V}{d} = \frac{0.6}{10^{-6}} = 6\times 10^5\ \text{V/m}$

29. (3) At junction a potential barrier/depletion layer is formed, with n-side at higher potential and p-side at lower potential. Therefore, there is an electric field at the junction directed from the n-side to p-side.



(2) In forward biasing, the diffusion current increases and drift current remains constant, so no current is due to the diffusion. In reverse biasing, diffusion becomes more difficult, so net current (very small) is due to the drift.

(2) Due to the large concentration of electrons in n-side and holes in p-side, they diffuse from their own side to other side. Hence, depletion region produces.

(3) When polarity of the battery is reversed, the p-n junction becomes reverse biased. So no current flows.

(2) Potential across the pn junction varies symmetrically linear, having p side negative and n side positive.

(3) p-n junction has low resistance in one direction of potential difference $+V$, so a large current flows (forward biasing). It has a high resistance in the opposite potential difference direction $-V$, so a very small current flows (reverse biasing).

(3) $E = \dfrac{hc}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{hc}{E} = \frac{6.6\times10^{-34}\times3\times10^{8}}{57\times10^{-3}\times1.6\times10^{-19}} = 217100\ \text{Å}$$

(2) In forward biasing, p-side is connected with positive terminal and n-side with negative terminal of the battery.

(1) All others have cathode at higher potential than anode.

(2) Current flow is possible and $i = \dfrac{V}{R} = \dfrac{(4-1)}{300} = 10^{-2}\ \text{A}$

(1) The potential of p-side is more negative than that of n-side. Hence, diode is in reverse biasing. In reverse biasing, it acts as open circuit, hence no current flows.

(3) In forward biasing, both positive and negative charge carriers move towards the junction.

(3) Only in option (3), p-side is more negative as compared to n-side.

(2) Because p-side is more negative as compared to n-side.

(2) In Figs. (ii), (iv) and (v). p-crystals are more positive as compared to n-crystals.

(1) In forward biasing, resistance of pn junction diode is zero. So whole voltage appears across the resistance.

(3) Because n-side is more positive as compared to p-side.

(4) Arsenic has five valence electrons, so it is a donor impurity. Hence X becomes n-type semiconductor. Indium has only three outer electrons, so it is an acceptor impurity. Hence Y becomes p-type semiconductor. Also n (i.e., X) is connected to the positive terminal of battery and p (i.e., Y) is connected to the negative terminal of battery. So p-n junction is reverse biased.

(1) The diode is in reverse biasing. So current through it is zero.

(3) $V' = V + IR = 0.5 + 0.1 \times 20 = 2.5\ \text{V}$

(1) Since diode in upper branch is forward biased and in lower branch is reversed biased. So current through circuit $i = \dfrac{V}{R + r_d}$;

here r_d = diode resistance in forward biasing = 0

$$\Rightarrow \quad i = \frac{V}{R} = \frac{2}{10} = 0.2\ \text{A}$$

(1) The voltage drop across resistance = 8 – 0.5 = 7.5 V

$$\therefore \text{ Current, } i = \frac{7.5}{2.2\times10^{3}} = 3.4\,\text{mA}$$

(2) The diode in lower branch is forward biased and diode in upper branch is reverse biased.

$$\therefore \quad i = \frac{5}{20+30} = \frac{5}{50}\ \text{A}$$

52. (2) The current through circuit, $i = \dfrac{P}{V} = \dfrac{100\times10^{-3}}{0.5} = 0.2\ \text{A}$

$\therefore$ Voltage drop across resistance = 1.5 – 0.5 = 1 V

$$\Rightarrow \quad R = \frac{1}{0.2} = 5\,\Omega$$

53. (2) $V_{\text{rms}} = \dfrac{V_0}{2} = \dfrac{200}{2} = 100\ \text{V}$

54. (1) At knee point, voltage across the diode is 0.7 V.

Hence voltage across resistance R is 5 – 0.7 = 4.3 V.

Using $V = iR \Rightarrow 4.3 = 1 \times 10^{-3} \times R \Rightarrow R = 4.3\ \text{k}\Omega$

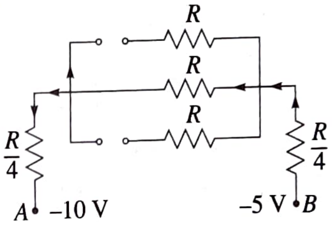
55. (4) In positive half cycle, one diode is in forward biasing and other is in reverse biasing, while in negative half cycle, their polarity reverses, and direction of current is opposite through R for positive and negative half cycles. So output is not rectified.

Since R_1 and R_2 are different, hence the peaks during positive half and negative half of the input signal will be different.

56. (2) In half-wave rectifier, $V_{\text{dc}} = \dfrac{V_0}{\pi} = \dfrac{10}{\pi}$

57. (3) (i) $V_A = -10\ \text{V}$ and $V_B = -5\ \text{V}$

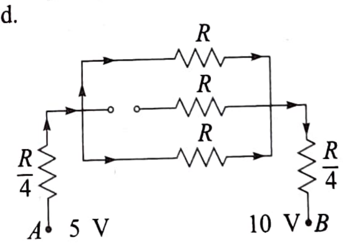
Diodes D_1 and D_3 are reverse biased and D_2 is forward biased.



$$\Rightarrow \quad R_{AB} = R + \frac{R}{4} + \frac{R}{4} = \frac{3}{2}R$$

(ii) When $V_A = -5\ \text{V}$ and $V_B = -10\ \text{V}$

Diodes D_2 is reverse biased and D_1 and D_3 are forward biased.



$$\Rightarrow R_{AB} = \frac{R}{4} + \frac{R}{2} + \frac{R}{4} = R$$

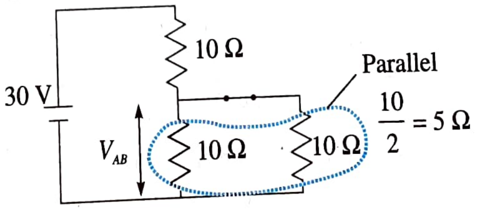
(iii) In this case, equivalent resistance between A and B is also R.

Hence correct order is (ii) = (iii) < (i).

58. (2) According to the given polarity, diode D_1 is forward biased, while D_2 is reverse biased. Hence, current will pass through D_1 only.

So current $i = \dfrac{6}{(150+50+100)} = 0.02\ \text{A}$

59. (1) Diode is in forward biasing. Hence, the circuit can be redrawn as follows:

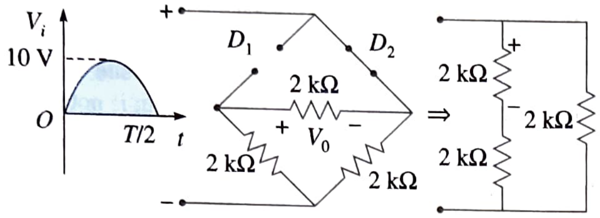


$$V_{AB} = \frac{30}{(10+5)} \times 5 = 10\,\text{V}$$

60. (4) The diode D will conduct for positive half cycle of a.c. supply because this is forward biased. For negative half cycle of a.c. supply, this is reverse biased and does not conduct. So output would be half wave rectified and for half wave rectified output

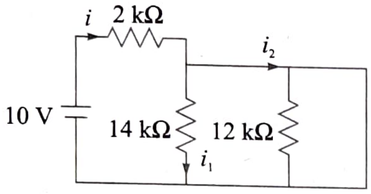
$$V_{\text{rms}} = \frac{V_0}{2} = \frac{200\sqrt{2}}{2} = \frac{200}{\sqrt{2}}$$

61. (2) For the positive half cycle of input, the resulting network is shown below:



$$\Rightarrow \quad (V_0)_{\max} = \frac{1}{2}(V_i)_{\max} = \frac{1}{2} \times 10 = 5\,\text{V}$$

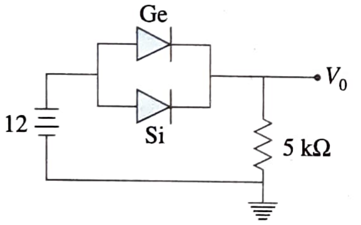
62. (4) The equivalent circuit can be redrawn as follows:



From figure, it is clear that current drawn from the battery

$$i = i_2 = \frac{10}{2} = 5\,\text{mA and } i_1 = 0$$

63. (2) Consider the case when Ge and Si diodes are connected as shown in the given figure.



Equivalent voltage drop across the combination Ge and Si diode = 0.3 V

$$\Rightarrow \text{Current, } i = \frac{12 - 0.3}{5\,k\Omega} = 2.34\,\text{mA}$$

∴ Output voltage, $V_0 = Ri = 5\text{ k}\Omega \times 2.34\text{ mA} = 11.7\text{ V}$

Now consider the case when diode connection are reversed. In this case voltage drop across the diode's combination = 0.7 V

$$\Rightarrow \text{Current, } i = \frac{12 - 0.7}{5\text{k}\Omega} = 2.26\,\text{mA}$$

$$\therefore \quad V_0 = iR = 2.26\text{ mA} \times 5\text{ k}\Omega = 11.3\text{ V}$$

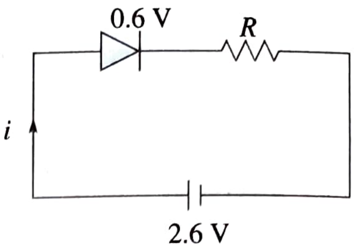
Hence charge in the value of $V_0 = 11.7 - 11.3 = 0.4$ V

64. (3) In space charge limited region, the plate current is given by Child's law, $i_p = KV_p^{3/2}$

$$\text{Thus, } \frac{i_{p_2}}{i_{p_1}} = \left(\frac{V_{p_2}}{V_{p_1}}\right)^{3/2} = \left(\frac{600}{150}\right)^{3/2} = (4)^{3/2} = 8$$

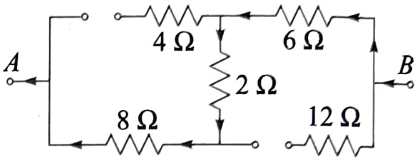
$$\text{or} \quad i_{p_2} = i_{p_1} \times 8 = 10 \times 8\text{ mA} = 80\text{ mA}$$

65. (3)



$$R = \frac{(2.6 - 0.6)}{5 \times 10^{-3}} = 400\,\Omega$$

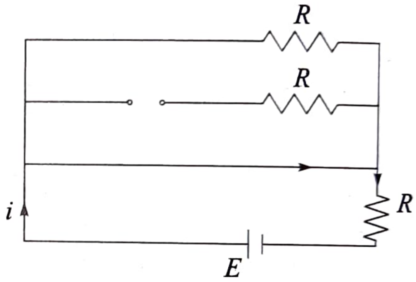
66. (3) According to the given figure, A is at lower potential w.r.t. Hence, both diodes are in reverse biasing. So equivalent circ can be redrawn as follows:



⇒ Equivalent resistance between A and B

$$R = 8 + 2 + 6 = 16\,\Omega$$

67. (1) Diodes D_1 and D_3 are forward biased and D_2 is reverse biase So the circuit can be redrawn as follows:



$$\Rightarrow \quad i = \frac{E}{R}$$

68. (4) The p-n junction will conduct only when it is forward biased i.e., when −5 V is fed to it. So it will conduct only for 3rd quarte part of signal shown and when it conducts potential drop 5 vol will be across both the resistors, so output voltage across R_2 i 2.5 V.

$$\therefore \quad V_0 = -2.5\text{ V}$$

69. (2) $V_b = i_b R_b \Rightarrow R_b = \dfrac{9}{35 \times 10^{-6}} = 257\text{ k}\Omega$

70. (4) In common emitter configuration, current gain

$$A_i = \frac{-h_{fe}}{1 + h_{oe}R_L} = \frac{-50}{1 + 25 \times 10^{-6} \times 10^3} = -48.78$$

71. (3) Voltage gain $= \dfrac{\text{Output voltage}}{\text{Input voltage}}$

$$\Rightarrow \quad V_{\text{out}} = V_{\text{in}} \times \text{Voltage gain}$$

$$\Rightarrow \quad V_{\text{out}} = V_{\text{in}} \times \text{Current gain} \times \text{Resistance gain}$$

$$= V_{\text{in}} \times \beta \times \frac{R_L}{R_{BE}} = 10^{-3} \times 100 \times \frac{10}{1} = 1\text{V}$$

72. (1) $n_e = 8 \times 10^{18}/\text{m}^3$, $n_h = 5 \times 10^{18}/\text{m}^3$

$$\mu_e = 2.3 \frac{\text{m}^2}{\text{volt-sec}}, \quad \mu_h = 0.01 \frac{\text{m}^2}{\text{volt-sec}}$$

∵ $n_e > n_h$, so semiconductor is n-type.

Also conductivity, $\sigma = \dfrac{1}{\text{Resistivity}(\rho)} = e(n_e\mu_e + n_h\mu_h)$

$\Rightarrow \quad \dfrac{1}{\rho}=1.6\times10^{-19}[8\times10^{18}\times2.3+5\times10^{18}\times0.01]$

$\Rightarrow \quad \rho=0.34\ \Omega\text{-m}$

(1) In common base mode, $\alpha=0.98$, $R=5\ \text{k}\Omega$, $R_{\text{in}}=70\ \Omega$

$\therefore$ Voltage gain, $A_v=\alpha\times\dfrac{R}{R_{\text{in}}}=0.98\times\dfrac{5\times10^3}{70}=70$

Power gain = Current gain × Voltage gain = 0.98 × 70 = 68.6

(3) $I_b=\dfrac{5-0.7}{8.6}=0.5\text{ mA}\Rightarrow I_c=\beta I_b=100\times0.5\text{ mA}$

By using $V_{CE}=V_{CC}-I_CR_L=18-50\times10^{-3}\times100=13\text{ V}$

(1) $I_e=10^{10}\times1.6\times10^{-19}\times\dfrac{1}{10^{-6}}=1.6\text{mA}$ $\quad\left(\because I=\dfrac{Q}{t}\right)$

Since 2% electrons are absorbed by base, hence 98% electrons reaches the collector, i.e., $\alpha=0.98$

$\Rightarrow \quad I_c=\alpha I_e=0.98\times1.6=1.568\text{ mA}\approx1.57\text{ mA}$

(2) $V=V_{CE}+I_CR_L \Rightarrow 15=7+I_C\times2\times10^3 \Rightarrow i_C=4\text{ mA}$

$\because \quad \beta=\dfrac{i_C}{i_B}\Rightarrow i_B=\dfrac{4}{100}=0.04\text{mA}$

7. (1) Here the emitter base junction of *npn* transistor is forward biased with battery V_{BB} through resistance R. When the value of R is reduced, then the emitter current i_e will increase. As a result, the collector current will also increase. ($i_c=i_e-i_b$). Due to increase in i_c, the potential difference across L increases and hence the reading of voltmeter also increases.

8. (4) "Single input" not acceptable.

9. (4) (1) $\overline{\overline{A}.\overline{B}}=\overline{\overline{A}}+\overline{\overline{B}}=A+B$

(2) $\overline{\overline{A}+\overline{B}}=\overline{\overline{A}}.\overline{\overline{B}}=A.B$

(3) $\overline{\overline{(A+B)}\cdot\overline{(A.B)}}=A.B+A.B=AB+AB$

(4) $0+0=0$

Clearly, (4) is incorrect.

80. (1) When $X_1=X_3=0$ and $X_2=1$, inputs to AND gate are 1 and 1 so, $y=0$ due to inversion of NOT gate.

81. (2) $\overline{(\overline{A}\cdot\overline{B})}\cdot A=(\overline{\overline{A}}+\overline{\overline{B}})\cdot A=(A+B)\cdot A$

$=A\cdot A+AB=A+AB=A(1+B)=A$

82. (1) (1 = high, 0 = low)

Input to A is in the sequence 1, 0, 1, 0.

Input to B is in the sequence 1, 0, 0, 1.

Sequence is inverted by NOT gate.

Thus, input to OR gate becomes 0, 1, 0, 1 and output of OR gate becomes 0, 1, 1, 1.

Since for OR gate 0 + 1 = 1, hence choice (1) is correct.

83. (3) The following arguments shall help us to arrive at the right choice.

(i) AND gate shall not respond to zero input.

(ii) Since R is zero and S is 1, therefore NOR gate gives zero. NOT gives '1' correspond to 0

(iii) NOR gate would not respond to 0 and 1.

84. (3) $A+B=1;\ C=1$

So, AND gate responds.

85. (4) The output of both AND gates will be zero. So, OR gate does not respond.

86. (2) Output of OR gate is $A+B$. Output of NAND gate is $\overline{A.B}$

Now, $Y=(A+B)\overline{A.B}=(A+B).(\overline{A}+\overline{B})$

If $A=1$ and $B=1$, then $A+B=1$ and $(\overline{A}+\overline{B})=0$

So, $Y=0$

If $A=0$ and $B=0$, then $Y=0$

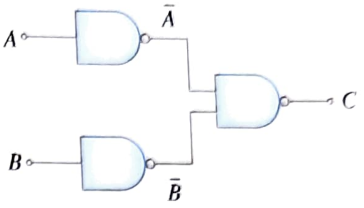
If $A=1$ and $B=0$, then $A+B$ gives 1

$(\overline{A}+\overline{B})=$ gives 1, So, $Y=1$

If $A=1$ and $B=1$, then $Y=1$

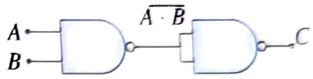
So, the given combination is XOR gate.

87. (1)



$C=\overline{\overline{A}.\overline{B}}=\overline{\overline{A}}+\overline{\overline{B}}=A+B$ (De Morgan's theorem)

Hence, output C is equivalent to OR gate.



$C=\overline{\overline{AB}.\overline{AB}}=\overline{\overline{AB}}+\overline{\overline{AB}}=AB+AB=AB$

In this case, output C is equivalent to AND gate.

88. (4) This is a truth table for OR logic gate because for OR gate. $X=A+B$.

89. (4) The output D for the given combination

$D=\overline{(A+B).C}=\overline{(A+B)}+\overline{C}$

If $A=B=C=0$, then $D=\overline{(0+0)}+\overline{0}=\overline{0}+\overline{0}=1+1=1$

If $A=B=1$, $C=0$, then $D=\overline{(1+1)}+\overline{0}=\overline{1}+\overline{0}=0+1=1$

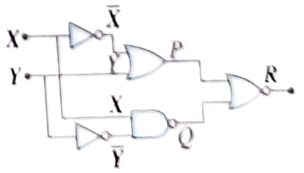
90. (3) The Boolean expression for the given combination is output $Y=(A+B).C$

The truth table is

A	B	C	$Y=(A+B)\cdot C$
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
0	1	1	1
1	0	1	1
1	1	1	1

Hence $A=1$, $B=0$, $C=1$.

91. (3)

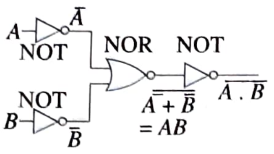


The truth table can be written as

X	Y	$\overline{X}$	$\overline{Y}$	$P=\overline{X}+Y$	$Q=\overline{X\cdot\overline{Y}}$	$R=\overline{P+Q}$
0	1	1	0	1	1	0
1	1	0	0	1	1	0
1	0	0	1	0	0	1
0	0	1	1	1	1	0

Hence $X=1$, $Y=0$ gives output $R=1$.

92. (4)



Hence option (4) is correct.

93. (2) The truth table of the circuit is given

A	B	C	$X=\overline{AB}$	$Y=\overline{BC}$	$Z=\overline{X+Y}$
0	0	0	1	1	0
1	0	0	1	1	0
0	0	1	1	1	0
1	0	1	1	1	0
0	1	0	1	1	0
1	1	0	0	1	0
0	1	1	1	0	0
1	1	1	0	0	1

Output Z of single three input gate is that of AND gate.

94. (1) With battery polarity, D_2 is off. So, $I=\dfrac{12}{4+2}=2\text{ A}$

95. (2) Here Y is the output of G.

96. (2) $I=\dfrac{2\text{ V}}{10\,\Omega}=0.2\text{ A}$

97. (2) Since the operational amplifier is assumed to be ideal, it (Q) does not sink nor source current.

By Kirchhoff's current law, $I_f = I$.

98. (3) The network can be represented by a single EX-OR gate

A	B	Output
0	0	0
1	0	0
0	1	1
1	1	0

99. (3) The truth table of the resulting logic circuit by connecting X to Y is as follows:

P	Q	R	R
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

Hence, form the truth table, the combination is equivalent to a single NAND gate.

(OR X is an AND gate and Y is a NOT gate, thus the combination is NOT-AND gate, i.e., a NAND gate)

Archives

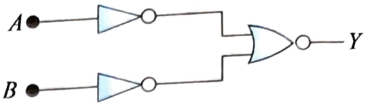
JEE Main

Single Correct Answer Type

1. (3) The given figure is a half-wave rectifier.

2. (1) Truth table

A	B	Y
1	1	1
1	0	0
0	1	0
0	0	0



3. (1) $\psi=\overline{\left(\overline{A.\overline{A.B}}\right).\left(\overline{B.\overline{A.B}}\right)}=\left(\overline{\overline{A.\overline{A.B}}}\right)\left(\overline{\overline{B.\overline{A.B}}}\right)$

$$=A.\left(\bar{A}+\bar{B}\right)+B.\left(\bar{A}+\bar{B}\right)=A.\bar{A}+A.\bar{B}+B.\bar{A}+B.\bar{B}$$

$$\psi=0+A.\bar{B}+B.\bar{A}+0$$

Alternative method

The truth table of the given circuit is as shown in the table

A	B	y	y_1	y_2	y
0	0	1	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	1	1	0

4. (3) $f_C=\dfrac{1}{2\pi RC}=\dfrac{2}{2\times3.14\times100\times10^3\times250\times10^{-12}}=6.37\text{ kHz}$

f_C = cut off frequency

As we know that $f_m=f_C$

$\therefore$ Option (3) is correct.

Note: The maximum frequency of modulation must be less than f_m, where

$$f_m=f_C\frac{\sqrt{1-\mu^2}}{\mu}\qquad[\because \mu\Rightarrow \text{modulation index}]$$

5. (4) For LED, in forward bias, intensity increases with voltage.

6. (3) $5=e^{1000\frac{V}{T}}-1$

$$\Rightarrow\quad e^{1000\frac{V}{T}}=6 \qquad \ldots(\text{i})$$

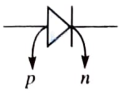
Again, $I=e^{1000\frac{V}{T}}-1$

$$\frac{dI}{dV}=e^{\frac{1000}{T}V}\,\frac{1000}{T}$$

$$dI=\frac{1000}{T}e^{\frac{1000}{T}V}\,dV$$

Using (i) $\Delta I=\dfrac{1000}{T}\times6\times0.01=\dfrac{60}{T}=\dfrac{60}{300}=0.2\text{mA}$

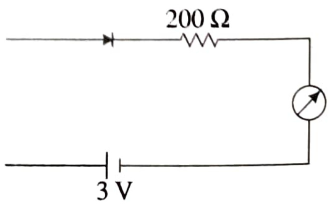
7. (3)



For forward bias, p-side must be at higher potential than n-side.

8. (2) In common emitter amplifier circuit input and output voltage are out of phase. When input voltage is increased then i_b is increased, i_c also increases so voltage drop across R_c is increased. However, increase in voltage across R_c is in opposite sense.

9. (4) Silicon diode is in forward bias. Hence across diode potential barrier



ΔV = 0.7 volts

$$I=\frac{V-\Delta V}{R}=\frac{3-0.7}{200}=\frac{2.3}{200}=11.5\text{ mA}$$

Chapter 9

Concept Application Exercises

xercise 9.1

1. (b) $n_{eff} = n_0\sqrt{1-\left(\frac{80.5N}{v^2}\right)} = 1\sqrt{1-\frac{80.5\times(400\times10^6)}{(55\times10^6)^2}} \approx 1$

Also $n_{eff} = \frac{\sin i}{\sin r}$

$\Rightarrow \sin r = \sin i \Rightarrow r = i = 45°$

2. (b) For demodulation, $\frac{1}{f_c} << RC$

$$\frac{1}{f_c} = \frac{1}{100\times10^3} = 10^{-5}\,\text{s}$$

$$RC = 10^3 \times 10 \times 10^{-12}\,\text{s} = 10^{-8}\,\text{s}$$

We see that $1/f_c$ here is not less than RC as required by the above condition. Hence, this is not good.

3. (b) Limiting value of $h\nu$ is E_g, such that $h\nu = \frac{hc}{\lambda} = E_g$

or $$\lambda = \frac{hc}{E_g} = \frac{6.63\times10^{-34}\,\text{J-s}\times3\times10^8\,\text{ms}^{-1}}{0.73\times1.6\times10^{-19}\,\text{J}} = 1703\ \text{nm}$$

4. (b) $P_t = P_c\left[1+\frac{m^2}{2}\right] = 9\left[1+\frac{(0.4)^2}{2}\right] = 9\left[1+\frac{0.16}{2}\right]$

$(\because m = 40\% = 0.4)$

$= 9\,(1.08) = 9.72\ \text{kW}$

5. (d) We know that $\left(\frac{I_t}{I_c}\right)^2 = 1+\frac{m^2}{2}$

Here, $I_t = 8.96$ A and $I_c = 8$ A

$\therefore \quad \left(\frac{8.96}{8}\right)^2 = 1+\frac{m^2}{2}$

or $\quad 1.254 = 1+\frac{m^2}{2}$

or $\quad \frac{m^2}{2} = 0.254$

or $\quad m^2 = 0.508$ or $m = 0.71 = 71\%$

6. (d) $\frac{P_t}{P_c} = 1+\frac{m^2}{2}$ or $P_c = P_t\left[\frac{2}{2+m^2}\right]$

$\therefore \quad P_c = 1500\left[\frac{2}{2+1}\right] = 1000\ \text{W}$

$\because \quad m = 100\% = 1$

7. (c) $P_c = P_t\left[\frac{2}{2+m^2}\right] = 900\left[\frac{2}{2+1}\right] = 600\ \text{W}$

Now, $P_{\text{LSB}} = \frac{m^2}{4}\times P_c = \frac{1}{4}\times600 = 150\ \text{W}$

8. (b) CS = 2 × Δf or Δf = CS/2

$\therefore \quad \Delta f = \frac{200}{2} = 100\,\text{kHz}$

Now $\quad m_f = \frac{\Delta f}{f_m} = \frac{100}{10} = 10$

9. (b) $m_f = \frac{\delta}{f_m} = \frac{2250}{500} = 4.5$

$\therefore$ New deviation $= 2(m_f f_m) = 2 \times 4.5 \times 6 = 54$ kHz

10. (a) If n is the number of bits per sample, then number of quantisation level $= 2^n$

Since the number of quantisation level is 16

$\Rightarrow \quad 2^n = 16 \Rightarrow n = 4$

$\therefore$ Bit rate = Sampling rate × No. of bits per sample

$= 8000 \times 8 = 32000$ bits/sec

11. (c) $P_{sb} = P_c\left(\frac{m_a}{2}\right)^2 = P_c\frac{(0.5)^2}{4} = 0.0625P_c$

Also $\quad P = P_c\left(1+\frac{m_a^2}{2}\right) = P_c\left(1+\frac{(0.5)^2}{2}\right) = 1.125P_c$

$\therefore \quad \%\ \text{saving} = \frac{(1.125P_c - 0.0625P_c)}{1.125P_c}\times100 = 94.4\%$

12. (b) As $f = \frac{c}{\lambda} \Rightarrow f_m = \frac{c}{\lambda_m}$ and $f_s = \frac{c}{\lambda_s}$

$\because \quad \lambda_m > \lambda_s \Rightarrow f_m < f_s$

Also $\quad f_m = \frac{1}{2\pi\sqrt{L_mC}}$ and $f_s = \frac{1}{2\pi\sqrt{L_sC}}$

13. (b) Maximum distance covered by space wave communication

$\sqrt{2Rh} = 62$ km

Critical frequency $= f_c = 9(N_{\text{max}})1/2 \simeq 9$ MHz

5 MHz $< f_c$, sky wave propagation (ionospheric propagation)

14. (a) Using $\frac{1}{f_{\text{carrier}}} << RC$

We get time constant, $RC = 1000 \times 10^{-12} = 10^{-9}$ s

Now $f = \frac{1}{T} = \frac{1}{10^{-9}} = 10^9\,\text{Hz}$

Thus, the value of carrier frequency should be much less than 10^9 Hz, say 100 kHz.

$\Rightarrow \quad \frac{f_m}{f_s} = \sqrt{\frac{L_s}{L_m}} \Rightarrow L_s < L_m$

Exercises

Single Correct Answer Type

1. (1) By using $f_c \approx 9(N_{\text{max}})^{1/2} \Rightarrow f_c \approx 2$ MHz

2. (1) A maximum frequency deviation of 75 kHz is permitted for commercial FM broadcast stations in the 88 to 108 MHz VHF band.

3. (2) Velocity factor $= \frac{1}{\sqrt{k}} = \frac{1}{\sqrt{2.6}} = 0.62$

4. (4) Here $\frac{n_1 - n_2}{n_1} = \frac{0.88}{100} \Rightarrow \frac{n_2}{n_1} = 0.9912$

$\therefore$ Critical angle, $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}(0.9912) = 84°24'$

5. (2) The energy flux, $\phi = \frac{\text{Pulse power}}{\text{Area}} = \frac{10^{12}}{10^{-4}} = 10^{16}\ \frac{\text{W}}{\text{cm}^2}$

6. (3) $v = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{10\times10^{-6}\times1\times10^{-9}}} = 1592\ \text{kHz}$

7. (1) VHF (very high frequency) band having frequency range 30 MHz to 300 MHz is typically used for TV and radar transmission.

8. (3) $\text{MUF} = \frac{f_c}{\cos\theta} = \frac{60}{\cos 70^\circ} = 175\,\text{MHz}$

9. (4) Laser beams are perfectly parallel. So that they are very narrow and can travel a long distance without spreading. This is the feature of laser while they are monochromatic and coherent these are characteristics only.

10. (2) The formula for modulating index is given by

$$m_f = \frac{\delta}{v_m} = \frac{\text{Frequency variation}}{\text{Modulating frequency}} = \frac{10\times10^3}{2\times10^3} = 5$$

11. (4) Here, $V_{\max} = \frac{24}{2} = 12\,\text{mV}$ and $V_{\min} = \frac{8}{2} = 2\,\text{mV}$

Now, $m = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} = \frac{12-4}{12+4} = \frac{8}{16} = \frac{1}{2} = 0.5 = 50\%$

12. (1) Here, $f_c = 1.5$ MHz = 1500 kHz, $f_m = 10$ kHz

∴ Low side band frequency

$= f_c - f_m = 1500\ \text{kHz} - 10\ \text{kHz} = 1490\ \text{kHz}$

Upper side band frequency

$= f_c + f_m = 1500\ \text{kHz} + 10\ \text{kHz} = 1510\ \text{kHz}$

13. (1) Frequency modulation requires much wider channel (7 to 15 times) as compared to AM.

14. (2) $P_t = P_c\left(1 + \frac{m_a^2}{2}\right)$; Here $m_a = 1$

$\Rightarrow\quad 1800 = P_c\left(1 + \frac{(1)^2}{2}\right) \Rightarrow P_c = 1200\ \text{W}$

15. (3) An antenna is a metallic structure used to radiate or receive EM waves.

16. (1) Carrier swing $= \frac{\text{Frequency deviation}}{\text{Modulating frequency}} = \frac{50}{7} = 7.143$

17. (3) In optical fibre, light travels inside it, due to total internal reflection.

18. (4) Few advantages of optical fibres are that the number of signals carried by optical fibres is much more than that carried by the Cu wire or radio waves. Optical fibres are practically free from electromagnetic interference and problem of cross talks whereas ordinary cables and microwave links suffer a lot from it.

19. (4) The process of changing the frequency of a carrier wave (modulated wave) in accordance with the audio frequency signal (modulating wave) is known as frequency modulation (FM).

20. (4) Following are the problems which are faced while transmitting audio signals directly.

(i) These signals are relatively of short range.

(ii) If everybody started transmitting these low frequency signals directly, mutual interference will render all of them ineffective.

(iii) Size of antenna required for their efficient radiation would be larger i.e. about 75 km.

21. (4) Remote sensing is the technique to collect information about an object in respect of its size, colour, nature, location, temperature etc. without physically touching it. There are some areas or location which are inaccessible. So to explore these areas or locations, a technique known as remote sensing is used. Remote sensing is done through a satellite.

22. (1) The critical frequency of a sky wave for reflection from a layer of atmosphere is given by $f_c = 9(N_{\max})^{1/2}$

$\Rightarrow\quad 10 \times 10^6 = 9(N_{\max})^{1/2}$

$\Rightarrow\quad N_{\max} = \left(\frac{10\times10^6}{9}\right)^2 \simeq 1.2\times10^{12}\,\text{m}^{-3}$

23. (2) Core of acceptance angle, $\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$

24. (3) $m_a = \frac{E_m}{E_c} = \frac{15}{60}\times100 = 25\%$

25. (1) Using $Z = \sqrt{\frac{L}{C}}$, we get $Z = \sqrt{\frac{0.40\times10^{-6}}{10^{-11}}} = 2\times10^2\,\Omega$

26. (2) Optical source frequency, $f = \frac{c}{\lambda} = \frac{3\times10^8}{1.3\times10^{-6}} = 2.3\times10^{14}\,\text{Hz}$

∴ Number of channels or subscribers $= \frac{2.3\times10^{14}}{20\times10^3} = 1.15\times10^{10}$

27. (1) Carrier swing $= \frac{\text{Frequency deviation}}{\text{Modulating frequency}} = \frac{50}{7} = 7.143$

28. (3) $f_c \propto (N)^{1/2}$

$\Rightarrow\quad (f_c)_E : (f_c)_{F1} : (f_c)_{F2}$

$= (2\times10^{11})^{1/2} : (5\times10^{11})^{1/2} : (8\times10^{11})^{1/2}$

$= 2 : 3 : 4$

29. (1) Attenuation $= 10\log\frac{120}{4} = 10\log 30 = 10\times1.4771 = 14.77\ \text{dB}$

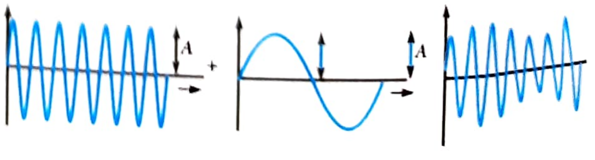
30. (1) $I_{\text{Carrier}} = \frac{I_{\text{rms}}}{\sqrt{1+\frac{m_a^2}{2}}} = \frac{11}{\sqrt{1+\frac{(0.5)^2}{2}}} = 10.35\,\text{A}$

31. (2) $P_c = \frac{P}{\left(1+\frac{m_a^2}{2}\right)} = \frac{10000}{\left(1+\frac{(0.5)^2}{2}\right)} = \frac{10000}{1.125} = 8.89\,\text{kW}$

32. (1) 1% of 10 GHz $= 10\times10^9\times\frac{1}{100} = 10^8\,\text{Hz}$

Number of channels $= \frac{10^8}{5\times10^3} = 2\times10^4$

33. (1) When signal amplitude is equal to the carrier amplitude, the amplitude of carrier wave varies between 2A and zero.



$m_a = \frac{\text{Amplitude charge of carrier}}{\text{Amplitude of normal carrier}} = \frac{2A - A}{A}\times100 = 100\%$

rchives

Main

gle Correct Answer Type

(4) Theory based.

(1) $r = R + h \cong R$

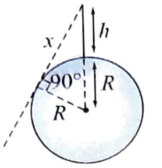
$$x = \sqrt{(R+h)^2 - R^2} = \sqrt{h^2 + 2hR}$$

$$x^2 = 250000 + 2 \times 500 \times 6.4 \times 100000$$

$$= 250000 + (64 \times 100000000)$$

$$= 10^4\,(640025)$$

$$\Rightarrow \quad x = 80 \text{ km}$$



3. (3) Modulated wave has frequency range.

$$\omega_c \pm \omega_m$$

$$\therefore \quad \omega_c >> \omega_m$$

$\therefore \quad \omega_m$ is excluded.

4. (4) As the carrier frequency is distributed as band width frequency, so available bandwidth $= 10\%$ of 10 GHz

$$= \frac{10}{100} \times 10 \times 10^9 = 10^9 \text{ Hz}$$

Bandwidth for each telephonic channel = 5 kHz

Number of channels $n = \dfrac{10^9}{5 \times 10^3}$

$n = 2 \times 10^5$ telephonic channels

APPENDIX 1

Chapterwise Solved January 2019 JEE Main Questions (All Sets)

Chapter 1
Geometrical Optics: Part 1

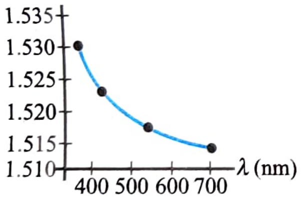
1. Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finally reflected from the second mirror (M_2) parallel to the first mirror (M_1). The angle between the two mirrors will be

(1) 90° (2) 45°
(3) 75° (4) 60°

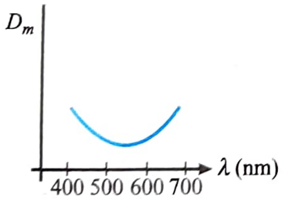
2. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is

(1) 30° (2) 45°
(3) 90° (4) 60°

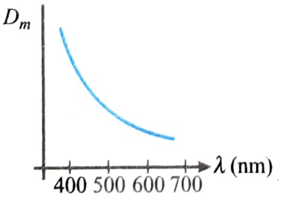
3. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation?



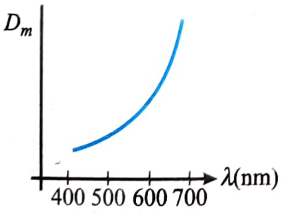
(1)



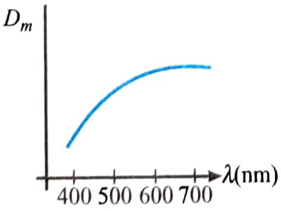
(2)



(3)

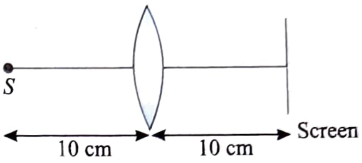


(4)



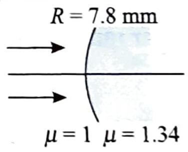
Chapter 2
Geometrical Optics: Part 2

4. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then d is



(1) 0.55 cm away from the lens
(2) 1.1 cm away from the lens
(3) 0.55 cm towards the lens
(4) 0

5. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

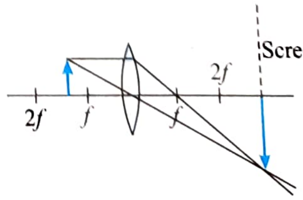


(1) 2 cm (2) 1 cm
(3) 3.1 cm (4) 4.0 cm

6. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be

(1) 0.92×10^{-3} m/s away from the lens
(2) 2.26×10^{-3} m/s away from the lens
(3) 1.16×10^{-3} m/s towards the lens
(4) 3.22×10^{-3} m/s towards the lens

7. Formation of real image using a biconvex lens is shown below:



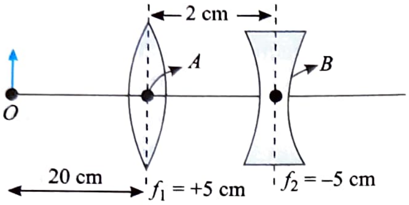
If the whole setup is immersed in water without disturbing the object and the screen position, what will one observe on the screen?

(1) Image disappears
(2) No change
(3) Erect real image
(4) Magnified image

8. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be

(1) $f_1 - f_2$
(2) $f_1 + f_2$
(3) $\dfrac{R}{\mu_2 - \mu_1}$
(4) $\dfrac{2f_1f_2}{f_1 + f_2}$

9. What is the position and nature of image formed by lens combination shown in figure? (f_1, f_2 are focal lengths)



(1) 70 cm from point B at left; virtual
(2) 40 cm from point B at right; real
(3) $\dfrac{20}{3}$ cm from point B at right, real
(4) 70 cm from point B at right, real

Chapter 3
Wave Optics

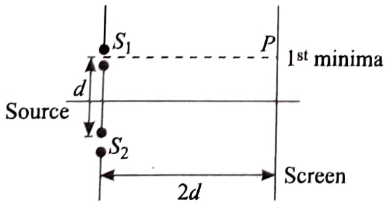
10. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-36° \le \theta \le 30°$ is

(1) 320
(2) 641
(3) 321
(4) 640

11. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:

(1) 4 : 1
(2) 25 : 9
(3) 16 : 9
(4) 5 : 3

12. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength X such that the first minima occurs directly in front of the slit (S_1)?



(1) $\dfrac{\lambda}{2(5-\sqrt{2})}$
(2) $\dfrac{\lambda}{(5-\sqrt{2})}$
(3) $\dfrac{\lambda}{(\sqrt{5}-2)}$
(4) $\dfrac{\lambda}{2(\sqrt{5}-2)}$

13. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\dfrac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same setup. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are

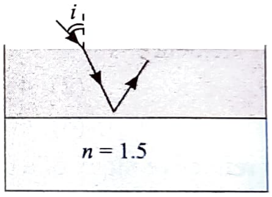
(1) 380 nm, 500 nm
(2) 625 nm, 500 nm
(3) 380 nm, 525 nm
(4) 400 nm, 500 nm

14. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $\dfrac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to

(1) 0.94
(2) 0.74
(3) 0.85
(4) 0.80

Chapter 4
Diffraction and Polarization

15. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is



(1) $\dfrac{3}{\sqrt{5}}$
(2) $\dfrac{5}{\sqrt{3}}$
(3) $\sqrt{\dfrac{5}{3}}$
(4) $\dfrac{4}{3}$

16. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is

(1) 09
(2) 10
(3) 04
(4) 05

Chapter 5
Dual Nature of Radiation and Matter

17. The magnetic field associated with a light wave is given, at the origin, by $B = B_0\,[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$. If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photoelectrons? ($c = 3 \times 10^8\ \text{ms}^{-1}$, $h = 6.6 \times 10^{-34}$ J-s)

(1) 7.72 eV
(2) 8.52 eV
(3) 12.5 eV
(4) 6.82 eV

18. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, by light of wavelength $\lambda_2 = 540$ nm. It is found that the maximum speed of the photoelectrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to

(Energy of photon $= \dfrac{1240}{\lambda(\text{in mm})}$ eV)

(1) 1.8 (2) 1.4
(3) 2.5 (4) 5.6

19. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2.The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photoelectrons. The number of emitted photoelectrons per second and their maximum energy, respectively, will be [1 eV = 1.6×10^{-19}J]

(1) 10^{10} and 5 eV (2) 10^{14} and 10 eV
(3) 10^{12} and 5 eV (4) 10^{11} and 5 eV

20. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to $\left(\dfrac{hc}{e} = 1240\,\text{nm-V}\right)$

(1) 0.5 V (2) 1.0 V
(3) 2.0 V (4) 1.5 V

21. If the de Broglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to

(Speed of light $= 3\times10^8$ m/s,

Planck's constant $= 6.63\times10^{-34}$ J·s,

Mass of electron $= 9.1\times10^{-31}$ kg)

(1) 1.45×10^6 m/s (2) 1.7×10^6 m/s
(3) 1.8×10^6 m/s (4) 1.1×10^6 m/s

22. When a certain photosensitive surface is illuminated with monochromatic light of frequency v, the stopping potential for the photo current is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $v/2$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is

(1) $\dfrac{3v}{2}$ (2) $2v$
(3) $\dfrac{4}{3}v$ (4) $\dfrac{5v}{3}$

23. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to

(1) 2020 nm (2) 2200 nm
(3) 2500 nm (4) 1700 nm

24. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of 50 V. Another particle B of mass '$4\,m$' and charge 'q' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\dfrac{\lambda_A}{\lambda_B}$ is close to

(1) 10.00 (2) 14.14
(3) 4.47 (4) 0.07

Chapter 6
Atomic Physics

25. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state, in terms of Bohr radius a_0, will be (hc = 12500 eV-Å)

(1) $9a_0$ (2) $25a_0$
(3) $4a_0$ (4) $16a_0$

26. In a hydrogen-like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be

(1) $\dfrac{27}{20}\lambda$ (2) $\dfrac{16}{25}\lambda$
(3) $\dfrac{20}{27}\lambda$ (4) $\dfrac{25}{16}\lambda$

27. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \dfrac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as

(1) $r_n \propto n^2, E_n \propto \dfrac{1}{n^2}$ (2) $r_n \propto \sqrt{n}, E_n \propto \dfrac{1}{n}$
(3) $r_n \propto n, E_n \propto n$ (4) $r_n \propto \sqrt{n}, E_n \propto n$

Chapter 7
Nuclear Physics

28. At a given instant, say t = 0, two radioactive substances A and B have equal activities. The ratio $\dfrac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. [If the half-life of A is m_2, the half-life of B is

(1) $\dfrac{\ln 2}{2}$ (2) $2\ln 2$
(3) $\dfrac{\ln 2}{4}$ (4) $4\ln 2$

29. A sample of radioactive material A, that has an activity of 10 mCi (1 Ci = 3.7×10^{10} decays/s), has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would then be respectively

(1) 20 days and 5 days (2) 20 days and 10 days
(3) 5 days and 10 days (4) 10 days and 40 days

30. Consider the nuclear fission: $\text{Ne}^{20} \rightarrow 2\text{He}^4 + \text{C}^{12}$

Given that the binding energy/nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement:

(1) 8.3 MeV energy will be released
(2) energy of 12.4 MeV will be supplied
(3) energy of 11.9 MeV has to be supplied
(4) energy of 3.6 MeV will be released

31. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At $t = 0$ it was 1600 counts per second and $t = 8$ seconds it was 100 counts per second. The count rate observed, as counts per second, at $t = 6$ seconds is close to

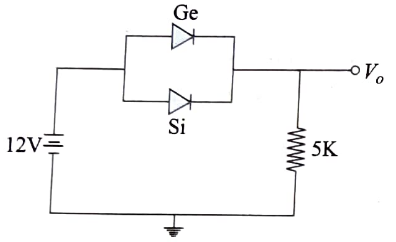
(1) 150 (2) 360
(3) 200 (4) 400

32. In a radioactive decay chain, the initial nucleus is $^{232}_{90}\text{Th}$. At the end there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus is $^{A}_{Z}X$ and Z are given by

(1) $A = 208$; $Z = 80$ (2) $A = 202$; $Z = 80$
(3) $A = 200$; $Z = 81$ (4) $A = 208$; $Z = 82$

Chapter 8
Semiconductor Devices and Digital Circuits

33. Ge and Si diodes start conducting at 0.3 V and 0.7 V respectively. In the following figure if Ge diode connection are reversed, the value of V_o changes by (assume that the Ge diode has large breakdown voltage)

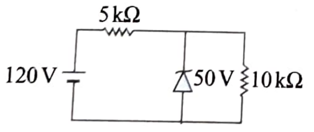


(1) 0.6 V (2) 0.8 V
(3) 0.4 V (4) 0.2 V

34. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19}m^{-3} and their mobility is 1.6 $\text{m}^2/(\text{V.s})$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to

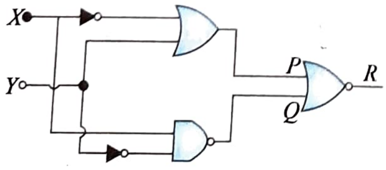
(1) 2 Ωm (2) 0.4 m
(3) 4 Ωm (4) 0.2 Ωm

35. For the circuit shown below, the current through the Zener diode is



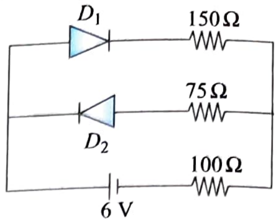
(1) 5 mA (2) Zero
(3) 14 mA (4) 9 mA

36. To get output '1' at R, for the given logic gate circuit the input values must be



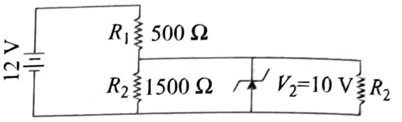
(1) $X = 0$, $Y = 1$ (2) $X = 1$, $Y = 1$
(3) $X = 0$, $Y = 0$ (4) $X = 1$, $Y = 0$

37. The circuit shown below contains two ideal diodes, each with a forward resistance of 50 Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in amperes) is



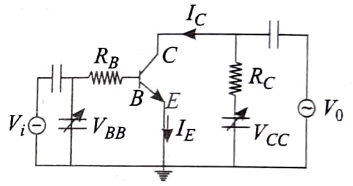
(1) 0.027 (2) 0.020
(3) 0.030 (4) 0.036

38. In the given circuit the current through Zener Diode is close to



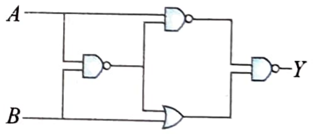
(1) 6.0 mA (2) 4.0 mA
(3) 6.7 mA (4) 0.0 mA

39. In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC} = 5$ V, $\beta_{dc} = 200$, $R_B = 100\Omega$, $R_C = 1\text{k}\Omega$ and $V_{BE} = 1.0$ V, The minimum base current and the input voltage at which the transistor will go to saturation, will be, respectively



(1) 20 μA and 3.5 V (2) 25 μA and 3.5 V
(3) 25 μA and 2.5 V (4) 20 μA and 2.8 V

40. The output of the given logic circuit is



(1) $\overline{A}B$ (2) $A\overline{B}$
(3) $AB + \overline{AB}$ (4) $A\overline{B} + \overline{A}B$

Chapter 9
Communication Systems

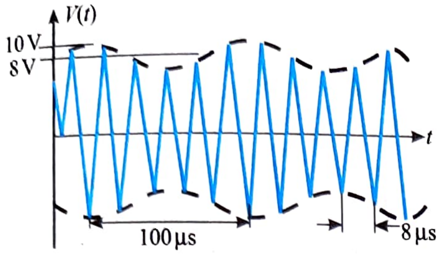
41. In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accomodated for transmitting TV signals of band width 6 MHz are (Take velocity of light $c = 3 \times 108$ m/s, $h = 6.6 \times 10^{-34}$ J-s)

(1) 3.75×10^6 (2) 4.87×10^5
(3) 3.86×10^6 (4) 6.25×10^5

42. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license, what broadcast frequency will you allot?

(1) 2750 kHz
(2) 2000 kHz
(3) 2250 kHz
(4) 2900 kHz

43. An amplitude modulated signal is plotted below:
Which one of the following best describes the above signal?



(1) $(9+\sin(2.5\pi\times10^5 t))\sin(\pi\times10^4 t)\,\text{V}$
(2) $(9+\sin(4\pi\times10^4 t))\sin(5\pi\times10^5 t)\,\text{V}$
(3) $(1+9\sin(2\pi\times10^4 t))\sin(2.5\pi\times10^5 t)\,\text{V}$
(4) $(9+\sin(2\pi\times10^4 t))\sin(2.5\pi\times10^5 t)\,\text{V}$

44. An amplitude modulated signal is given by $V(t) = 10[1 + 0.3\cos(2.2 \times 10^4)]\sin(5.5 \times 10^5 t)$. Here t is in seconds. The sideband frequencies (in kHz) are [Given $\pi = 22/7$]

(1) 1785 and 1715
(2) 892.5 and 857.5
(3) 89.25 and 85.75
(4) 178.5 and 171.5

45. To double the converging range of a TV transmission tower, its height should be multiplied by

(1) $\frac{1}{\sqrt{2}}$
(2) 4
(3) $\sqrt{2}$
(4) 2

46. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index?

(1) 0.6
(2) 0.5
(3) 0.3
(4) 0.4

Answers Key

1. (4) **2.** (4) **3.** (2) **4.** (1) **5.** (3)
6. (3) **7.** (1) **8.** (3) **9.** (4) **10.** (2)
11. (2) **12.** (4) **13.** (2) **14.** (3) **15.** (1)
16. (4) **17.** (1) **18.** (1) **19.** (4) **20.** (2)
21. (1) **22.** (1) **23.** (3) **24.** (2) **25.** (4)
26. (3) **27.** (4) **28.** (3) **29.** (1) **30.** (3)
31. (3) **32.** (4) **33.** (3) **34.** (2) **35.** (4)
36. (4) **37.** (2) **38.** (4) **39.** (2) **40.** (2)
41. (4) **42.** (2) **43.** (4) **44.** (3) **45.** (2)
46. (1)